Pragmatic Reasoning through Semantic Inference

Leon Bergen, Roger Levy, and Noah Goodman

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1 Introduction

Theories of natural language semantics aim to provide a simple account of how people interpret expressions in their language. Attempts to provide such an account face a basic challenge: the interpretation of expressions frequently varies with linguistic and social context. An obvious response to such contextual variation is to posit that natural language expressions are highly polysemous. A naive implementation of this idea will have at least two deficiencies: the theory will need to be extremely complex to accommodate all of the possible meanings of each expression; and it will miss the systematic relationship between an expression's context and its interpretation.

Gricean theories of pragmatics provide an elegant solution to these problems. They posit that the interpretation of an expression is not necessarily identical to its semantic content. Rather, this semantic content plays a specific role in the derivation of the expression's interpretation. In typical circumstances, speakers and listeners regard each other as rational agents who share the goal of communicating information to each other. A speaker chooses an utterance by reasoning about the beliefs that a listener would form if they interpreted utterances according to their semantic content; the speaker will be more likely to choose an utterance that is effective at communicating their intended meaning. The listener, in turn, interprets an utterance by reasoning about which intended meanings would have made the speaker most likely to choose this utterance. Gricean pragmatic accounts thus factor the interpretation of an expression into two parts: its semantic content, which determines its literal meaning, and cooperative social reasoning, which builds on this literal interpretation to determine the expression's inferred meaning. By factoring out the role of semantic content in this manner, Gricean pragmatic accounts reduce the explanatory burden of semantic theories. Many facts about an expression's interpretation will be determined by the communicative setting in which the expression is used, and not simply the expression's semantic content.

Despte the promise and apparently broad empirical coverage of these theories, attempts at formalizing them (e.g., Gazdar, 1979) have historically met with less success than formalization in other linguistic domains such as phonology, syntax, or semantics. Nevertheless, there is strong reason to believe that formal accounts of Gricean pragmatic reasoning has substantial potential scientific value. First, all Gricean theories assume that multiple factors—most famously Grice's quality, quantity, relevance, and manner—jointly guide the flexible relationship between literal semantic content and understood meaning, and in all Gricean theories these factors can potentially come into conflict (e.g., the opposition between Horn's (1984) Q and R principles). Our success at cooperative communication implies that a calculus of how different factors' influence is resolved in each communicative act is broadly shared within every speech community, yet extant theories generally leave this calculus unspecified and are thus unsatisfactory in predicting preferred utterance interpretation when multiple factors come into conflict. Mathematical formalization can provide such a calculus. Second, in the decades since Grice's original work there has been a persistent drive toward conceptual unification of Grice's original maxims into a smaller set of principles (e.g., Horn, 1984; Levinson, 2000; Sperber & Wilson, 1986). Mathematical formalization can help rigorously evaluate which such efforts are sound, and may reveal new possibilities for unification. Third, the appropriate mathematical formalization may bring pragmatics into much closer contact with empirical data, by making clear (often quantitative) and falsifiable predictions regarding communicative behavior in specific situations that may be brought under experimental control. This kind of payoff from formalization has been seen in recent years in related fields including psycholinguistics (R. L. Lewis & Vasishth, n.d.; Smith & Levy, 2013) and cognitive science (Tenenbaum, Kemp, Griffiths, & Goodman, 2011). Fourth, the development of pragmatic theory necessarily has a tight relationship with that of semantic theory. A precise, formalized pragmatic theory may contribute to advances in semantic theory by revealing the nature of the literal meanings that are exposed to Gricean inference and minimizing the possibility that promissory appeals to pragmatics may leave key issues in semantics unresolved.

The last several years have, in fact, seen a number of recent accounts which formalize Gricean pragmatic reasoning using game theory or related utility-based decision-theoretic frameworks that are beginning to realize this potential (Degen, Franke, & Jäger, 2013; Frank & Goodman, 2012; Franke, 2009; Franke & Jäger, 2013; Goodman & Stuhlmüller, 2013; Jäger, 2012; Parikh, 2000; Rothschild, 2013). These accounts find conceptual unification in grounding cooperative communicative behavior in simple principles of efficient information exchange by rational agents that can reason about each other. These accounts provide a precise specification of the reasoning that leads conversational partners to infer conversational implicatures either by using the notion of a game-theoretic equilibrium to define conditions that the agents' reasoning must meet or by providing a computational or procedural description of the reasoning itself. They characteristically provide formal proposals of the division between semantic content and pragmatic inference in which the semantic content of each linguistic expression is determined outside of the model, by a separate semantic theory. This semantic content serves as input to the pragmatics model, which in turn, specifies how agents use this semantic content, in addition to facts about their conversational setting, in order to infer enriched pragmatic interpretations of the expressions. Finally, by bringing in linking assumptions regarding the relationship between probabilistic beliefs and action from mathematical psychology, some of these models have been tested against empirical data far more rigorously than has been seen in previous work (Degen et al., 2013; Frank & Goodman, 2012; Goodman & Stuhlmüller, 2013).

This paper continues these efforts, using recursive probabilistic models to formalize Gricean explanations of a sequence of increasingly complex pragmatic phenomena. We will begin by providing an account, in line with previous game-theoretic models, of scalar implicatures and generalized class of these implicatures, which we refer to as *specificity implicatures*. We will also demonstrate how this *rational speech acts model* provides a solution to the symmetry problem for scalar implicatures.

We will next turn to M-implicatures, inferences that assign marked interpretations to complex expressions. We will show that the simple model of specificity implicatures does not derive M-implicatures, for reasons that are closely related to a well-known problem in game theory, the multiple equilibrium problem for signalling games. In order to derive even the simplest types of M-implicatures, we need to relax the traditional Gricean factorization of semantic content and pragmatic inference. In particular, the semantic content of expressions will not be determined in advance of pragmatic inference. Rather, the participants in a conversation will jointly infer this semantic content, as they are performing pragmatic reasoning.

Semantic inference plays an essential role in the derivation of M-implicatures. In order to represent the speaker and listener's inferences about the semantic content of their language's expressions, we will introduce *lexical uncertainty*, according to which the speaker and listener begin their pragmatic reasoning uncertain about how their their language's lexicon maps expressions to literal meanings. An important part of pragmatic reasoning thus involves resolving this semantic uncertainty, in addition to determining the non-literal content of the expressions. By extending the rational speech acts model with lexical uncertainty, we will be able to derive simple M-implicatures, in which complex expressions are assigned low probability interpretations. We will be able to derive a larger class of M-implicatures, in which complex utterances are assigned more generally marked interpretations, by relaxing the assumption that the speaker is knowledgeable.

Finally, we will consider a difficult class of embedded implicatures, which have not yet been derived within game-theoretic models of pragmatics. These implicatures cannot be derived by the rational speech acts model or the simple extension of this model with lexical uncertainty. In order to derive these implicatures, our model will need to be sensitive to the compositional structure of the expressions that it is interpreting. We will extend the model so that it respects the compositional structure of expressions, and represents uncertainty about the semantic content of genuine elements of the lexicon — i.e., atomic expressions — rather than whole expressions. When the model is extended in this manner, it will derive the embedded implicatures in question.

Though the determination of semantic content cannot be separated from pragmatic reasoning under our proposal — indeed, semantic inference will drive the derivation of the more interesting implicatures that we will consider — we will not have to abandon all of the explanatory advantages that factored Gricean accounts provide. Under our proposal, the explanatory burden of semantic theories will still be limited: they will need to account for approximately the same interpretive phenomena as they do under more traditional Gricean theories. As we will describe in more detail below, this is because the semantic content provided by semantic theories will still only play a limited functional role within our models. Our models primarily depart from traditional Gricean theories in their account of what role this semantic content will play.

2 The baseline rational speech-act theory of pragmatics

We begin by introducing the baseline rational speech-act theory of pragmatics (Frank & Goodman, 2012; Goodman & Stuhlmüller, 2013), built on a number of simple foundational assumptions about speakers and listeners in cooperative communicative contexts. We assume first a notion of COMMON KNOWLEDGE (Clark, 1996; D. Lewis, 1969; Stalnaker, 1978)—information known by both speaker and listener, with this shared knowledge jointly known by both speaker and listener, knowledge of the knowledge of shared knowledge jointly known by both speaker and listener, and so on *ad infinitum* (or at least as many levels of recursion up as necessary in the recursive pragmatic inference). Communication involves the transmission of knowledge which is not common knowledge: we assume that the speaker, by virtue of some observation that she has made, is in a particular belief state regarding the likely state of some conversationally relevant aspect of the world (or, more tersely, regarding the world). In engaging in a cooperative communicative act, the speaker and listener have the joint goal of bringing the listener's belief state as close as possible to that of the speaker, by means of the speaker formulating and sending a not-too-costly signal to the listener, who interprets it. The lexicon and grammar of the speaker and listener's language serve as resources by which literal content can be formulated. As pragmatically sophisticated agents, the speaker and the listener recursively model each other's expected production decisions and inferences in comprehension.

More formally, let O be the set of possible speaker observations, $\mathcal{W}$ the set of possible worlds, and $\mathcal{U}$ the set of possible utterances. Observations $o \in O$ and worlds $w \in \mathcal{W}$ have joint prior distribution $P(o,w)$, shared by listener and speaker. The literal meaning of each utterance $u \in \mathcal{U}$ is defined by a lexicon $\mathcal{L}$, which is a mapping from each possible utterance-world pair to the truth value of the utterance in that world. That is,

$$\mathcal{L}(u,w) = \begin{cases} 0 & \text{if } w \notin [\![u]\!] \\ 1 & \text{if } w \in [\![u]\!] \end{cases} \tag{1}$$

where $[\![u]\!]$ is the intension of u.[1] The first and simplest component of the model is the LITERAL LISTENER, who interprets speaker utterance u by conditioning on it being true and computing via Bayesian inference a belief state about speaker observation state o and world w. This updated distribution L_0 on w is defined by:

$$L_0(o,w|u,\mathcal{L}) \propto \mathcal{L}(u,w)P(o,w). \tag{2}$$

Social reasoning enters the model through a pair of recursive formulas that describe how the speaker and listener reason about each other at increasing levels of sophistication. We begin with the speaker, who plans a choice of utterance based on the EXPECTED UTILITY of each utterance, with utterances being high in utility insofar as they bring the listener's belief distribution about world and speaker observation close to that of the speaker, and low in utility insofar as they are costly to produce. Discrepancy in belief distribution is measured by the standard information theoretic quantity of KULLBACK-LEIBLER DIVERGENCE from the listener's posterior distribution $L(o,w|u)$ on observation and world given utterance, to the speaker's distribution given observation $P(o,w|o)$, which reduces to $P(w|o)$ and we denote more compactly as P_o. The Kullback-Leibler divergence from distribution Q to distribution P is in general defined as

$$D_{\text{KL}}(P||Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)}. \tag{3}$$

Therefore the discrepancy in speaker and listener belief distributions is quantified as

$$D_{\text{KL}}(P_o||L^u) = \sum_w P(w|o) \log \frac{P(w|o)}{L(o,w|u)}. \tag{4}$$

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[1] Note that this definition of the lexicon departs from standard usage, as it assigns meanings to whole utterances rather than atomic subexpressions. This is a provisional assumption which will be revised in Section 5.

where L^u is shorthand for $L(\cdot|u)$. We define the expected utility of utterance u for a recursion-level n speaker who has observed o as

$$U_n(u|o) = -D_{\mathrm{KL}}(P_o||L_{n-1}) - c(u) \tag{5}$$

where $c(u)$ is the cost of utterance u. Intuitively, utterances are costly insofar as they are time-consuming or effortful to produce; in this paper, we remain largely agnostic about precisely what determines utterance cost, assuming only that utterance cost is strictly monotonic in utterance lengths (as measured in words).

In the first part of this paper, we assume that for each world $w \in \mathcal{W}$, there is a unique observation $o \in O$ consistent with this world. In this special case, it is common knowledge that the speaker knows the true world w with probability 1, so that $P(w|o)$ is 1 for that world and 0 for all other worlds. This entails that we can ignore the world variable w in the speaker and listener equations, and the discrepancy in speaker and listener beliefs reduces to the negative log-probability or SURPRISAL of the observation for the listener given the utterance:

$$D_{\mathrm{KL}}(P_o||L^u) = \log \frac{1}{L(o|u)} = -\log L(o|u). \tag{6}$$

Under these conditions, (expected) utterance utility can be written as simply

$$U_n(u|o) = \log L_{n-1}(o|u) - c(u) \tag{7}$$

The assumption of speaker knowledgeability is relaxed in Section 4.5.

We are now ready to state the speaker's formula. The speaker's conditional distribution over utterances given the world w under consideration as the listener's possible interpretation is defined as

$$S_n(u|o) \propto e^{\lambda U_n(u|o)}, \tag{8}$$

where $\lambda > 0$. This specification of the speaker formula uses the SOFTMAX FUNCTION or LUCE-CHOICE RULE (Sutton & Barto, 1998) to map from a set of utterance utilities to a probability distribution over utterance choice. The INVERSE-TEMPERATURE parameter λ governs the speaker's degree of "greedy rationality". When $\lambda = 1$, the probability that the speaker chooses utterance u is proportional to the exponentiated utility of u. As λ increases, the speaker's distribution over utterance choices becomes increasingly more strongly peaked toward utterances with high exponentiated utility. The Luce-choice rule is used extensively in psychology and cognitive science as a model of human decision-making, and in reinforcement learning in order design algorithms that balance maximizing behavior that is optimal in the short-run and exploratory behavior that is beneficial in the long-run (Sutton & Barto, 1998).

Finally, we turn to the listener's recursive formula for interpreting utterances by reasoning about likely speaker choices. The listener's higher-order interpretations are simply defined as

$$L_n(o,w|u) \propto P(o,w) S_n(u|o). \tag{9}$$

That is, the listener uses Bayes' rule to reconcile their prior expectations about world state to be described with their model of the speaker. Equations (2), (5), (8), and (9) constitute the heart of this basic model. Note the relationship between recursion levels of the speaker and listener in Equations (5): the first speaker S_1 reasons about the literal listener L_0, the first pragmatic listener L_1 reasons about S_1, the second speaker S_2 reasons about the first pragmatic listener L_1, and so forth. The model we present here generalizes the rational speech-act model presented in Goodman and Stuhlmüller (2013) by adding utterance costs and the possibility of recursion beyond S_1.

2.1 Auxiliary assumptions: alternative sets, but no lexical scales

As in much previous work in pragmatics (Gazdar, 1979; Grice, 1975; Horn, 1984; Levinson, 2000), our models of pragmatic reasoning will rely heavily the set of alternative utterances available to the speaker. That is, in deriving the implicatures for an utterance, our models will reason about why the speaker did not use the other utterances available to them. We will not be providing a general theory of the alternative utterances that are reasoned about during the course of pragmatic inference. Rather, as is done in most other work in pragmatics, we will posit the relevant set of utterances on a case-by-case basis. As is discussed below, however, there are certain cases for which our models require fewer restrictions on the set of alternatives than most other models. These examples will provide suggestive — though not decisive — evidence that no categorical restrictions need to be placed on the alternatives set within our models, i.e. that every grammatical sentence in a language can be considered as an alternative during pragmatic reasoning. The mechanisms by which this may be made possible are discussed below.

Our models' treatment of lexical scales will represent a larger departure from the norm. By a "scale," we are referring to a totally ordered set of lexical items which vary along a single dimension; a typical example is the set of lexical items <"some", "most", "all">, where each item (when used in a sentence) is logically stronger than all of the items that fall below it on the scale. Such scales play an important role in many theories of pragmatic reasoning, where they constrain the set of alternative utterances available to the speaker. In such theories, it is assumed that the set of alternative utterances can be totally ordered along a relevant dimension (e.g. along the dimension of informativeness for ordinary scalar implicatures), so that this set forms a scale. Our models will not use scales in order to derive pragmatic inferences. In certain cases, the set of alternatives used by the model will include multiple utterances which are logically equivalent to each other. In other cases, the set of alternatives will include utterances which are jointly logically inconsistent. In general, the global constraints on the alternatives set which are described by scales will not be required by our models.

3 Specificity implicature in the baseline theory

To demonstrate the value of the baseline theory presented in Section 2, we show here how it accounts for a basic type of pragmatic inference: specificity implicatures, a generalization of scalar implicatures, in the case where it is common knowledge that the speaker knows the relevant world state. Specificity implicatures describe the inference that less specific utterances imply the negation of more specific utterances. For example, "Some of the students passed the test" is strictly less specific than "All of the students passed the test," and therefore the use of the first utterance implicates that not all of the students passed. This is of course an example of a scalar implicature, in that there is a canonical scale, ordered according to logical strength, which both "some" and "all" fall on.

Not all specificity implicatures are naturally described as scalar implicatures. For example, consider the utterance "The object that I saw is green" in a context in which there are two green objects, one of which is a ball and one of which has an unusual and hard-to-describe shape. In this context, the utterance will be interpreted as describing the strangely shaped object, because the speaker could have said "The object that I saw is a ball" to uniquely pick out the ball (see Frank &

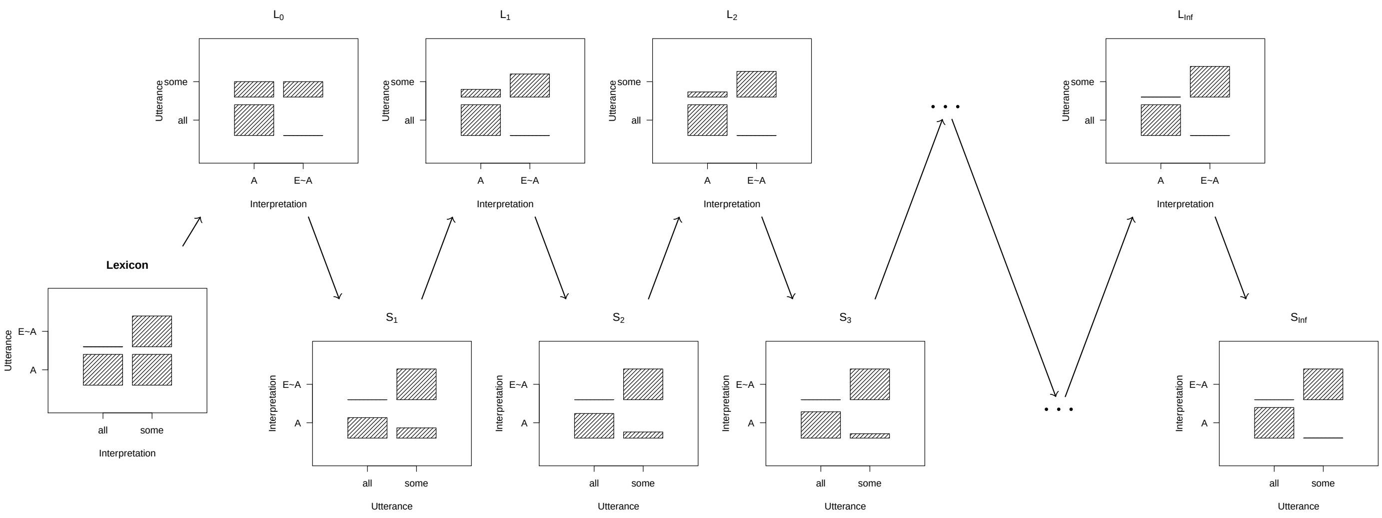

Figure 1: *Some* strengthening with $P(\forall) = \frac{1}{2}$, $P(\exists\neg\forall) = \frac{1}{2}$, $c(\text{all}) = c(\text{some}) = 0$, $\lambda = 1$

Goodman, 2012 for experimental evidence for these implicatures). That is, in this context, there is an available utterance which is more specific than “green”, and as a result “green” receives a specificity implicature which is the negation of the more specific utterance. It is important to note that neither “green” nor “ball” is strictly logically stronger than the other; it is only in a particular context that one can be strictly more descriptive than the other. Thus, these utterances do not fall on a scale which is ordered according to logical strength.[2]

In general, specificity implicatures will arise in contexts in which there is a pair of utterances such that one utterance is more contextually specific than the other. To a first approximation, an utterance “A” is more contextually specific than “B” when the contextually-salient meanings consistent with “A” are a subset of those consistent with “B.” The use of the less specific utterance “B” will result in the inference that “A” is false. It is this more general phenomenon that the model will be explaining.

3.1 Derivation of specificity implicatures

This model can be used to derive specificity implicatures as follows. A rational speaker will use as specific of an utterance as possible in order to communicate with the literal listener; a more specific utterance is more likely to be interpreted correctly by the literal listener. If the speaker does not use a specific utterance, then this is evidence that such an utterance would not have communicated her intended meaning. The listener L_1 knows this, and (given the assumption of speaker knowledgeability) infers that the speaker must know that the more specific utterance is false. Therefore, a less specific utterance implies the negation of a more specific utterance for this listener.

To illustrate this reasoning, we will consider the simplest possible example in which specificity

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[2] Though these utterances are logically incommensurable, it may still be possible to describe them as falling on an *ad-hoc* scale, as in Hirschberg (1985). While we will not be providing a direct argument against this analysis, our model obviates the need for a scalar representation in cases like this.

implicatures are possible. In this example, there are two utterances,

$$\mathcal{U} = \{\text{some, all}\},$$

and two meanings,

$$\mathcal{W} = \{\forall, \exists\neg\forall\},$$

where the intensions of the utterances are as usual:

$$\begin{aligned} [\![\text{some}]\!] &= \{\forall, \exists\neg\forall\}; \\ [\![\text{all}]\!] &= \{\forall\} \end{aligned}$$

Since it is common knowledge that the speaker knows the relevant world state, we can without loss of generality consider the observation and world variables to be equal, so that $o = w$, and drop w from the recursive equations (2)– (9). This allows the baseline model to be expressed as

$$\begin{aligned} L_0(o|u,\mathcal{L}) &\propto \mathcal{L}(u,o)P(o), & (10) \\ U_n(u|o) &= \log L_{n-1}(o|u) - c(u), & (11) \\ S_n(u|o) &\propto e^{\lambda U_n(u|o)}, & (12) \\ L_n(o|u) &\propto P(o)S_n(u|o), & (13) \end{aligned}$$

for integers $n > 0$. For illustration, we take the prior on observations as uniform—$P(\exists\neg\forall) = P(\forall) = \frac{1}{2}$—the cost $c(u)$ of both utterances as identical (the specific value has no effect, and we treat it here as zero), and the softmax parameter $\lambda = 1$.[3]

Figure 1 depicts the listener and speaker posteriors $L_n(\cdot|u)$ and $S_n(\cdot|o)$ at increasing levels of recursion n for these parameter values. The lexicon matrix depicts the mapping of each possible utterance–world pair to a 0/1 value; each speaker (respectively listener) matrix should be read as a conditional distribution of utterances given interpretations (respectively interpretations given utterances), with bar height proportional to conditional probability (hence each row in each speaker or listener matrix sums to probability mass 1):

	Listener n	
all	$L_n(\forall\mid\text{all})$	$L_n(\exists\neg\forall\mid\text{all})$
some	$L_n(\forall\mid\text{some})$	$L_n(\exists\neg\forall\mid\text{some})$
	$\forall$	$\exists\neg\forall$

	Speaker n	
$\forall$	$S_n(\text{all}\mid\forall)$	$S_n(\text{some}\mid\forall)$
$\exists\neg\forall$	$S_n(\text{all}\mid\exists\neg\forall)$	$S_n(\text{some}\mid\exists\neg\forall)$
	all	some

Crucially, while the literal listener interprets *some*, which rules out no worlds, entirely according to the prior (and hence as equiprobable as meaning $\forall$ and $\exists\neg\forall$), the speaker and listener both associate *some* increasingly strongly with $\exists\neg\forall$ as the pragmatic recursion depth increases.

One way to understand the fundamental reason for this behavior—the signature pattern of specificity implicature—is by considering the effect on one level of recursive inference on the listener's tendency to interpret *some* with unstrengthened meaning $\forall$. Let us denote $L_{n-1}(\forall|\text{some})$ by

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[3] Changes in the prior on observations, utterance costs, and the softmax parameter change the precise values of the speaker and listener posteriors at various levels of recursion, but do not change the signature specificity-implicature pattern that the model exhibits.

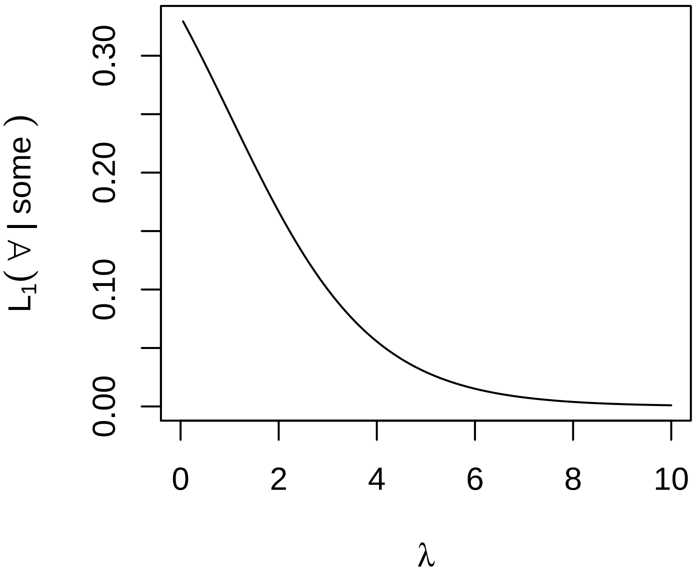

Figure 2: The degree of *some* strengthening as a function of the ”greedy rationality” parameter λ, with $\mathrm{P}(\forall) = \frac{1}{2}$, $\mathrm{P}(\exists\neg\forall) = \frac{1}{2}$, $c(\text{all}) = c(\text{some}) = 0$

the probability p. Further, note that lexical constraints on the literal listener mean that $L_n(\exists\neg\forall|\text{all}) = 0$ always. This means that we can write, following Equations (11)–(13):

$$
\begin{array}{lcc} \multicolumn{3}{c}{L_{n-1}} \\ \hline \text{all} & 1 & 0 \\ \text{some} & p & 1-p \\ & \forall & \exists\neg\forall \end{array}
\qquad
\begin{array}{lcc} \multicolumn{3}{c}{U_n} \\ \hline \forall & 0 & \log p \\ \exists\neg\forall & -\infty & \log(1-p) \\ & \text{all} & \text{some} \end{array}
\qquad
\begin{array}{lcc} \multicolumn{3}{c}{S_n} \\ \hline \forall & \frac{1}{1+p} & \frac{p}{1+p} \\ \exists\neg\forall & 0 & 1 \\ & \text{all} & \text{some} \end{array}
\qquad
\begin{array}{lcc} \multicolumn{3}{c}{L_n} \\ \hline \text{all} & 1 & 0 \\ \text{some} & \frac{p}{2p+1} & \frac{1+p}{2p+1} \\ & \forall & \exists\neg\forall \end{array}
$$

For all $p > 0$, the strict inequality $\frac{p}{2p+1} < p$ holds; therefore L_n is less inclined than L_{n-1} to interpret *some* as meaning $\forall$.

The above analysis assumed a uniform prior and $\lambda = 1$. The precise values of listener and speaker inferences are affected by these choices. A more exhaustive analysis of the behavior of this recursive reasoning system under a range of parameter settings is beyond the scope of the present paper, but the qualitative pattern of specificity implicature—that when pragmatic reasoning is formalized as recursive speaker–listener inference, more specific terms like *all* guide more general terms like *some* toward meanings not covered by the specific term—is highly general and robust to precise parameter settings. It is worth noting, however, that the value “greedy rationality” parameter λ affects the strength of the implicature when recursion depth is held constant. Figure 2 shows the tendency of the first pragmatic listener L_1 to interpret *some* as meaning $\forall$ (recall that for the literal listener, $L_0(\forall|\text{some}) = L_0(\exists\neg\forall|\text{some}) = \frac{1}{2}$ when the prior is uniform). This dependence on λ is due to L_1 modeling the first speaker S_1's degree of “greedy rationality”. As greedy rationality increases, the strength of specificity implicature increases, to the extent that the possibility of $\forall$ interpretation for *some* can all but disappear after just one round of iteration with sufficiently high λ.

3.2 The symmetry problem

In addition to explaining specificity implicatures, the model provides a straightforward solution to the symmetry problem for scalar implicatures. As previously noted, on the standard account of scalar implicatures, implicatures are computed with reference to a scale; lower utterances on the scale imply the negation of higher utterances on the scale. For example, the implicature for "some" is computed using the scale <"some", "all">, so that "some" implies the negation of "all." The symmetry problem describes a problem with constructing the scales for the implicature computations: there are multiple consistent ways of constructing the scales, and different scales will give rise to different implicatures. The only formal requirement on a scale is that items higher on it be logically stronger than those lower on it. A possible scale for "some" is therefore <"some", "some but not all">. If this scale is used, "some" will imply that "some but not all" is not true, i.e. that "all" is true.

Fox and Katzir (2011) break the symmetry between "all" and "some but not all" by providing a theory of the alternative utterances which are considered during the computation of scalar implicatures. This theory posits that the set of scalar alternatives is computed via a set of combinatorial operations. That is, only the utterances which are constructed through these operations will be placed on the scale. The definition of these operations ensures that for each utterance on a scale, the set of utterances higher on the scale are consistent with each other. As a result, a consistent set of implicatures will be computed for each utterance.

The rational speech act model provides a different solution to this problem, which places weaker requirements on the set of alternative utterances. For the previous example, the model can include both "all" and "some but not all" as alternatives, and still derive the correct implicatures. It does so by assigning higher cost to "some but not all" than to "all." Because "some but not all" is assigned a higher cost, it is less likely to be used to communicate *not all* than "all" is to communicate *all*. Thus, when the listener hears the utterance "some," they will reason that the speaker was likely to have intended to communicate *not all*: if the speaker had intended to communicate *all*, they would have used the utterance "all," but if they had intended to communicate *not all*, they would have been less likely to use "not all."

In general, this approach allows arbitrary sets of grammatical utterances to be considered as alternatives, without resulting in contradictory inferences, and while still preserving attested implicatures. The model will do this by assigning more complex utterances higher cost, and as a result weighing these more costly utterances less during pragmatic inference. Utterances that are more costly to the speaker are less likely to be used, because the speaker is rational. As an utterance becomes more and more costly, it becomes less and less salient to the speaker and listener as an alternative, and has less and less of an effect on the interpretation of other utterances.

4 Lexical uncertainty

We will next consider a different type of pragmatic inference: M-implicatures. An M-implicature arises when there are two semantically equivalent utterances that differ in complexity. In general, the more complex utterance will receive a marked interpretation. The most straightforward way for an interpretation to be marked is for it to have low probability. Consider, for example, the following two sentences:

(i) John can finish the homework.

(ii) John has the ability to finish the homework.

These two sentences (plausibly) have the same literal semantic content, but they will typically not be interpreted identically. The latter sentence will usually be interpreted to mean that John will not finish the homework, while the former example does not have this implicature. Horn (1984) and Levinson (2000) cite a number of other linguistic examples which suggest that the assignment of marked interpretations to complex utterances is a pervasive phenomenon, in cases where there exist simpler, semantically equivalent alternatives.

Though M-implicatures describe a linguisic phenomenon, the reasoning that generates these implicatures applies equally to ad-hoc communication games with no linguistic component. Consider a one-shot speaker-listener signaling game with two utterances, SHORT and LONG (the costs of these utterances reflect their names), and two meanings, FREQ and RARE; nothing distinguishes the utterances other than their cost, and neither is assigned a meaning prior to the start of the game (so that effectively both have the all-*true* meaning). The speaker in this game needs to communicate one of the meanings; which meaning the speaker needs to communicate is sampled according to the prior distribution on these meanings (with the meaning FREQ having higher prior probability). The listener in turn needs to recover the speaker's intended meaning from their utterance. The speaker and listener will communicate most efficiently in this game if the speaker uses LONG in order to communicate the meaning RARE, and SHORT in order to communicate FREQ, and the listener interprets the speaker accordingly. That is, if the speaker and listener coordinate on this communication system, then the speaker will successfully transmit their intended meaning to the listener, and the expected cost to the speaker will be minimized. Bergen, Goodman, and Levy (2012) find that in one-shot communication games of this sort, people do in fact communicate efficiently, suggesting that the pragmatic knowledge underlying M-implicatures is quite general and not limited to specific linguistic examples.[4]

Perhaps surprisingly, our baseline rational speech-act model of Sections 2–3 is unable to account for speakers' and listeners' solution to the one-shot M-implicature problem. The behavior of the baseline model is shown in Figure 3; the model's qualitative failure is totally general across different settings of prior probabilities, utterance costs, and λ. The literal listener L_0 interprets both utterances identically, following the prior probabilities of the meanings. Crucially, L_0's interpretation distribution provide no information that speaker S_1 can leverage to associate either utterance with any specific meaning; the only thing distinguishing the utterances' expected utility is their cost. This leads to an across-the-board dispreference on the part of S_1 for LONG, but gives no starting point for more sophisticated listeners or speakers to break the symmetry between these utterances.

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[4] The communication game considered in that paper differs slightly from the one considered here. In the experiments performed in that paper, there were three utterances available to the speaker, one of which was expensive, one of intermediate cost, and one cheap, and three possible meanings, one of which was most likely, one of intermediate probability, and one which was least likely. Participants in the experiment coordinated on the efficient mapping of utterances to meanings, i.e. the expensive utterance was mapped to the least likely meaning, and so on.

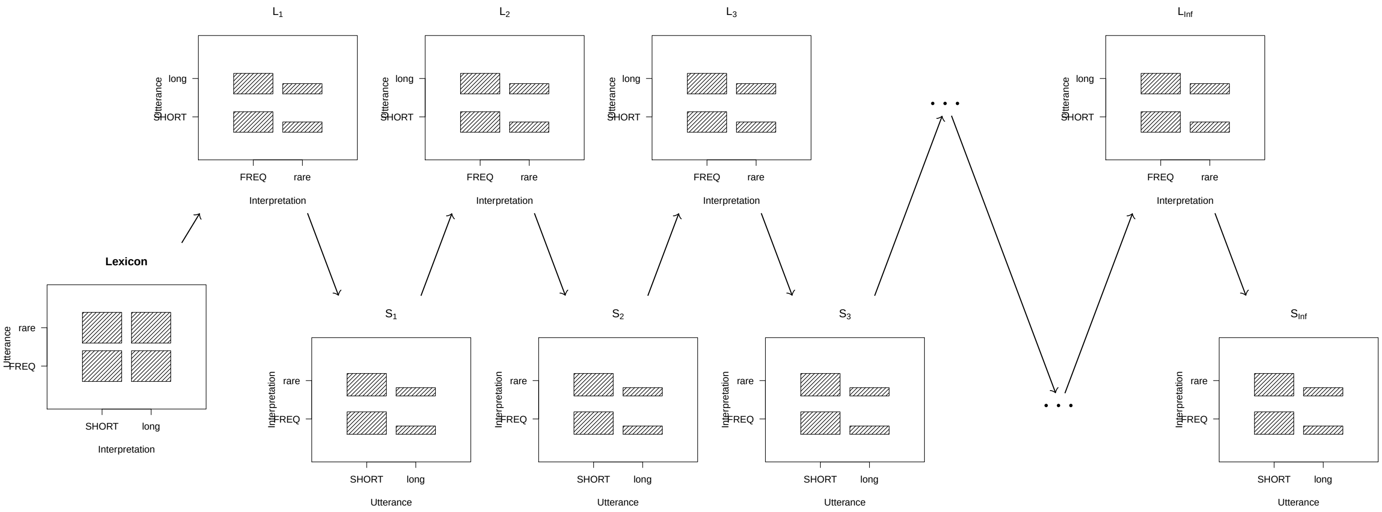


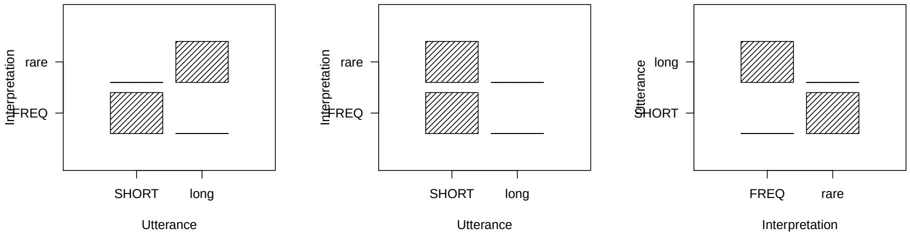
Figure 3: The failure of the basic model to derive M-implicature (illustrated here for $P(\text{FREQ}) = \frac{2}{3}$, $P(\text{rare}) = \frac{1}{3}$, $\lambda = 3$, $c(\text{SHORT}) = 1$, $c(\text{long}) = 2$)

4.1 The multiple equilibrium problem

Our baseline model's failure for M-implicature is in fact closely related to a more general problem from game theory, the multiple equilibrium problem for signalling games (Cho & Kreps, 1987; Rabin, 1990). In a typical signalling game, a subset of the agents in the game each receive a type, where this type is revealed only to the agent receiving it; in the settings being considered in this paper, each speaker has a type, which is the meaning that they want to communicate. The goal of the listener is to correctly guess the type of the speaker based on the signal that they send.

To describe the multiple equilibrium problem for such games, we first need to introduce the relevant notion of equilibrium. Loosely speaking, the equilibria for a game describe the self-consistent ways that the game can be played. The simplest equilibrium concept in game theory is the Nash equilibrium (Fudenberg & Tirole, 1991; Myerson, 2013; Nash et al., 1950). For games with two agents A and B, a pair of strategies (σ_A, σ_B), which describe how each agent will play the game, are a Nash equilibrium if neither agent would benefit by unilaterally changing their strategy; that is, the strategies are an equilibrium if, fixing σ_B, there is no strategy for A that would improve the outcome of the game for A, and vice-versa.

The relevant notion of equilibrium for signalling games is the Bayesian Nash equilibrium (Harsanyi, 1967), which in addition to the requirements imposed by the definition of the Nash equilibrium also imposes consistency constraints on the beliefs of the agents. In particular, given the prior distribution over types, and the agents' strategies (which define the likelihood of taking actions given a player type), the agents must use Bayes' rule to compute their posterior distribution over types after observing an action. Each agent's strategy must also be rational given their beliefs at the time that they take the action, in the sense that the strategy must maximize their expected utility. The multiple equilibrium problem arises in a signalling game when the game has multiple Bayesian Nash equilibria. This occurs when the agents can devise multiple self-consistent communication systems given the constraints of the game. That is, given the assumption that the other agents are using the communication system, it will not be rational for one agent to unilaterally start



(a) Separating Pareto-optimal (b) Pooling 1 (c) Separating Pareto-suboptimal

Figure 4: Multiple equilibria (speaker matrices) for the M-implicature signaling game

using a different communication system.

The multiple equilibrium problem can be illustrated concretely using the game above. This game has two general classes of equilibria, illustrated in Figure 4. In the first class, which are called the *separating equilibria*, successful communication occurs between the speaker and listener, but their communication system may be suboptimal from an information-theoretic perspective. In the first such equilibrium, the speaker chooses LONG when they want to communicate RARE, and SHORT when they want to communicate FREQ (Figure 4a). Given these strategies, the listener knows how to interpret each utterance: LONG will be interpreted as RARE— conditional on hearing LONG, the only possibility is that it was produced by the agent wanting to communicate RARE— and similarly SHORT will be interpreted as FREQ. This is clearly an equilibrium, because neither speaker will successfully communicate their intended meaning if they unilaterally change their strategy; for example, if the speaker wanting to communicate RARE switches to using SHORT, then they will be interpreted as intending FREQ. A second separating equilibrium is also possible in this game. Under this equilibrium, the speaker-utterance pairs are reversed, so that the agent intending to communicate RARE uses SHORT, and the agent intending FREQ uses LONG (Figure 4c). This is inefficient — in expectation, it will be more expensive than the previous equilibrium for the speaker — but it is nonetheless an equilibrium, because neither speaker can unilaterally change strategies without failing to communicate.

The second type of equilibrium in this game, known as a *pooling equilibrium*, is still more deficient than the inefficient separating equilibrium, and it is the one that is most closely related to the problem for our initial model of pragmatic inference. In one pooling equilibrium, the speaker chooses the utterance SHORT, independent of the meaning that they want to communicate (Figure 4b). Because the speakers always choose SHORT, this utterance communicates no information about the speaker's intended meaning, and the listener interprets this utterance according to the prior distribution on meanings. Assuming that the utterance LONG is also interpreted according to the prior, it will never be rational for the speaker to choose this utterance. .[5] Thus this is indeed an equilibrium.

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[5] Note that because in this equilibrium the speaker never uses one of the two utterances, the listener cannot interpret the never-used utterance by Bayesian conditioning, because it is not possible to condition on a probability 0 event. As a result, standard game-theoretic models need to separately specify the interpretation of probability 0 signals. We will return to this issue below.

These arguments demonstrate that under the standard game-theoretic signalling model, speakers and listeners are not guaranteed to arrive at the efficient communication equilibrium. Rather, there is the possibility that they will successfully communicate but do so inefficiently, with cheaper utterances interpreted as referring to less likely meanings. There is also the possibility that they will fail to communicate at all, in the case that all speakers choose the cheapest available utterance. However, M-implicatures demonstrate that at least in certain cases, people are able to systematically coordinate on the efficient strategies for communication, even when semantics provides no guide for breaking the symmetries between utterances. Thus, there is something to account for in people's strategic and pragmatic reasoning beyond what is represented in standard game-theoretic models or in our initial model of pragmatic reasoning.

In recent work in linguistics, there have generally been three approaches to accounting for these reasoning abilities. The first approach uses the notion of a *focal point* for equilibria (Parikh, 2000). On this approach, people select the efficient equilibrium in signalling games because it is especially salient; the fact that it is salient makes each agent expect other agents to play it, which in turn makes each agent more likely to play it themselves. While this approach does derive the efficient equilibrium for communication games, it is not entirely satisfactory, since it does not provide an independent account of salience in these games — precisely the feature which allows the agents to efficiently communicate under this approach.

An alternative approach has been to derive the efficient equilibrium using evolutionary game theory, as in De Jaegher (2008); Van Rooy (2004). These models show that given an appropriate evolutionary dynamics, inefficient communication systems will evolve towards more efficient systems among collections of agents. While these models may demonstrate how efficient semantic conventions can evolve among agents, they do not demonstrate how agents can efficiently communicate in one-shot games. Indeed, in the relevant setting for M-implicatures, the agents begin with an inefficient communication system — one in which the semantics of their utterances does not distinguish between the meanings of interest — and must successfully communicate within a single round of play. There is no room for selection pressures to apply in this setting.

Finally, Franke (2009), Jäger (2012), and Franke and Jäger (2013) have derived M-implicatures in the Iterated Best Response (IBR) and Iterated Quantal Response (IQR) models of communication, which are closely related to the rational speech act model considered in the previous section. The naive versions of these models do not derive M-implicatures, for reasons that are nearly identical to why the rational speech act model fails to derive them. In the IBR model, players choose strategies in a perfectly optimal manner. Because the expensive utterance in the Horn game is strictly worse than the cheap utterance — it is more expensive and has identical semantic content – an optimal speaker will never use it. As a result, in the naive IBR model, the speaker chooses the expensive utterance with probability 0, and no coherent inference can be drawn by the listener if they hear this utterance; interpreting this utterance would require them to condition on a probability 0 event. Franke (2009) and Jäger (2012) show how to eliminate this problem in the IBR model and correctly derive M-implicatures. They propose a constraint on how listeners interpret probability 0 utterances, and show that this constraint results in the efficient equilibrium. This proposal cannot be extended to the rational speech acts model, because it relies on the expensive utterance being used with probability 0; in the rational speech acts model, agents are only approximately rational, and as a result, every utterance is used with positive probability.

As in the rational speech acts model, agents are only approximately rational in the IQR model, and the IBR derivation of M-implicatures similarly does not extend to this model. Franke and Jäger

(2013) therefore provide an alternative extension of the IQR model which derives M-implicatures. Under this proposal, agents who receive low utility from all of their available actions engage in more exploratory behavior. In a Horn game, the speaker who wants to communicate the meaning RARE starts out with a low expected utility from all of their actions: no matter which utterance they choose, the listener is unlikely to interpret them correctly. As a result, this speaker will engage in more exploratory behavior — i.e., behave less optimally with respect to their communicative goal — and will be more likely to choose the suboptimal expensive utterance. This is sufficient to break the symmetry between the cheap and expensive utterances, and derive the M-implicature.

Unlike the proposed modification of the IBR model, Franke and Jäger (2013)'s proposed derivation of M-implicatures within the IQR model would extend straightforwardly to the rational speech acts model. We will nonetheless be proposing an alternative extension to the rational speech acts model. This is for several reasons. First, the derivation within the IQR model depends on the empirical assumption that agents with worse alternatives available to them will choose among these alternatives less optimally than agents with better alternatives available. Though this is a reasonable assumption, it may turn out to be empirically false; to our knowledge, it has not been experimentally evaluated. Second, the derivation of M-implicatures which we present can be extended to explain a number of other phenomena, which will be discussed in later sections. These explanations will hinge on features which are distinctive to our proposed extension of the rational speech acts model.

4.2 The lexical-uncertainty model

In the previous version of the model, it was assumed that the lexicon $\mathcal{L}$ used by the speaker and listener was fixed. For every utterance u, there was a single lexical entry $\mathcal{L}_u$ that gave the truth function for u. This fixed lexicon determined how the literal listener would interpret each utterance.

In the current version of the model, we introduce *lexical uncertainty*, so that the fixed lexicon is replaced by a set of lexica Λ over which a there is a probability distribution $P(\mathcal{L})$. This distribution represents sophisticated listeners' and speakers' uncertainty about how the literal listener will interpret utterances.

Introducing lexical uncertainty generalizes the previous model; the base listener L_0 remains unchanged from equation 2, i.e. the literal listener is defined by:

$$L_0(o,w|u,\mathcal{L}) \propto \mathcal{L}(u,w)P(o,w) \tag{14}$$

for every lexicon $\mathcal{L} \in \Lambda$. The more sophisticated speakers and listeners, S_n and L_n for $n \geq 1$, are defined by:

$$U_1(u|o,\mathcal{L}) = -D_{KL}(P_o||L_0^{u,\mathcal{L}}) - c(u), \tag{15}$$

where $L_k^{u,\mathcal{L}}$ is the level-k listener's posterior distribution on o and w conditional on utterance u and lexicon $\mathcal{L}$,

$$S_1(u|o,\mathcal{L}) \propto e^{\lambda U_1(u|o,\mathcal{L})}, \tag{16}$$

$$L_1(o,w|u) \propto P(o,w) \sum_{\mathcal{L}\in\Lambda} P(\mathcal{L}) S_1(u|o,\mathcal{L}), \tag{17}$$

$$U_n(u|o) = -D_{KL}(P_o||L_{n-1}^u) - c(u) \qquad \text{for } n > 1, \tag{18}$$

$$S_n(u|o) \propto e^{\lambda U_n(u|o)} \qquad \text{for } n > 1, \tag{19}$$

$$L_n(o,w|u) \propto P(o,w) S_n(u|o) \qquad \text{for } n > 1.^6 \tag{20}$$

The set of lexica Λ must satisfy several desiderata:

I. Every lexicon must be an ENRICHMENT of the language's semantics. Formally, if $\mathcal{L}_S$ represents the base SEMANTIC LEXICON for the language (i.e. the lexicon that maps each utterance to its truth function under the language's semantics), then every $\mathcal{L} \in \Lambda$ must be such that for all utterances u and all world states w, $\mathcal{L}(u,w) = 1$ only if $\mathcal{L}_S(u,w) = 1$.

II. Each utterance must receive a non-contradictory interpretation in every $\mathcal{L}$. Formally, for each utterance u and each lexicon $\mathcal{L} \in \Lambda$ there must exist a world w such that $\mathcal{L}(u,w) = 1$.

III. Every lexicon $\mathcal{L} \in \Lambda$ must contain all utterances in the base semantic lexicon, plus a distinguished utterance u_{null} such that $\mathcal{L}(u_{null}, w) = 1$ for every world w (and not subject to Criterion I). That is, u_{null} must receive a trivial interpretation under every lexicon. Informally, the speaker can decide to stay silent rather than say anything.

We further require that u_{null} be the most expensive utterance available.[7] This corresponds to the

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[6] It is possible to define the lexical-uncertainty model more concisely by replacing Equations (15)–(20) with the following three equations:

$$U_n(u|o,w,\mathcal{L}) = -D_{KL}(P_o||L_{n-1}^{u,\mathcal{L}}) - c(u). \tag{i}$$

$$S_n(u|o,w,\mathcal{L}) \propto e^{\lambda U_n(u|o,w,\mathcal{L})}, \tag{ii}$$

$$L_n(o,w|u,\mathcal{L}) \propto \sum_{\mathcal{L}'\in\Lambda} P(o,w) P(\mathcal{L}') S_n(u|o,w,\mathcal{L}'), \tag{iii}$$

Once the first marginalization over lexica occurs at the L_1 level, higher-level speaker and listener distributions lose their dependence on the lexicon $\mathcal{L}$ being conditioned on, since there is no dependence on $\mathcal{L}$ in the right-hand side of equation (iii). In this paper we rely on the less concise definitions provided in the main text, however, on the belief that they are easier to follow than those in Equations (i)–(iii).

[7] The utterance u_{null} may be arbitrarily expensive, so that the speaker is arbitrarily unlikely to use it. We require it for the model in order to ensure that there is always an utterance which includes the speaker's intended meaning in its support. If such an utterance does not exist, then with probability 1 the speaker will be unable to communicate their intended meaning, and the speaker's distribution over utterances will be undefined.

We have considered two other approaches to solving this problem, both of which result in qualitatively similar predictions for all of the models considered in this paper. In the first of these approaches, a global constraint is imposed on each lexicon $\mathcal{L} \in \Lambda$: for each world state w, there must exist an utterance u such that $\mathcal{L}(u,w) = 1$. Given this condition, there will always exist an utterance which allows the speaker to communicate their intended meaning with positive probability. In the second of these approaches, the truth-conditional semantics of each utterance is slightly weakened. When an utterance u is false at a world state w, we define $\mathcal{L}(u,w) = 10^{-6}$ (or any smaller, positive number), and when it is true, we define $\mathcal{L}(u,w) = 1 - 10^{-6}$. In this case, each utterance always assigns at least a small amount of mass to each world state, so that the well-definedness of the speaker's distribution is no longer an issue.

cost that the speaker must incur to successfully communicate to the listener their intent to remain silent; from the listener's perspective, the speaker may be intending many other things at first — they may be formulating their next utterances, or clearing their throats — and this ambiguity will only be resolved after a substantial amount of time has passed. In the models we consider in the remainder of this paper, u_{null} never becomes a preferred speaker choice due to its high cost, though it is possible that for other problems u_{null} may turn out to be an effective communicative act. We leave the question of whether this is a desirable feature of our model for future work.

4.3 Specificity implicature under lexical uncertainty

Before demonstrating how the lexical-uncertainty model derives M-implicature (which we do in Section 4.4), in this section we walk the reader through the operation of the lexical-uncertainty model for a simpler problem: the original problem of specificity implicature, which the revised lexical-uncertainty model also solves. The setup of the problem remains the same, with (equal-cost) utterance set $\mathcal{U} = \{\text{some}, \text{all}\}$, meanings $\mathcal{W} = \{\forall, \exists\neg\forall\}$, and literal utterance meanings—semantic lexicon $\mathcal{L}_S$ in the terminology of Section 4.2—$[\![\text{some}]\!] = \{\forall, \exists\neg\forall\}$, $[\![\text{all}]\!] = \{\forall\}$. To complete the specification of the lexical-uncertainty model of specificity implicature, we must simply state the set of lexica Λ, and the distribution $P(\mathcal{L})$ over the set. We make the minimal assumption that Λ is the set of all possible lexica admissible under Criteria I–III:

$$\mathcal{L}_1 = \left\{ \begin{array}{ll} [\![\text{all}]\!] & = \{\forall\} \\ [\![\text{some}]\!] & = \{\exists\neg\forall, \forall\} \\ [\![u_{null}]\!] & = \{\exists\neg\forall, \forall\} \end{array} \right\} \quad \mathcal{L}_2 = \left\{ \begin{array}{ll} [\![\text{all}]\!] & = \{\forall\} \\ [\![\text{some}]\!] & = \{\exists\neg\forall\} \\ [\![u_{null}]\!] & = \{\exists\neg\forall, \forall\} \end{array} \right\} \quad \mathcal{L}_3 = \left\{ \begin{array}{ll} [\![\text{all}]\!] & = \{\forall\} \\ [\![\text{some}]\!] & = \{\forall\} \\ [\![u_{null}]\!] & = \{\exists\neg\forall, \forall\} \end{array} \right\}$$

and a uniform distribution over Λ: $P(\mathcal{L}_1) = P(\mathcal{L}_2) = P(\mathcal{L}_3) = \frac{1}{3}$. Note that *some* can be enriched to either $\exists\neg\forall$ or to $\forall$, and before pragmatic inference gets involved there is no preference among either those two or an unenriched meaning.

We can now compute the behavior of the model. Since it is common knowledge that the speaker knows the relevant world state, we can once again let $o = w$ and drop w from the recursive equations, so that the lexical-uncertainty model of Equations (14)–(20) can be expressed as

$$L_0(o|u,\mathcal{L}) \propto \mathcal{L}(u,o)P(o), \tag{21}$$
$$U_1(u|o,\mathcal{L}) = \log L_0(o|u,\mathcal{L}) - c(u), \tag{22}$$
$$S_1(u|o,\mathcal{L}) \propto e^{\lambda U_1(u|o,\mathcal{L})}, \tag{23}$$
$$L_1(o|u) \propto P(o) \sum_{\mathcal{L}\in\Lambda} P(\mathcal{L}) S_1(u|o,\mathcal{L}), \tag{24}$$
$$U_n(u|o) = \log L_{n-1}(o|u) - c(u) \qquad \text{for } n > 1, \tag{25}$$
$$S_n(u|o) \propto e^{\lambda U_n(u|o)} \qquad \text{for } n > 1, \tag{26}$$
$$L_n(o|u) \propto P(o) S_n(u|) \qquad \text{for } n > 1. \tag{27}$$

Figure 5 shows the listener and speaker posterior distributions at varying levels of recursion. At the L_0 literal-listener and S_1 first-speaker levels, different inferences are drawn conditional on the lexicon entertained: the three lexica $\mathcal{L}_1$ through $\mathcal{L}_3$ are stacked top to bottom in the leftmost panel, and the dependencies among lexicon-specific inferences are indicated with arrows between panels.

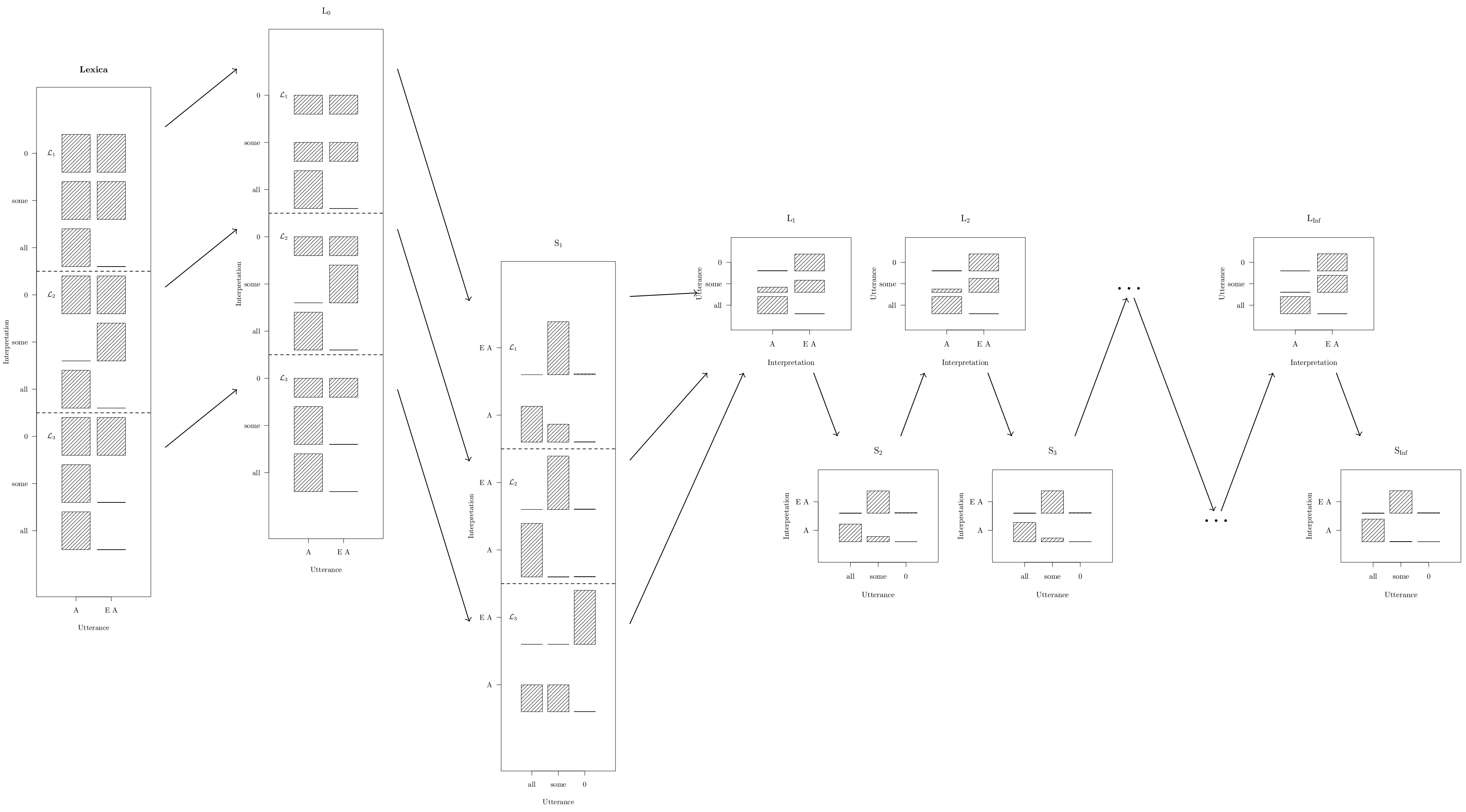

Figure 5: Specificity implicatures under lexical uncertainty, shown here with $\mathrm{P}(\forall) = \frac{1}{2}$, $\mathrm{P}(\exists\neg\forall) = \frac{1}{2}$, $c(\text{all}) = c(\text{some}) = 1$, $c(\emptyset) = 5$, $\lambda = 1$

Up through S_1, each lexicon-specific recursive inference chain operates indistinguishably from that of the baseline model, except that an enriched lexicon rather than the base semantic lexicon of the language is used throughout.

The specificity implicature first appears at the level of the listener L_1, who is reasoning about the speaker S_1. The listener computes their posterior distribution over the speaker's intended meaning by marginalizing over the possible lexica that the speaker may have been using (Equation (24)). As can be seen in the second column of the third panel of Figure 5, S_1's posterior supports three different possible interpretations of *some*. Under the lexicon in which *some* has been enriched to mean $\forall$ (bottom subpanel), *some* should be interpreted to categorically mean $\forall$; under the lexicon in which *some* has been enriched to mean $\exists\neg\forall$ (middle subpanel), *some* should be interpreted to categorically mean $\exists\neg\forall$. Under the lexicon in which *some* remains unenriched, *some* should be preferentially interpreted as $\exists\neg\forall$ due to blocking of $\forall$ by *all*, exactly as in the baseline model. Thus in the final mixture of lexica determining the overall interpretive preferences of L_1, there is an overall preference of *some* to be interpreted as $\exists\neg\forall$; this preference can get further strengthened through additional speaker–listener iterations, exactly as in the baseline model. Thus specificity implicatures still go through under lexical uncertainty.

4.4 Derivation of M-implicature under lexical uncertainty

We now show how lexical uncertainty allows derivation of one-shot M-implicature. We consider the simplest possible M-implicature problem of two possible meanings to be communicated—one higher in prior probability (FREQ) than the other (RARE)—that could potentially be signaled by two uttearnces—one less costly (SHORT) than the other (LONG). The semantic lexicon of the language is completely unconstrained:

$$\mathcal{L}_S = \left\{ \begin{array}{ll} [\![\text{SHORT}]\!] & = \{\text{FREQ}, \text{RARE}\} \\ [\![\text{LONG}]\!] & = \{\text{FREQ}, \text{RARE}\} \end{array} \right\}$$

Each utterance has three possible enrichments—$\{\text{FREQ}, \text{RARE}\}$, $\{\text{FREQ}\}$, and $\{\text{RARE}\}$—leading to nine logically possible enriched lexica. We make the minimal assumption of taking Λ to be this complete set of nine, illustrated in the first panel of Figure 6, and putting a uniform distribution over Λ.

Because utterance costs play no role in the literal listener's inferences, L_0 is completely symmetric in the behavior of the two utterances (second panel of Figure 6. However, the variety in lexica gives speaker S_1 resources with which to plan utterance use efficiently. The key lexica in question are the four in which the meaning of only one of the two utterances is enriched: $\mathcal{L}_2$, $\mathcal{L}_3$, $\mathcal{L}_4$, and $\mathcal{L}_7$. $\mathcal{L}_2$ and $\mathcal{L}_7$ offer the speaker the partial associations LONG–RARE and SHORT–FREQ, respectively, whereas $\mathcal{L}_3$ and $\mathcal{L}_4$ offer the opposite: LONG–FREQ and SHORT–RARE, respectively. Crucially, the former pair of associations allows greater expected speaker utility, and thus undergo a stronger specificity implicature in S_1, than the latter pair of associations.

This can be seen most clearly in the contrast between $\mathcal{L}_2$ and $\mathcal{L}_3$. The speaker S_1 forms a stronger association of LONG to RARE in $\mathcal{L}_2$ than of LONG to FREQ in $\mathcal{L}_3$. This asymmetry arises because the value of precision varies with communicative intention. A speaker using $\mathcal{L}_2$ can communicate RARE precisely by using LONG, and will not be able to effectively communicate this meaning by using the vague utterance SHORT. Thus, this speaker will be relatively likely to use

LONG to communicate RARE. In contrast, LONG will communicate FREQ precisely under $\mathcal{L}_3$, but this meaning can also be communicated effectively with the utterance SHORT. Thus, the speaker using $\mathcal{L}_3$ will be less likely to choose LONG.

When the first pragmatic listener L_1 takes into account the variety of S_1 behavior across possible lexica (through the marginalization in Equation (24)), the result is a weak but crucial LONG–RARE association. Further levels of listener–speaker recursion amplify this association toward increasing categoricality. (The parameter settings in Figure 6 are chosen to make the association at the L_1 level relatively visible, but the same qualitative behavior is robust for all finite $\lambda > 1$.) Simply by introducing consideration of multiple possible enrichments of the literal semantic lexicon of the language, lexical uncertainty allows listeners and speakers to converge toward the M-implicature equilibrium that is seen not only pervasively in natural language but also in one-shot rounds of simple signaling games.

4.5 Ignorance as a marked state

The lexical-uncertainty model introduced earlier in this section provided a novel means by which speakers and listeners in one-shot communication games align forms and meanings in terms of what can be thought of as two different types of *markedness*: cost of forms and prior probabilities, or frequencies, of meanings. Perhaps remarkably, a third type of markedness emerges as a side effect of this model that can explain a particularly vexing class instance of implicature, most famously exemplified by the sentence pair below:

(i) Some or all of the students passed the test.

(ii) Some of the students passed the test.

As discussed in Section 3, (ii) has a specificity implicature that strengthens the literal meaning of "some" to an understood meaning of "some but not all". The implicatures of (i) differ crucially in two ways. First, as noted by Gazdar (1979, see also Chierchia, Fox, & Spector, 2012), (i) lacks the basic specificity implicature of (ii). Second, (i) seems to possess an *ignorance* implicature: namely, that the speaker is not sure whether or not all the students passed the test. Accounting for why the specificity implicature is lacking and how the ignorance implicature comes about has become a problem of considerable prominence in recent semantic and pragmatic theory (Fox, 2007, 2014; Meyer, 2013; Russell, 2012).

4.5.1 An empirical test of ignorance implicature

Before proceeding further, a note regarding the available data is called for. To the best of our knowledge, the only data adduced in the literature in support of the claim that sentences like (i) possess ignorance implicatures have been introspective judgments by the authors of research articles on the phenomenon in question. It is therefore worth briefly exploring exactly how this claim might be more objectively tested and thus verified or disconfirmed. In our view, the claim that "some or all" sentences such as (i) possess an ignorance implicature that corresponding sentences such as (ii) do not should make the following empirically testable prediction: that of sentence pairs like ((i)–(ii)) differing only in TARGET QUANTIFIER "some or all" versus "some", comprehenders should be less likely to conclude that the speaker knows, and/or conclude that the speaker is less

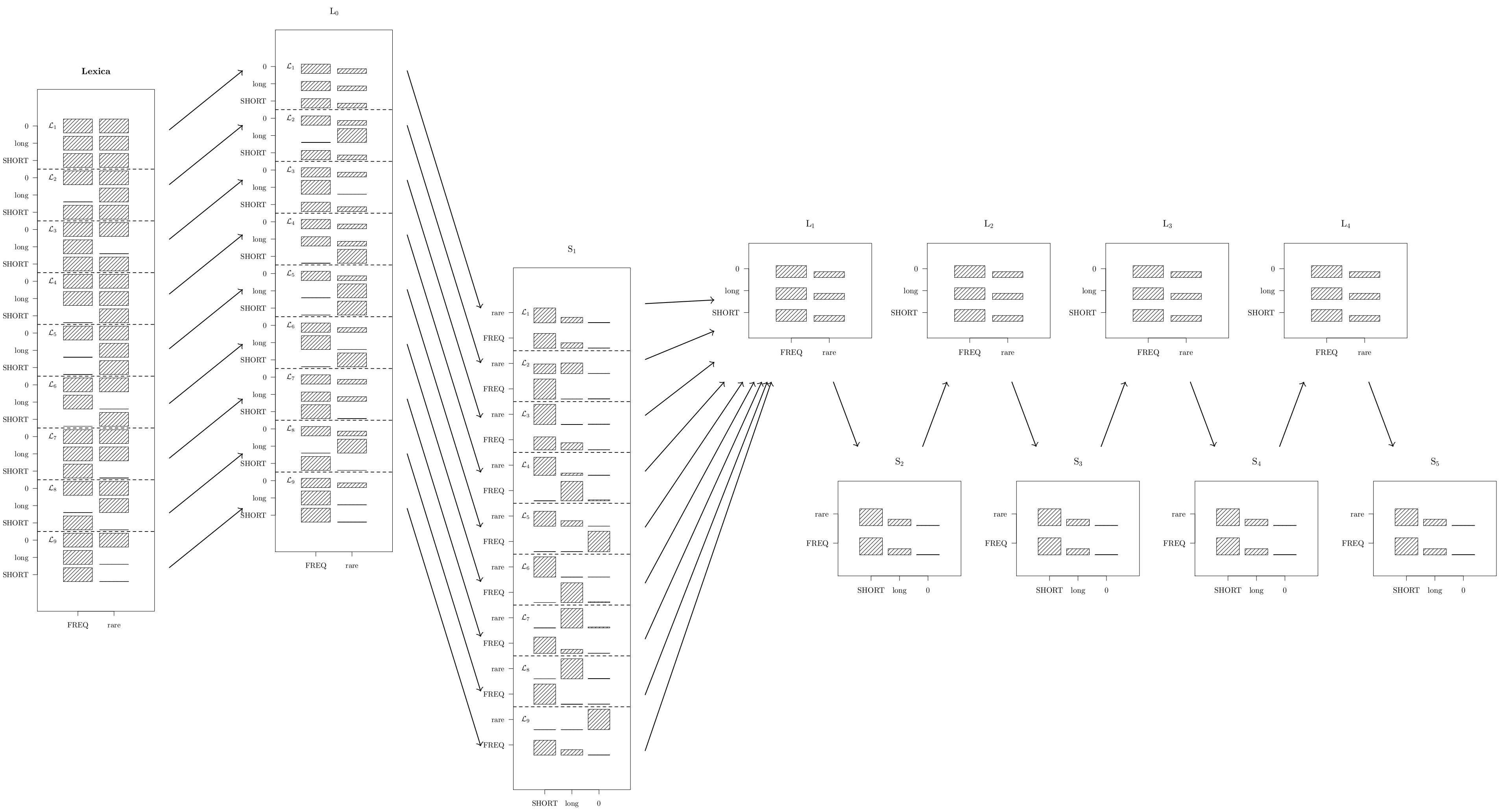

Figure 6: Deriving M-implicature with $P(\text{FREQ}) = \frac{2}{3}$, $P(\text{rare}) = \frac{1}{3}$, $\lambda = 10$, $c(\text{SHORT}) = 1$, $c(\text{long}) = 2$, $c(\emptyset) = 5$.

likely to know, that substitution of the target quantifier with both (a) “all”, and (b) “not all”, for the “some or all” variant than for the “some” variant. To test this prediction, we ran a brief experiment that involved presenting speakers with paragraphs of the following type, each in one of two variants:

> Letters to Laura’s company almost always have checks inside. Today Laura received 10 letters. She may or may not have had time to check all of the letters to see if they have checks. You call Laura and ask her how many of the letters have checks inside. She says, ”{Some/Some or all} of the letters have checks inside.”

Participants were asked two questions:

- *How many letters did Laura look inside?* Answers to this question confirmed (a) above: significantly fewer participants answered *10* in the “some” condition than in the “some or all” condition.
- *Of the letters that Laura looked inside, how many had checks in them?* Answers to this question confirmed (b) above: significantly fewer participants gave the same number as an answer to both this and the preceding question in the “some” condition than in the “some or all “condition.

We are now on more solid ground in asserting that “some or all” triggers an ignorance implicature that is lacked by “some” and that needs to be explained, and proceed to derive this ignorance implicature within our lexical-uncertainty model. (Further details of this experiment can be found in Appendix A.)

4.5.2 Deriving ignorance implicature

To show how our model derives ignorance implicature for the “some or all” case, we first lay out assumptions about the set of world and observation states, the prior over these states, the contents of the semantic lexicon, and utterance costs:

		w	
	$P(o,w)$	$\forall$	$\exists\neg\forall$
	$\forall$	$\frac{1}{3}$	0
o	?	$\frac{1}{6}$	$\frac{1}{6}$
	$\exists\neg\forall$	0	$\frac{1}{3}$

$$\mathcal{L}_S = \left\{ \begin{array}{ll} [\![\text{all}]\!] & = \{\forall\} \\ [\![\text{some}]\!] & = \{\exists\neg\forall, \forall\} \\ [\![\text{some or all}]\!] & = \{\exists\neg\forall, \forall\} \end{array} \right\}$$

u	$c(u)$
all	0
some	0
some or all	1

Exactly as before in our treatment of specificity implicature in Sections 3 and 4.3, we assume two possible world states: $\mathcal{W} = \{\forall, \exists\neg\forall\}$. In order to capture the notion of possible speaker ignorance, however, we have relaxed the assumption of a one-to-one mapping between speaker observation state and world state, and allow three observation states: $\exists\neg\forall$, $\forall$, and a third, “ignorance” observation state denoted simply as ?. For the prior over $\langle o,w\rangle$ state pairs we assume a uniform distribution over the three possible observations and a uniform conditional distribution over world states given the ignorance observation state. We follow standard assumptions regarding literal compositional semantics in assigning identical unrefined literal meanings to “some” and “some or all” in the semantic lexicon. However, the more prolix “some or all” is more costly than both “some” and “all”, which are of equal cost.

Following our core assumptions laid out in Section 4.2, the set of possible lexica generated under lexical uncertainty involves all possible refinements of the meaning of each utterance: "all" cannot be further refined, but "some" and "some or all" each have three possible refinements (to $\{\forall\}$, $\{\exists\neg\forall\}$, or $\{\forall,\exists\neg\forall\}$), giving us nine lexica in total. Also following our core assumptions, each possible lexicon includes the null utterance u_{null} with maximally general meaning $[\![u_{null}]\!] = \{\exists\neg\forall,\forall\}$ and substantially higher cost than any other utterance; here we specify that cost to be $c(u_{null}) = 4$.

Figure 7 shows the results of the lexical uncertainty model under these assumptions, with greedy rationality parameter $\lambda = 4$.[8] (We chose the above parameter values to make the model's qualitative behavior easy to visualize, but the fundamental ignorance-implicature result seen here is robust across specifications of the prior probabilities, "greedy" rationality parameter, and utterance costs, so long as $c(\text{all}) = c(\text{some}) < c(\text{some or all}) < c(u_{null})$.) The key to understanding how the ignorance implicature arises lies in the S_1 matrices for lexica $\mathcal{L}_3$ and $\mathcal{L}_7$. In each of these lexica, one of *some* and *some or all* has been refined to mean only $\exists\neg\forall$, while the other remains unrefined. For a speaker whose observation state is ignorance, an utterance with a refined meaning has infinitely negative expected utility and can never be used; hence, this speaker near-categorically selects the unrefined utterance (*some* in $\mathcal{L}_3$, *some or all* in $\mathcal{L}_7$; the null utterance being ruled out due to its higher cost in both cases). But crucially, while in $\mathcal{L}_7$ the informed speaker who has observed $\exists\neg\forall$ prefers the refined utterance "some", in $\mathcal{L}_3$ that speaker prefers the *unrefined* utterance—again "some"—due to its lower cost. This asymmetry leads to an asymmetry in the marginalizing listener L_1, for whom the association with $\exists\neg\forall$ is crucially stronger for "some" than for "some or all". Further rounds of pragmatic inference strengthen the former association, which in turn drives an ignorance interpretation of "some or all" through the now-familiar mechanics that give rise to scalar implicature.

5 Compositionality

In the previous section we introduced the lexical uncertainty extension of the rational speech-act model, which surmounted a general class of challenges: explaining why two utterances with identical literal content but different form complexity receive different interpretations. In each case, lexical uncertainty led to an alignment between utterances' formal complexity and some kind of markedness of the interpretations they receive. The first example was that of M-implicatures such as the difference in interpretation between *Sue smiled* and *The corners of Sue's lips turned slightly upwards* (Levinson, 2000), where the relevant notion of markedness is the prior probability of the meaning: ordinary smiles are more common than smirks and grimaces. The second example was that of ignorance implicatures for disjunctions such as *some or all*, in which the relevant notion of markedness is the degree of speaker ignorance about the world state: the more complex utterance is interpreted as indicating a greater degree of speaker ignorance.

However, there are even more challenging cases than these: cases in which non-atomic utterances with identical literal content *and* identical formal complexity receive systematically different

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[8] Note that interpretations in listener functions L_i are given as observation states, not pairs of observation and world states. This is a presentational shorthand; the full listener functions $L_0(o,w|u,\mathcal{L})$ and $L_i(o,w|u)$ can always be recovered by multiplying the posterior distribution on observations by the conditional distribution $P(w|o)$.

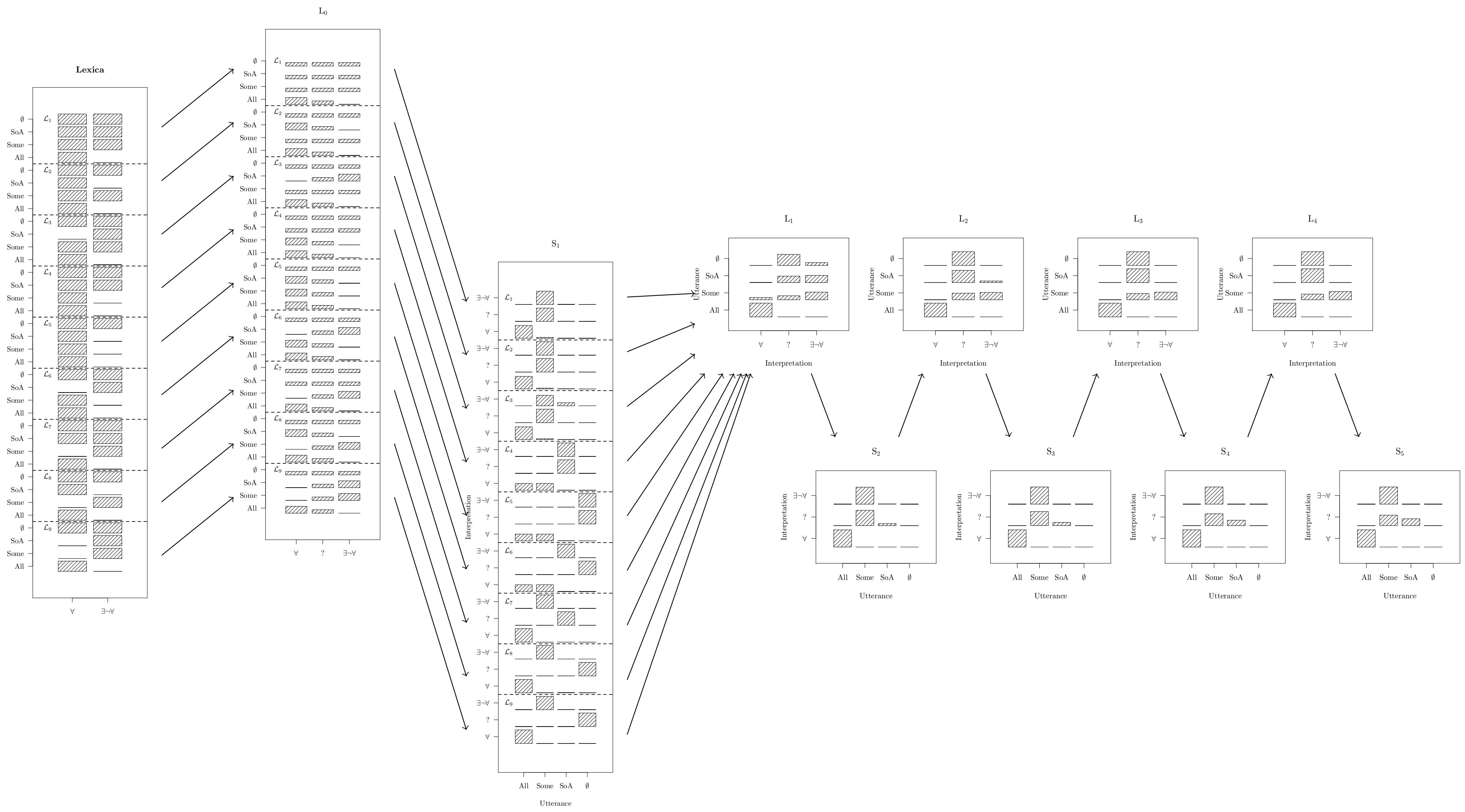

Figure 7: Markedness

interpretations. A general class of these cases can be constructed from entailment scales containing more than two items, by creating a disjunction out of two non-adjacent terms on the scale:

(i) Context: A and B are visiting a resort but are frustrated with the temperature of the springs at the resort they want to bathe in.
A: The springs in this resort are always warm or scalding. [Understood meaning: *but never hot.*]

(ii) Context: A is discussing with B the performance of her son, who is extremely smart but blows off some classes, depending on how he likes the teacher.
A: My son's performance in next semester's math class will be adequate or stellar. [Understood meaning: *but not good.*]

(iii) Context: there are four people in a dance class, and at the beginning of each class, the students are paired up with a dance partner for the remainder of the class. A, who is not in the class, learns that one of the students in the class did not have a dance partner at a particular session, and encounters B.
B: Any idea how many of the students attended the class?
A: One or three of the students showed up to the class. [Understood meaning: *it wasn't the case that either exactly two students or exactly four students showed up.*]

These disjunctive expressions—*warm or scalding*, *decent or stellar*, *one or three*—pose two serious challenges for neo-Gricean theories. First, in each case there are alternatives disjunctive expressions with identical formal complexity (in the sense of having the same syntactic structure and number of words) and literal meaning under standard assumptions that the literal meanings of such expressions are lower bounds in the semantic space of the scale, but different understood meaning: *warm or hot*, *decent or good*, *one or two*.[9] It is not at all clear on a standard neo-Gricean account how these pairs of alternatives come to have different pragmatic interpretations. Second, these expressions have the property that their understood meanings are NON-CONVEX within the semantic space of the scale. This property poses a serious challenge for standard neo-Gricean accounts: since all the alternatives whose negation could be inferred through pragmatic reasoning have literal meanings that are upper bounds in the semantic space, it is unclear how the resulting pragmatically strengthened meaning of the utterance could ever be non-convex.

The basic lexical uncertainty framework developed in Section 4 does not provide an explanation for these cases, which we will call NON-CONVEX DISJUNCTIVE EXPRESSIONS. That framework can only derive differences in pragmatic interpretation on the basis of differences in literal meaning or complexity; in the current cases, the utterance pairs receive distinct interpretations despite sharing the same literal meaning and complexity. It turns out, however, that these cases can be elegantly handled by an extension of the framework that is justifiable on independent grounds. Productive language use requires the computation of novel meanings from familiar constituent parts. But for one to adapt one's language use effectively across contexts, it would be natural for speakers and listeners to maintain some flexibility in the meanings of the constituent parts themselves, whereas

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[9] Explaining the difference in meaning between *one or three* and *one or two* is only a challenge for pragmatic theories if numerals have a lower-bound semantics; if numerals have an exact semantics, then these disjunctive utterances will receive different literal interpretations. However, this objection does not hold for non-numeric scales such as ¡*warm, hot, scalding*¿, in which each lexical item has an uncontroversial lower-bound semantics. We will be using the numerical examples for illustrative purposes, but our claims will be equally applicable to the non-numeric examples.

basic lexical uncertainty introduces that flexibility only at the level of the complete utterance. Extending the framework to account for this possibility gives us what we will call COMPOSITIONAL LEXICAL UNCERTAINTY: we partition the set of utterances in the language into those that are ATOMIC and those that are COMPLEX, and we allow a lexicon to arbitrarily and independently refine the meanings of the atomic utterances from those in the semantic lexicon, but constrain the "refined" meanings of a complex utterance to be equal to that determined by the application of ordinary rules of semantic composition to the refined meanings of its atomic parts. In Section 5.1 we formally define compositional lexical uncertainty, and in Section 5.2 we show how it correctly derives the understood meanings of non-convex disjunctive expressions.

Before introducing the compositional lexical uncertainty framework, it is worth noting that alternative game-theoretic frameworks do not derive the appropriate interpretations of non-convex disjunctive expressions. While the IBR model is able to derive the distinction between *some* and *some or all*, it cannot derive the distinction between *one or two* and *one or three*.[10] The IBR model only derives different pragmatic interpretations based on differences in semantic content or cost; the version of the IBR model which derives the ignorance implicature for *some or all* relies on the difference in cost between *some* and *some or all* in its derivation. Because the utterances *one or two* and *one or three* have identical semantic content and complexity, the IBR model will assign these utterances identical interpretations.

5.1 Compositional lexical uncertainty

Compositional lexical uncertainty involves only one change to the original lexical-uncertainty model introduced in Section 4: a change to how the literal listener interprets utterances. We now assume that the set of utterances $\mathcal{U}$ in the language is generated from a set $\mathcal{U}_A$ of atomic utterances. For our purposes, the utterances in $\mathcal{U}_A$ do not have any internal structure; they are simply Boolean-valued on each possible world.[11] That is, for every $u \in \mathcal{U}_A$, the lexicon $\mathcal{L}$ associates u with a lexical entry that is a truth function on worlds: $\mathcal{L}_u(w) = 1$ if u is true in w and $\mathcal{L}_u(w) = 0$ otherwise. As in the previous models, the lexicon is constrained in how it assigns lexical entries to utterances. In particular, the lexicon must assign each atomic utterance a meaning that is at least as precise as its meaning in English.

The meanings of non-atomic utterances will be built up compositionally using the lexicon. First, for an atomic utterance u, we define its denotation $[\![u]\!]_{\mathcal{L}}$ relative to lexicon $\mathcal{L}$ by:

$$[\![u]\!]_{\mathcal{L}}(w) = \mathcal{L}(u,w) \tag{28}$$

That is, the denotation of an atomic utterance relative to a lexicon is identical to its entry in the lexicon.

We can now build up the denotations of complex utterances in the obvious inductive manner.

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[10] The IQR model does not provide an account of the difference in interpretation between "some" and "some or all." It is strictly more difficult to derive the appropriate implicatures in the current example — because there are strictly fewer asymmetries for the model to exploit — and therefore the IQR model will also not derive these implicatures.

[11] We will not be dealing with composition of non-Boolean typed expressions in the current work. Lexical uncertainty for Boolean expressions is sufficient to derive the phenomena considered in this paper, but future work will look at extending this technique to a wider range of semantic types.

For the disjunction $u_1 \vee u_2$, we define its denotation $[\![u_1 \vee u_2]\!]_{\mathcal{L}}$ relative to lexicon $\mathcal{L}$ by:

$$[\![u_1 \vee u_2]\!]_{\mathcal{L}}(w) = \begin{cases} 1 & \text{if } [\![u_1]\!]_{\mathcal{L}}(w) = 1 \text{ or } [\![u_2]\!]_{\mathcal{L}}(w) = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{29}$$

We define the denotation of utterances built up from conjunctions and other Boolean connectives similarly.

The desiderata constraining the set of allowable lexica Λ are accordingly modified from those of basic lexical uncertainty (c.f. desiderata I–III in Section 4.2; changes to the original definitions are given here in bold):

- I′. Every lexicon must be an ENRICHMENT of **the semantics of the language's atomic utterances**. Formally, if $\mathcal{L}_S$ represents the base SEMANTIC LEXICON for the language (i.e. the lexicon that maps each utterance to its truth function under the language's semantics), then every $\mathcal{L} \in \Lambda$ must be such that for all **atomic** utterances $u \in \mathcal{U}_A$ and all world states w, $\mathcal{L}(u,w) = 1$ only if $\mathcal{L}_S(u,w) = 1$.
- II′. Each **atomic** utterance must receive a non-contradictory interpretation in every $\mathcal{L}$. Formally, for each utterance $u \in \mathcal{U}_A$ and each lexicon $\mathcal{L} \in \Lambda$ there must exist a world w such that $\mathcal{L}(u,w) = 1$.
- III′. Every lexicon $\mathcal{L} \in \Lambda$ must contain all utterances in the base semantic lexicon, plus a distinguished utterance u_{null} such that $\mathcal{L}(u_{null},w) = 1$ for every world w (and not subject to Criterion I). That is, u_{null} must receive a trivial interpretation under every lexicon. Informally, the speaker can decide to stay silent rather than say anything.

The literal listener interprets utterances according to their denotations:

$$L_0(w,o|u,\mathcal{L}) \propto [\![u]\!]_{\mathcal{L}}(w)P(w,o) \tag{30}$$

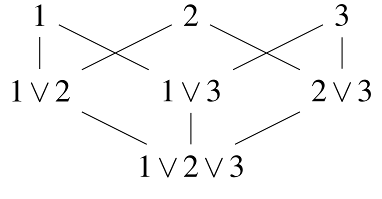
In other words, the literal listener filters out worlds that are inconsistent with the denotation of the utterance. The definitions of the higher-order speakers and listeners are unchanged from the previous versions of the model.

5.2 Derivation of non-convex disjunctive expressions

We demonstrate the account of non-convex implicatures afforded by compositional lexical uncertainty using the running example of *one or three*, though the same account would hold for non-convex disjunctions on other scales such as *warm or scalding* and *decent or stellar*. For discursive simplicity we limit the range of the space to the integers $\{1,2,3\}$, though the account generalizes to arbitrary convex subsets of the integers. The set of ATOMIC UTTERANCES U_A and possible observation states O are, respectively:

$$U_A = \{one, two, three\}$$

$O =$



where the join-semilattice relationship among the seven members of O is depicted for expository convenience. The set of world states W contains what we will call only BASIC world states—in this case, 1, 2, and 3—and the mapping between world states and speaker observation states is not one-to-one. Under these circumstances, an observation state is compatible with all basic world states above it on the lattice, and observation states thus vary in the degree of speaker ignorance.

Since utterance meanings are defined as sets of world states, the literal meaning of each atomic utterance can easily be picked out as the set of world states that lie above a particular node on the join semilattice. In our running example, these nodes are $1 \vee 2 \vee 3$ for *one,* $2 \vee 3$ for *two*, and 3 for *three*. Hence we have

$$\mathcal{L}_S = \left\{ \begin{array}{ll} \llbracket one \rrbracket & = \{1,2,3\} \\ \llbracket two \rrbracket & = \{2,3\} \\ \llbracket three \rrbracket & = \{3\} \end{array} \right\}$$

for the simple indicative case.[12]

The set of possible lexica consists of all logically possible combinations of refinements (i.e., subsets) of each atomic utterance's meaning. In the simple indicative case, *one* has seven possible refinements, *two* has three possible refinements, and *three* has one, hence there are twenty-one logically possible lexica, a few of which are shown below:

$$\left\{ \begin{array}{ll} \llbracket one \rrbracket & = \{1,2,3\} \\ \llbracket two \rrbracket & = \{3\} \\ \llbracket three \rrbracket & = \{3\} \\ \llbracket one\ or\ two \rrbracket & = \{1,2,3\} \\ \llbracket two\ or\ three \rrbracket & = \{3\} \\ \llbracket one\ or\ three \rrbracket & = \{1,2,3\} \\ \llbracket one\ or\ two\ or\ three \rrbracket & = \{1,2,3\} \end{array} \right\} \left\{ \begin{array}{ll} \llbracket one \rrbracket & = \{3\} \\ \llbracket two \rrbracket & = \{2,3\} \\ \llbracket three \rrbracket & = \{3\} \\ \llbracket one\ or\ two \rrbracket & = \{2,3\} \\ \llbracket two\ or\ three \rrbracket & = \{2,3\} \\ \llbracket one\ or\ three \rrbracket & = \{3\} \\ \llbracket one\ or\ two\ or\ three \rrbracket & = \{2,3\} \end{array} \right\} \left\{ \begin{array}{ll} \llbracket one \rrbracket & = \{1\} \\ \llbracket two \rrbracket & = \{2\} \\ \llbracket three \rrbracket & = \{3\} \\ \llbracket one\ or\ two \rrbracket & = \{1,2\} \\ \llbracket two\ or\ three \rrbracket & = \{2,3\} \\ \llbracket one\ or\ three \rrbracket & = \{1,3\} \\ \llbracket one\ or\ two\ or\ three \rrbracket & = \{1,2,3\} \end{array} \right\}$$

To show how this account correctly derives understood meanings for non-convex disjunctive utterances, we need to complete the model specification by choosing utterance costs and prior probabilities. Similar to the approach taken in Section 4.5.2, we make the minimally stipulative assumptions of (i) a uniform distribution over possible observations, (ii) a uniform conditional distribution for each observation over all worlds compatible with that observation; and (iii) a constant, additive increase in utterance cost for each disjunct added to the utterance. We set the cost per disjunct arbitrarily at 0.05 and set λ to 5, though our qualitative results are robust to precise choices of (i–iii) and of λ.

Here we examine in some detail how the model correctly accounts for interpretations of non-convex disjunctive expressions in the simple indicative case. Even in this case there are 21 lexica, which makes complete visual depiction unwieldy; for simplicity, we focus on the twelve lexica in which the denotation of *one* has not been refined to exclude 1, because it is in this subset of lexica in which *one* has already been distinguished from *two* and we can thus focus on the inferential dynamics leading to different interpretations for *one or two* versus *one or three*. Figure 8 shows the behavior of this pragmatic reasoning system. The three leftmost panels show the twelve lexica and the resulting literal-listener L_0 and first-level speaker S_1 distributions respectively; the three rightmost panels show the marginalizing listener L_1 and the subsequent speaker and listener S_2 and

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[12] We skirt the issue of whether the semantic lexicon includes non-atomic utterances, as it has no impact on the model's predictions.

Figure 8: Non-convex disjunction, for uniform marginal distribution $P(O)$, uniform conditional distributions $P(W|O)$, cost per disjunct of 0.05, and $\lambda = 5$. Only lexica (and L_0 and S_1 distributions) in which the refined meaning of *one* contains the world state 1 are shown.

L_2 respectively; by the L_2 level, pragmatic inference has led both atomic and disjunctive utterances to be near-categorically associated with interpretations such that each atomic term in an utterance has an exact meaning at the lower bound of the term's unrefined meaning (and such that disjunctive utterances are thus disjunctions of exact meanings). The key to understanding why this set of interpretations is obtained can be found in the asymmetries among possible refinements of atomic terms in the lexica. Observe that under lexical uncertainty both *two* and *three* can have refined meanings of $\{3\}$; but whereas *three* MUST have this meaning, *two* has other possible meanings as well ($\{2\}$ and $\{2,3\}$). Consequently, the set of lexica in which *one or two* has $\{1 \vee 3\}$ as its meaning ($\mathcal{L}_6$ and $\mathcal{L}_8$) is a strict subset of the set of lexica in which *one or three* has that meaning (which also includes $\mathcal{L}_2$, $\mathcal{L}_4$, $\mathcal{L}_{10}$, and $\mathcal{L}_{12}$). Pragmatic inference leads to a strong preference at the S_1 level in the latter four lexica for expressing observation state $\{1 \vee 3\}$ with *one or three*, even in $\mathcal{L}_4$ and $\mathcal{L}_{10}$ where that observation state is compatible with the utterance *one or two*. Furthermore, there are no lexica in which the reverse preference for expressing $\{1 \vee 3\}$ with *one or two* is present at the S_1 level. This asymmetry leads to a weak association between *one or three* and $\{1 \vee 3\}$ for the marginalizing L_1 listener, an association which is strengthened through further pragmatic inference.

5.3 *Some or all* ignorance implicatures with compositional lexical uncertainty

For completeness, we briefly revisit the ignorance implicatures of *some or all* originally covered in Section 4.5, now within the framework of compositional lexical uncertainty. In short, compositional lexical uncertainty derives ignorance implicature for *some or all* for similar reasons that it derives interpretations for the more difficult cases of non-convex disjunctive expressions: there are lexica in which *some* is refined to mean $\{\exists\neg\forall\}$, but no lexica in which *some or all* can be refined to have this meaning. This asymmetry leads to a weak association for the marginalizing L_1 listener between *some* and $\exists\neg\forall$ and between *some or all* and the ? ignorant-speaker observation state. Further pragmatic inference strengthens this association (S_2 and L_2).[13]

6 Discussion

We have introduced the notion of lexical uncertainty in order to account for a sequence of increasingly complex pragmatic phenomena. This technique was first used in order to explain simple M-implicatures, in which complex utterances are assigned less probable interpretations. By relaxing the knowledgeability assumption on speakers, we were able to use this technique to account for a more general class of M-implicatures, which assign *marked* interpretations to complex utterances. The notion of markedness here includes interpretations that are low probability as well as interpretations under which the speaker is ignorant. Finally, we were able to account for a range of embedded implicatures by using lexical uncertainty to form meanings compositionally.

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[13] It is worth remarking that this asymmetry resulting from the constraints across denotations of utterances imposed by compositional lexical uncertainty is strong enough to derive the empirically observed interpretations and associated ignorance implicatures of disjunctive expressions even without any differences in utterance costs. Thus compositional lexical uncertainty can be viewed as a fully-fledged alternative to the "ignorance as a marked state" view of the basic ignorance implicatures of Section 4.5.

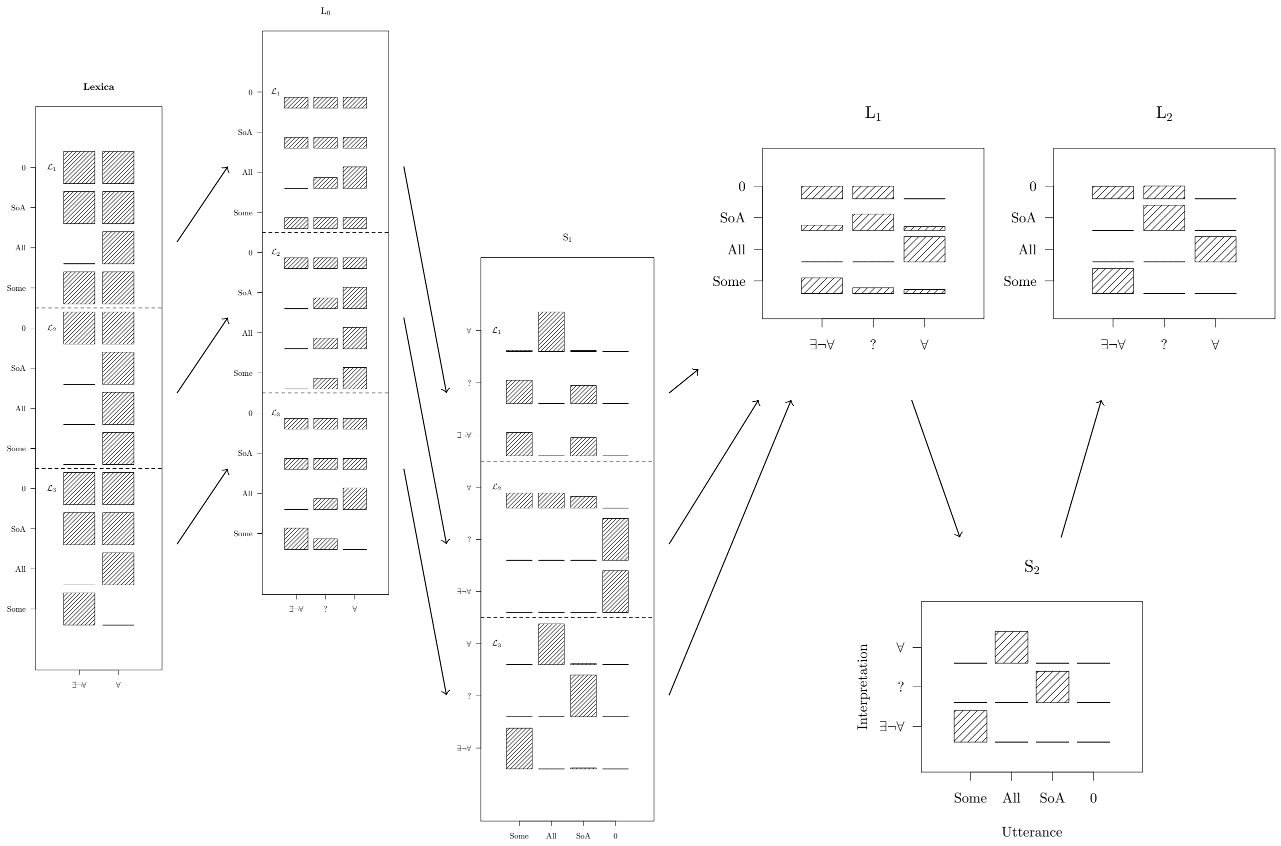

Figure 9: *Some or all* ignorance implicature under compositional lexical uncertainty.

6.1 Semantic free variables

The core conceptual and technical idea involved in lexical uncertainty is to model the speaker and listener as uncertain about the semantic content of their language's utterances. As implemented in this paper, lexical uncertainty assumes that the lexicon fully specifies the semantic content of each lexical item; in order to model the speaker and listener's semantic uncertainty, we model them as being uncertain about the lexicon that is being used. However, there is nothing essential about the assumption that the lexicon fully determines each utterance's semantic content. In particular, there is an alternative way of formalizing semantic uncertainty, which preserves all of the modeling results presented in this paper but which pushes the semantic uncertainty into the lexicon itself.

This alternative formalization uses *semantic free variables* in order to represent semantic uncertainty.[14] These semantic free variables are used to assign underspecified semantic content to utterances. This is done in the obvious manner: certain variables in a lexical entry are abstracted out, and replaced with free variables. The semantic content of a lexical item is fully specified once all of the free variables in its lexical entry have been assigned values. Semantic uncertainty can be represented as uncertainty about the values of the relevant variables. For example, we might assign

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[14] Lexical uncertainty, as first proposed in Bergen et al. (2012), assumes that the lexicon fully specifies the semantic content of each lexical item. The reinterpretation in terms of semantic free variables was proposed in Lassiter and Goodman (2013).

the adjective "tall" the lexical entry $\lambda x \lambda y.\text{height}(y) > x$, where the value of the threshold variable x is left unspecified in the lexicon. The value of this variable is inferred during pragmatic inference, in a manner nearly identical to the lexical inference procedure described in this paper.

Though lexical uncertainty and the semantic free variable technique use distinct representations to derive pragmatic inferences, it is fairly straightforward to show that each technique can be used to simulate the other. We will provide an informal sketch of this equivalence. Given a set of underspecified lexical items, we can fully fix the semantic interpretation of these items by assigning values to all of their semantic free variables. Thus, a complete assignment of values to the lexical items' semantic free variables can be represented by a lexicon which assigns each lexical item a fully specified semantic content. The joint distribution over the values of semantic free variables therefore can be represented by a distribution over lexicons, and it follows almost immediately that lexical uncertainty can simulate semantic free variables.

It is similarly straightforward to use semantic free variables to simulate lexical uncertainty. For every lexical item with lexical entry $\lambda y.C(y)$, where C is a fully specified predicate, we can create an abstracted version of this entry by conjoining a free predicate variable to it, as in $\lambda F \lambda y.C(y) \wedge F(y)$. This conjoined free variable will represent the enriched semantic content that this lexical item will convey, in addition to the invariant semantic content conveyed by the predicate C. We can represent a distribution over lexical entries assigned to this lexical item by a suitable distribution over values to assign the free variable of the abstracted lexical entry. Therefore we can represent a distribution over lexicons by abstracting all of the lexical entries in this manner, and defining suitable distribution over values to assign to the variables in these lexical entries.

This demonstrates that there is no substantive commitment involved in using lexical uncertainty rather than semantic free variables, or vice-versa. Nonetheless, the choice of representation will have an effect on how simple it is to describe certain models. It is natural, for example, to describe uncertainty about the meaning of "tall" as uncertainty about the height threshhold that an object is required to meet. As already noted, it is straightforward to represent this by substituting a free variable for this threshhold, and placing the appropriate distribution over the value of this variable. It is less natural to represent this using lexical uncertainty; one can define the required distribution over lexical entries for "tall," but lexical uncertainty does not itself explain why this distribution should be preferred over the many other possible distributions over pragmatic enrichments of "tall." In contrast, lexical uncertainty provides a more natural representation for deriving M-implicatures, such as the difference in interpretation between "John can finish the homework" and "John has the ability to finish the homework." Lexical uncertainty works in these cases by defining a distribution over arbitrary pragmatic enrichments of these utterances. This distribution can be defined, using the technique described above, in terms of semantic free variables, but it is less natural; there is nothing in this case analogous to a height threshhold for the free variable to represent.

6.2 Semantic uncertainty

In models that derive pragmatic inferences from recursive social reasoning — e.g., the rational speech acts, iterated best response, and iterated quantal response models — the semantic content of an utterance plays a specific functional role. For the recursive reasoning in these models to be well-defined, the recursion needs to ground out in the responses of an agent who is not reasoning about other agents. As they are usually set up, these models ground out in a listener who receives an utterance, and provides it with an interpretation which is not a function of the behavior of any other agent. The semantic content of the utterance, along with the prior beliefs of the listener, determines this interpretation: this listener interprets the utterance by using its semantic content to filter their prior beliefs. This can be considered a formal implementation of a traditional conception of how semantic content and pragmatic inferences relate to each other, i.e. that semantic content serves as the initial input for pragmatic reasoning.

This broad conception of the relationship between semantic content and pragmatic inference is consistent with a number of theories of semantics and pragmatics. It is consistent, for example, with both neo-Gricean and grammatical theories of scalar implicatures and related phenomena. These theories make distinct claims about whether particular phenomena — e.g., inferences associated with embedded scalar terms — arise from pragmatic reasoning or from grammatical computations. For the present purposes, however, these can be seen as different claims about what the semantic input to pragmatic inference consists of. These theories share the basic architecture described above, according to which the semantic content of an utterance is fixed before serving as an input to pragmatic inference.

It is also worth noting that this architecture makes no claims about what non-pragmatic factors influence an utterance's semantic content. It is consistent, for example, with contextualist theories of semantic content, according to which the semantic content of a linguistic expression may shift depending on the context. The architecture under discussion only requires that the semantic content of an utterance be fixed in advance of pragmatic reasoning.

The concept of lexical uncertainty introduced in this paper is not obviously consistent with this architecture. Under lexical uncertainty, there is not a unique base-case for the speaker-listener recursion. Rather, there is a distribution over base-cases, corresponding to the possible lexicons that the literal listener may be using in order to interpret utterances. The consequence of this is that there does not exist a unique assignment of semantic content to each utterance which determines its interpretation by the literal listener. The assignment of semantic content to each utterance will instead be partially determined during pragmatic inference itself. Conditional on the speaker having chosen a particular utterance, a pragmatically sophisticated listener will have a posterior distribution over what lexicon the speaker is using, and this distribution will not, in general, be identical to the prior distribution over lexicons. Thus the semantic content of each utterance is coupled with the pragmatic reasoning of the speaker and listener; the utterances' semantic content is neither conceptually nor probabilistically prior to the pragmatic reasoning.

Given that lexical uncertainty, as a formal technique, appears to require positing a non-standard relationship between semantic content and pragmatic reasoning, it is worth considering the possible interpretations of this technique. The first of these interpretations is the least radical, but also the least relevant for the phenomena that we have considered in this paper. Under this interpretation, a language assigns each utterance (in a context) a definite semantic semantic content, in line with traditional accounts of how semantic content is assigned to utterances. The speaker is using a particular language, but the listener has uncertainty about which language the speaker is using, the speaker knows that the listener is uncertain, and so on. This would describe, for example, a situation in which the speaker is using a foreign language which is not spoken by the listener, such that the speaker knows that the listener does not know this language, and so on.

This is not, however, a plausible interpretation of the pragmatic inferences described in this paper. In all of the cases considered, the speaker and listener are using a fixed language, English, and neither has uncertainty about the language being spoken by the other. In the derivation of ignorance implicatures, for example, both the speaker and listener know the meaning of "some" and “some or all.” Indeed, the English semantic content of these utterances plays a crucial role in the derivation of the appropriate implicatures. This is because the English semantic content of these utterances constrains the possible lexicons that are considered by the speaker and listener; a lexicon is only assigned positive probability if it is an enrichment of the English language lexicon. It is straightforward to show that if the English semantic content of the utterances is not taken into account in defining the distribution over lexicons — e.g., if the distribution over lexicons is uniform over the set of logically possible lexicons — then the model will not derive the correct implicatures.

Given that the derivations presented in this paper require the speaker and listener to know the semantics of English, lexical uncertainty cannot require uncertainty about the semantics of the language being spoken, at least in the normal sense of this term. We therefore propose an alternative interpretation of lexical uncertainty. This interpretation will be more clearly understood after considering the interpretation of more traditional iterated response models, such as the iterated best response and rational speech acts models.

These iterated response models begin with a listener who interprets utterances literally, and does not reason strategically about the likely intent of the speaker. Such a listener does not correspond to an actual agent in the world, as the model does not require us to posit that there actually exists an agent with this interpretive behavior; rather, this non-strategic listener is merely hypothetical, playing a particular role in the model's computation of utterance interpretations. The iterated response models then consider a speaker who optimizes their utterances with respect to the literal listener. Such a speaker also does not correspond to an actual agent in the world. These models thus use a sequence of increasingly sophisticated speakers and listeners in order to compute pragmatic interpretations, but until the most sophisticated speaker and listener are computed, none of these speakers or listeners represent actual agents.

The most sophisticated speaker and listener do represent actual agents, namely the actual speaker and listener whose conversation is being modeled. The motivation for using hypothetical agents describe their reasoning can be explained by considering the challenges involved in formalizing Gricean reasoning. Formal models of Gricean pragmatics try to capture the intuition that a listener interprets an utterance by reasoning about what the speaker would say under different circumstances, and that a speaker chooses an utterance by reasoning about how the listener will interpret different utterances. However, there is a clear barrier to making this idea mathematically precise: the speaker is defined in terms of the listener, and the listener is defined in terms of the speaker, leading to a regress. There are generally two strategies for eliminating the problems stemming from this regress. First, as is standardly done in game theory, we can compute a fixed point of the speaker-listener recursion, and stipulate that the speaker and listener act according to this fixed point.[15] Second, as is done in the iterated response models, we can posit that the sophisticated speaker is reasoning about a listener who is strictly less sophisticated than them, and that this listener is reasoning about a speaker who is still less sophisticated, and so on.[16] This second approach avoids regress because the speaker is not reasoning about a listener who has a correct

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[15] This main problem with this approach is that, as noted in the discussion of the multiple equilibrium problem, there will typically be many fixed points for the speaker-listener recursion. A great deal of work is required in order to cull the fixed points to those that are relevant for pragmatics.

[16] This approach is not, strictly speaking, incompatible with the fixed-point approach, as the iterated response models can be viewed as specifying which fixed point of the speaker-listener recursion should be distinguished. However, we do not find this perspective to be very illuminating about how to properly interpret the iterated response models.

model of the speaker; the speaker is only being defined in terms of less sophisticated speakers, thus avoiding proper self-definition.

This perspective leads straightforwardly to an interpretation of the hypothetical agents in the iterated response models, as being objects of the higher-order beliefs of the speaker and listener. The most sophisticated speaker believes that their utterances will be interpreted by a particular type of listener. For the speaker's beliefs to be well-defined, this listener must be less sophisticated than the speaker. This listener, in turn, believes that the utterance that they received was chosen by a particular type of speaker. For this listener's beliefs to be well-defined, this speaker must in turn be less sophisticated than the listener. Continuing in this manner, we can see that the actual representational claims of the model can be formulated in terms of the higher-order beliefs of the speaker and listener. The model represents the speaker (and similarly the listener) as holding beliefs that can be paraphrased as follows: *The speaker believes that the listener believes that ... the speaker believes that the listener will interpret utterances literally*. Thus, all of the hypothetical agents considered by the model are in fact objects of the higher-order beliefs of the most sophisticated speaker and listener. Any claim about the properties of, e.g., a hypothetical listener is in fact a claim that the most sophisticated speaker (or listener) believes that the listener believes that ... the speaker believes that the listener holds these properties.

This interpretation of the iterated response models can be extended to provide an interpretation of lexical uncertainty. As defined in Equation 19, only the least sophisticated speaker and listener, S_1 and L_0, are parameterized by lexicons; these are the only speakers and listeners represented as having beliefs about the lexicon. Building on the interpretation of the iterated response models, lexical uncertainty can be interpreted as a claim about the higher-order beliefs of the most sophisticated speaker and listener. In particular, it refers to the most sophisticated speaker's (or listener's) belief that the listener believes that ... the listener a) believes that the least sophisticated speaker is using a particular lexicon and b) is uncertain about what lexicon this speaker is using. That is, lexical uncertainty attributes to the speaker and listener the higher-order belief that there is uncertainty about the lexicon being used by the least sophisticated speaker and listener.

Under the proposed interpretation, lexical uncertainty does not need to posit that any actual agents interpret utterances according to a non-standard lexicon, just as the iterated response models do not need to posit that any actual agents interpret utterances according to the standard English lexicon. Instead, in both cases, the models are describing properties of agents who are the objects of the actual speaker and listener's higher-order beliefs. As such, under both types of models, the agents' lexicons are theoretical entities that play a particular functional role in deriving utterance interpretations. Though there may be alternative methods for evaluating hypotheses about the nature of the agents' lexicons — e.g., by querying the speaker and listener's higher-order beliefs in some other manner — the primary method for evaluating such hypotheses will be by determining if, as part of the model, they provide the correct predictions about how utterances will be used and interpreted.

6.3 Utterance cost and complexity

The notion of utterance cost plays an important role in the explanations of a number of phenomena discussed in this paper. The proposed solution to the symmetry problem relies on assigning non-salient alternatives a higher cost than salient alternatives; the derivation of M-implicatures requires a cost asymmetry between the utterance that will be assigned a high-probability meaning and the one that will be assigned a low-probability meaning; and the more general treatment of markedness requires that utterances receiving marked interpretations be more costly.

The most natural interpretation of the cost parameter in our models is that it represents how much effort is required for the speaker to convey an utterance. This effort may reflect the length of the utterance (in, e.g., syllables); the difficulty of correctly pronouncing it; the amount of energy required to produce the sounds required for the utterance; or still other possible factors. An interpretation of the cost parameter in this manner constitutes a theory of how the speaker chooses utterances, as well as a theory of how *the listener* believes the speaker chooses utterances. That is, if cost represents usage effort, then the speaker will derive lower utility from more effortful utterances, and will be less likely to choose such utterances (all things being equal).

In providing a theory of how the speaker chooses utterances, and of how the listener believes the speaker chooses utterances, we may also wish to account for an additional feature of utterances, one which does not perfectly track effortfulness: the complexity of the utterance under the speaker's theory of their language. That is, the speaker may be less likely to use a particular utterance, not neccesarily because it is difficult to say, but because it is a complex utterance according to their grammar. For example, the speaker may be unlikely to use the locative-inversion construction, e.g. "Onto the table jumped the cat," even though by all appearances it is no more difficult to say than, "The cat jumped onto the table"; this is attested in the corpus frequencies for these constructions, where the locative inversion is much less common. A theory of how the speaker chooses utterances should thus be sensitive to the speaker's judgments of linguistic complexity.

The distinction between difficulty and complexity may, in addition, be important for correctly describing the pragmatic phenomena that have been discussed in this paper. It is not clear, for example, whether the difference between "John can do his homework" and "John has the ability to do his homework" derives from the greater length of the latter utterance, or from its use of a less common construction. If it is the latter, then it is necessary for our models to be able to represent the difference in linguistic complexity of these utterances, and to correctly derive implicatures from this difference.

Though we will not discuss this in detail in this paper, it is straightforward to represent linguistic complexity in our models, and to derive exactly the same predictions starting from differences in complexity rather than differences in difficulty. Indeed, it is possible to show that a natural representation of linguistic complexity is mathematically equivalent, under our models, to the representation of effort in terms of disutility. This can be demonstrated using the planning as inference equivalence, and can be illustrated using the implementation of our models in the probabilistic programming language Church Goodman, Mansinghka, Roy, Bonawitz, and Tenenbaum (2008). Under this approach, linguistic complexity is represented by the speaker's prior distribution over natural language expressions. Utterances will be assigned low probability under this distribution when they are judged to be complex by the speaker's grammar. Such a distribution will fall out automatically if the grammar is represented in terms of a probabilistic generative process, but alternative representations of the grammar are compatible with this formalism. Our models will thus predict an M-implicature both when an utterance is difficult for the speaker to use, and when it is assigned low probability under the speaker's grammatical model.

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A Experimental validation of ignorance implicature

Here we will describe an experimental evaluation of the linguistic judgments discussed in Section 4.5. For ease of exposition, we will reproduce the examples from that section here:

(i) Some or all of the students passed the test.

(ii) Some of the students passed the test.

The experiment evaluated two claims about the interpretation of example (i). The first claim is that while example (ii) implicates that not all of the students passed the test, example (i) does not carry this implicature. The second claim is that this example carries an ignorance implicature: it implicates that the speaker does not know whether all of the students passed.

A.1 Methods

Participants Thirty participants were recruited from Amazon's Mechanical Turk, a web-based crowdsourcing platform. They were provided with a small amount of compensation for participating in the experiment.

Materials We constructed six items of the following form:

> Letters to Laura's company almost always have checks inside. Today Laura received 10 letters. She may or may not have had time to check all of the letters to see if they have checks. You call Laura and ask her how many of the letters have checks inside. She says, "{Some/Some or all} of the letters have checks inside."

The name of the speaker (e.g. "Laura") and the type of object being observed (e.g. checks inside letters) were varied between items. The speaker's utterance was varied within items, giving two conditions for each item, "Some" and "Some or all." Each participant was shown every item in a randomly assigned condition.

After reading an item, participants were asked two questions:

A: *How many letters did Laura look inside?*

B: *Of the letters that Laura looked inside, how many had checks in them?*

Question **A** was used to assess whether the speaker knows *all*, which in this example would mean that Laura knows that all of the letters have checks inside of them. This question assesses whether the speaker meets a necessary condition on knowing *all*. If, for example, Laura has not looked inside each letter, then she cannot know that all of the letters have checks inside. Question **B** was used to assess whether the speaker knows *not all*, which in this example would mean that Laura looked inside letters which did not have checks in them. If the numerical response to the first question exceeds the response to the second question, then Laura knows that not all of the letters have checks in them.

A.2 Results

We first analyzed the effect of the speaker's utterance on judgments of whether the speaker observed the full world state, as measured by responses to Question **A**. In particular, we analyzed the effect on the probability that the speaker examined all 10 objects, which we denote by $P(\mathbf{A} = 10)$. This analysis was performed using a logistic mixed-effects model, with random intercepts and slopes for items and participants. Responses in the "Some or all" condition were significantly less likely to indicate that the speaker examined all 10 objects than in the "Some" condition ($\beta = -5.81$; $t = -2.61$; $p < 0.01$). This result is shown in Figure 10.

We next analyzed the effect of the speaker's utterance on judgments of whether the speaker knows *not all*. This was measured using the probability that the number of total observations (as measured by the response to Question **A**) was greater than the number of positive observations (as measured by Question **B**). This probability is denoted by $P(\mathbf{A} > \mathbf{B})$. The analysis was performed

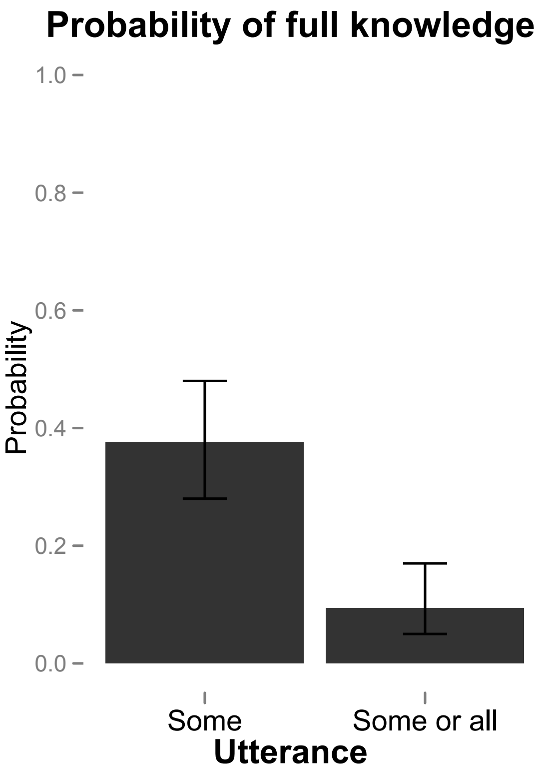

Figure 10: $P(\mathbf{A} = 10)$ as a function of the speaker's utterance. Error bars are 95% confidence intervals.

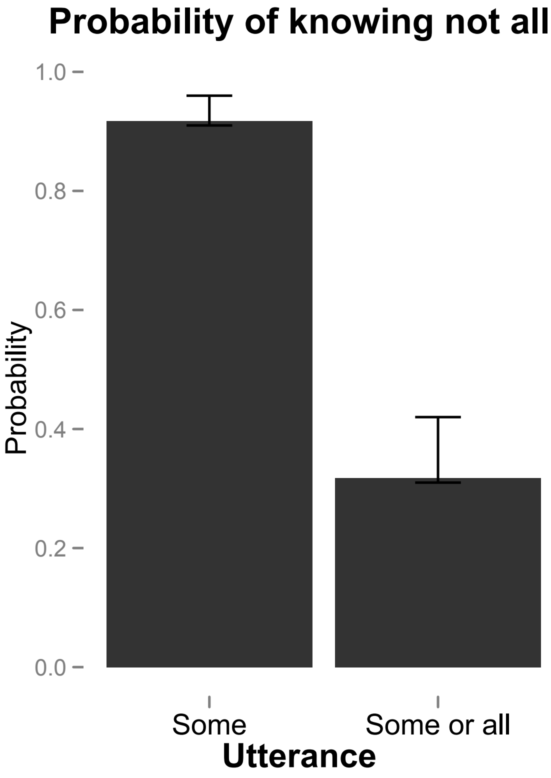

Figure 11: $P(\mathbf{A} > \mathbf{B})$ as a function of the speaker's utterance.

using a logistic mixed-effects model, with random intercepts for participants, and random intercepts and slopes for items.[17] Responses in the "Some or all" condition were significantly less likely to indicate that $\mathbf{A} > \mathbf{B}$ than those in the "Some" condition ($\beta = -4.73$; $t = -7.22$; $p < 0.001$). This result is shown in Figure 11.

These results provide evidence for the two claims about the interpretation of "Some or all." First, while "Some" carries a specificity implicature, and indicates that the speaker knows *not all*, "Some or all" does not carry this implicature, and instead indicates that the speaker does not know *not all*. Second, "Some or all" indicates that the speaker also does not know *all*. Together, this provides evidence that "Some or all" carries an ignorance implicature, providing information that the speaker does not know the full state of the world.

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[17] The model which included random slopes for participants did not converge.