

Gesture Projection and Cosuppositions*

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Preliminary draft - August 17, 2015

Abstract: In dynamic theories of presupposition, a trigger pp' (e.g. *it stopped raining*) with presupposition p (*it rained*) and assertive component p' (*it doesn't now rain*) comes with a requirement that p should be entailed by the local context of pp' . We argue that some co-speech gestures should be analyzed within a presuppositional framework, but with a twist: an expression p co-occurring with a co-speech gesture G with content g comes with the requirement that the local context of p should guarantee that p entails g ; we call such assertion-dependent presuppositions 'cosuppositions'. We show that this analysis can be combined with a predictive analysis of local contexts (e.g. Schlenker 2009) to account for complex patterns of gesture projection in quantified and in attitudinal contexts, and compare our account to two potential alternatives: one based on supervaluations, and one that treats co-speech gestures as supplements (Ebert and Ebert 2014). We argue that the latter is correct, but for 'post-speech' gestures (= gestures that come after the expressions they modify), rather than for co-speech gestures.

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*I wish to thank the following colleagues for discussion and objections: Amir Anvari, Diane Brentari, Emmanuel Chemla, Chris Kennedy, Manuel Kriz, Salvador Mascarenhas, Rob Pasternak, Benjamin Spector, as well as audiences at New York University and University of Chicago. Special thanks to Rob Pasternak and to Emmanuel Chemla for discussion of most aspects of this project. The research leading to these results received funding from the European Research Council under the European Union's Seventh Framework Programme (FP/2004-2013) / ERC Grant Agreement N°324115-FRONTSEM (PI: Schlenker). Research was conducted at

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1 Introduction




Co-speech gestures provide a way to enrich the meaning of spoken language utterances. A very simple example is provided in (1). Without a co-speech gesture, the sentence is neutral as to what kind of help was provided. With the upward movement of the palms co-occurring with *help*, chances are that John helped his son by somehow lifting him or by pushing him upwards.


- (1) John UP_  helped his son.

Important research has been conducted in psychology and linguistics about the typology of gestures (see for instance Kendon 2004, McNeill 2005 and Giorgolo 2010 for discussion)¹. Here we will be primarily concerned with *iconic co-speech gestures*, i.e. gestures that occur with spoken words and enrich their meaning by depicting an aspect of the denoted situations. We will not be concerned with how a gesture such as *UP* in (1) comes to have the content that it does. Rather, we will ask how co-speech gestures interact with the logical structure of a sentence. For although the example in (1) is very simple, gestures may appear in the scope of a variety of logical operators, hence a question: *how are the enrichments of expressions modified by co-speech gestures inherited by complex sentences?* This is the 'projection problem' for co-speech gestures;² to our knowledge, it was initially studied in pioneering work by Ebert and Ebert 2014, who took the line that the semantic contribution of co-speech gestures should be likened to that appositive relative clauses (we will call this a 'supplemental analysis').

In this piece, we argue instead that (some) co-speech gestures should be analyzed within a presuppositional framework, albeit with a twist. In standard theories, a presupposition trigger *pp'* (e.g. *it stopped raining*) with presupposition *p* (*it rained*) and assertive component *p'* (*it doesn't now rain*) comes with a requirement that *p* should be entailed by the local context of *pp'*. By contrast, an expression *p* co-occurring with a co-speech gesture *G* with content *g* comes with the requirement that the local context should guarantee that *p* entails *g*. In other words, the co-speech gesture triggers an assertion-dependent presupposition, something we call a 'cosupposition'. Importantly, no modification of presupposition theory is needed to handle such cases, since nothing prevents a trigger *pp'* to have a presupposition of the form ($p' \Rightarrow q$), which is thus conditionalized on the assertive component *p'*. We show that this analysis can be combined with a predictive analysis of local contexts (e.g. Schlenker 2009) to account for complex patterns of gesture projection in quantified and in attitudinal contexts, and compare our account to two potential alternatives: one based on supervaluations, and the supplemental analysis of Ebert and Ebert 2014. We argue that the latter is correct, but for 'post-speech' gestures (= gestures that come after the expressions they modify), rather than for co-speech gestures. Thus the *timing* of a gestural enrichment can significantly alter its semantic status.

The intuition we pursue can be described rather simply by inserting the example in (1) in various contexts, as in (2). Here and throughout, we adopt the convention of *writing co-speech gestures in capital letters (and sometimes with a picture illustrating them) right before the expression they modify* (a single word, unless the scope of the modification is marked by square brackets]).

- (2) a. John UP_  helped his son.
 b. John didn't UP_  help his son.
 c. Did John UP_  help his son?

¹ As summarized in Giorgolo 2010 (pp. 4-5), one prominent distinction is that between (i) 'emblems', which 'are typically culture specific gestures, associated with a fixed meaning' – for instance the 'thumb up'  gesture used in Western culture; (ii) 'pantomimes', which are 'usually sequences of movements that reproduce some kind of motor or physical activity, either by directly reenacting the described scene or by visualizing it through some form of iconic mapping'; and (iii) co-speech gestures, with which we are concerned in the present piece. Besides iconic gestures, the latter category also includes 'beats', which are 'short and simple movements that pattern quite closely with the prosodic peaks of the accompanying utterance'; deictic signs; and 'metaphorics', which 'spatially represent abstract entities'.

² This expression should be understood by analogy with the 'projection problem for presuppositions', which consists in determining how the presuppositions of complex sentences are inherited from the assertive and presuppositional contributions of their component parts.

As noted, one can get in (2)a the inference that John helped by some kind of upwards movement. Now in (2)b we infer that John didn't help his son, but that *if* he had this would have been through an upwards movement. As for the question in (2)c, it leaves open *whether* John helped his son, but suggests that *if* he did, it was by lifting him.

The intuition, then, is that the co-speech gesture further specifies the relevant action in case it was in fact performed. While in unembedded cases, one might want to posit that the relevant inference follows from the context of the conversation, we will see that in embedded cases a more sophisticated notion, that of a *local context*, is needed. This notion is standardly used in dynamic semantics (e.g. Heim 1983); informally, the local context of an expression recapitulates the semantic content already contributed by expressions that precede it. In many cases, we will take for granted the value of the local contexts. But in some cases we will base our discussion on the reconstruction of local contexts offered in Schlenker 2009, as it has the advantage of giving a *recipe* to compute the local context of an expression in any sentence once the bivalent (classical) behavior of the latter has been specified. In simple cases, nothing is lost in this way because Schlenker 2009 shows that, *modulo* some technical assumptions, standard results of dynamic semantics are derived by this procedure. But the algorithm developed in Schlenker 2009 will allow us to derive predictions for some non-standard cases (notably, embedding under factive expressions such as *be unaware that*, where we will need to compute the local context of the embedded clause).

Five notes might be useful at the outset.





- First, we are primarily concerned with *patterns of gesture projection*, defined as the way in which the gestural enrichments of elementary expressions are inherited by complex sentences. A separate issue is to determine which *epistemic status* these gestural enrichments have. The difference matters because in many cases co-speech gestures are informative. But by now there is a sizable literature on informative presuppositions, and hence the informative nature of co-speech gestures need not necessarily get in the way of a presuppositional account (e.g. Stalnaker 2002, von Stechow 2008, Schlenker 2012). Alternatively, one might take gestures to be evaluated with respect to what one might term the 'speaker's local context', obtained by computing local contexts not with respect to the standard Context Set (which corresponds to what is common belief among the speech act participants), but rather with respect to the speaker's belief state. We come back to this point below.
- Second, presuppositional phenomena notoriously give rise to patterns of local accommodation, whereby a presupposition essentially gets incorporated into the assertive component. This happens for instance in the sentence *John doesn't know that he is going to be hired – because he won't be!*: the factive inference triggered by *know* is in this case evaluated within the scope of negation. Gestural presuppositions also give rise to local accommodation, and this might explain why there is some amount of individual variation in the assessment of projection patterns – it might be that some speakers are more willing to apply local accommodation than others. We will focus most of our investigation on cases with gestural enrichments that are non-assertive; but it should be kept in mind that *modulo* local accommodation some assertive uses can be obtained as well.
- Third, as briefly mentioned above, the timing of co-speech gestures matters. For most of the discussion, we will focus on gestures that co-occur with certain constituents. But when similar gestures are uttered right *after* these constituents, and are thus 'post-speech gestures' (with their own timing slot) rather than 'co-speech gestures', they may give rise to different inferential patterns, rather reminiscent of appositive relative clauses – a claim made about all co-speech gestures by Ebert and Ebert 2014. We come back to this point in Section 4.2.2.
- Fourth, at this point we will use standard linguistic methodology in basing our generalizations on introspective judgments, both our own and those of other linguists we have consulted. There are obvious limitations to this method. While we are convinced that an experimental study ought to investigate patterns of gesture projection, we feel that it will be most useful if it can assess fine-grained predictions of well-developed theories. Our goal is to present such a theory in the present paper.
- Fifth, if our ideas are on the right track, they should in the end be combined with those of Lascarides and Stone 2009, who developed a framework in which gestures can be integrated into Logical Forms by way of explicit anaphoric and narrative relations. But since their emphasis was not on the projection problem *per se* (as they did not consider the interaction between co-speech gestures and logical operators³), we will leave the integration of these two frameworks for future research.

³ Stone and Lascarides do discuss the interaction between gestures and expressions that introduce discourse referents, in particular dynamic existential quantifiers. Their formal framework also contains standard logical

2 Patterns of Gesture Projection

2.1 The Problem

To introduce the projection problem for co-speech gestures, we start from a few examples without embedding, as in (3); in each case, we have explicitly written the inferences that we take to be licensed (preceded by: =>).

- (3) a. John brought LARGE_  [a bottle of beer].
=> John brought a bottle of beer, and it was large
- a'. John found LARGE_  [a bottle he liked]
=> John found a bottle he liked, and it was large
- b. John UP_  helped his son.
=> John helped his son by way of an upwards movement of his hands
- c. John SLAP_  punished his son.
=> John punished his son by slapping him

Now let us embed (3)a' under the quantifier *exactly one philosopher* so as to obtain a bound reading, as in (4)a. And we also consider a control sentence without a co-speech gesture, as in (4)b; and one in which the co-speech contribution has been made part of the assertive component with an explicit modifier, as in (4)c. We also included quantified versions of (3)b,c in (5).

- (4) a. Exactly one philosopher found LARGE [a bottle he liked].
=> exactly one philosopher found a bottle he liked, and that bottle was large.
b. Exactly one philosopher found a large bottle he liked.
c. Exactly one philosopher found a bottle like LARGE this that he liked.
- (5) a. Exactly one of these 10 guys UP helped his son.
=> exactly one of these 10 guys helped his son, and he did so by pushing him upwards
b. Exactly one of these 10 guys SLAP punished his son.
=> exactly one of these 10 guys punished his son, and he did so by slapping him

With the co-speech gesture in (4)a, the inference we obtain is that (i) a philosopher found a bottle he liked (with bound *he*), (ii) no other philosopher found a bottle *he* liked, and (iii) the bottle the first philosopher found was large. (i) and (ii) taken together are just the literal meaning of the sentence. What is interesting is that initially (iii) seems to only modify the positive part of the literal meaning, not its negative part. Importantly, the example crucially involves a bound variable, and we must thus explain how the gestural inference interacts with the compositional semantics of the sentence. This is of course also the case of the control sentences in (4)b and (4)c. But in the latter two examples, the modifier restricts the extension of the relevant Noun Phrase, and yields a meaning that does *not* entail that of the sentence without the modifier (this is because *exactly one philosopher* creates a non-monotonic environment). For instance, *Exactly one philosopher found a large bottle he liked* does not entail *Exactly one philosopher found a bottle he liked*: it could be that exactly one philosopher found a large bottle he liked, while another philosopher found a *small* bottle he liked, in which case the first sentence would be true but the second would be false. By contrast, we believe that (4)a can naturally be read in a way that implies that exactly one philosopher found a bottle he liked – with the additional implication that this bottle was large. We believe that analogous generalizations hold of (5). The challenge is to explain why this is. (As noted at the outset, we do not claim that the co-speech gesture in (4)a does not *also* allow for a reading that restricts the extension of the Noun Phrase. For us, this will be due to the possibility of applying local accommodation to the relevant presupposition.)

To know where we are going, it will be useful to state at the outset some simplified generalizations (the analysis we develop in Section 3 will make more precise and rigorous predictions, and thus this informal statement is just included to guide intuition).

connectives, but the examples they discuss do not involve embedding under any of the operators discussed in this piece; as a result, the issue of the assertive, presuppositional or supplemental status of co-speech enrichments does not arise in their article.

(6) **Initial generalizations [for simple cases]**

- a. [Setting aside local accommodation] Co-speech gestures usually strengthen the meaning of utterances.
 b. If a co-speech gesture G modifies an n -ary predicate p in a sentence $\dots p \dots$, then:

(i) if the sentence $\dots p \dots$ entails that at least one n -tuple of objects satisfies p , then one infers that those tuples satisfy the content of G ;

(ii) if the sentence $\dots p \dots$ fails to entail that at least one n -tuple of object satisfies p , then one infers that if there were such (relevant) tuples, they would satisfy the content of G .

To illustrate, (4) *Exactly one of these 10 guys punished his son* entails that there is one pair $\langle x, x$'s son \rangle that satisfies the predicate *punish*. This case falls under (6)b(i), and we infer that the pair that satisfies *punish* also satisfies the content of *SLAP* – and thus that the punishment involved some slapping.

We now turn to propositional, quantificational and attitudinal examples to establish and refine these generalizations.

2.2 Propositional examples

In questions and under negation, the co-speech gestures under consideration here trigger conditional inferences, as is illustrated in (7)-(8).

- (7) a. John won't bring LARGE [a bottle of beer].
 => if John brought a bottle of beer, it would be a large one
 b. John won't UP help his son.
 => If John helped his son, he would do so by pushing him upwards
 c. John won't SLAP punish his son.
 => If John punished his son, he would do so by slapping him

- (8) a. Will John bring LARGE [a bottle of beer]?
 => if John brings a bottle of beer, it will be a large one
 b. Will John UP help his son?
 => If John helps his son, he will do so by pushing him upwards
 c. Will John SLAP punish his son?
 => If John punishes his son, he will do so by slapping him

Both cases fall under (6)b(ii). To illustrate, we note that *John' won't punish his son* and *Will John punish his son* fail to entail that there is at least one pair $\langle x, y \rangle$ that satisfies *punish*, and as stated in (6)b(ii) we infer that *if* there were a pair $\langle x, y \rangle$ satisfying *punish*, it would satisfy the content of *SLAP* and would thus involve some slapping.

To study projection in upward-monotonic environments, it is useful to embed some of the examples in questions so as to distinguish the presuppositional component (which is preserved or 'projects' under questions) from the assertive component (which doesn't project). In (9)a, we obtain a (doubly) conditional inference: if John comes to our workshop, then if he brings a bottle of beer, it will be a large one. (These complex cases do not work well to illustrate the initial generalizations in (6), but they will follow from the analysis in Section 3.)

- (9) a. If John comes to our workshop, will he bring LARGE [a bottle of beer]?
 => if John comes to our workshop, if he brings a bottle of beer, it will be a large one
 b. If John shows up in a minute, will he UP help his son?
 => if John shows up in a minute, if he helps his son, he will do so by pushing him upwards
 c. If John shows up in a minute, will he SLAP punish his son?
 => if John shows up in a minute, if he punishes his son, he will do so by slapping him

In the antecedent of conditionals (a non-upward-monotonic environment – which obviates the need for the question test), we seem to get the same inferences as under negation and in questions, possibly in weakened form:

- (10) a. If John comes to our workshop and brings LARGE [a bottle of beer], people will talk about it.
 => if John comes to our workshop, if he brings a bottle of beer, it will be a large one
 b. If John shows up in a minute and UP helps his son, people will talk about it.
 => if John shows up in a minute, if he helps his son, he will do so by pushing him upwards
 c. If John shows up in a minute and SLAP punishes his son, people will talk about it.
 => if John shows up in a minute, if he punishes his son, he will do so by slapping him

Finally, under existential modals (which create an upward-monotonic environment), the facts are subtle and possibly controversial; as things stand, we believe that a weak inference is triggered, and we believe that it is preserved in questions, as is illustrated in (11)-(12).

- (11) a. John might bring LARGE [a bottle of beer].
=>? if John brings a bottle of beer, it will be a large one
b. John might UP help his son.
=>? if John helps his son, he'll do so by pushing him upwards
c. John might SLAP punish his son.
=>? if John punishes his son, he'll do so by slapping him
- (12) a. Could John bring LARGE [a bottle of beer]?
=>? if John brings a bottle of beer, it will be a large one
b. Could John UP help his son?
=>? if John helps his son, he will do so by pushing her upwards
c. Could John SLAP punish his son?
=>? if John punishes his son, he will do so by slapping him

2.3 Quantified examples

We turn to quantified examples, and as before place them in questions when this is needed to distinguish between presupposition and assertion. We start with cases involving a quantificational adverb, which has the advantage of making for particularly minimal pairs with the propositional examples discussed before (here too, the initial generalizations in (6) would be refined to be applicable, but the analysis of Section 3 will derive the desired results).

- (13) a. On Mondays, did John always bring LARGE [a bottle of beer]?
=> on Mondays, when John brought a bottle of beer, it was a large one
b. On Mondays, did John always UP help his son?
=> on Mondays, when John helped his son, he did so by pushing him upwards
c. On Mondays, did John always SLAP punish his son?
=> on Mondays, when John punished his son, he did so by slapping him

Similar generalizations hold of nominal quantifiers:

- (14) a. Did each of these 5 philosophers find LARGE [a bottle he liked]?
=> for each of these 5 philosophers, if he likes a bottle, it's a large one
b. Did each of these 10 guys UP help his son?
=> for each of these 10 guys, if he helped his son, he did so by pushing him upwards
c. Did each of these 10 guys SLAP punish his son?
=> for each of these 10 guys, if he punished his son, he did so by slapping him

As we noted at the outset, embedding under *exactly n* numerals is particularly informative, as these have both a positive and a negative component, which are differentially affected by gestural enrichments.

- (15) a. On exactly four occasions last month, John brought LARGE [a bottle of beer] to the seminar.
=> on exactly four occasions last month, John brought a bottle of beer to the seminar, and in each case it was a large bottle
b. On exactly four occasions last month, John UP helped his son.
=> on exactly four occasions last month, John helped his son, and in each case he did so by pushing him upwards.
c. On exactly four occasions last month, John SLAP punished his son.
=> on exactly four occasions last month, John punished his son, and in each case he did so by slapping him
- (16) a. Exactly one of these 5 philosophers found LARGE [a bottle he liked].
=> exactly one of these 5 philosophers found a bottle he liked, and the bottle he found was large
b. Exactly one of these 10 guys UP helped his son.
=> exactly one of these 10 guys helped his son, and he did so by pushing him upwards
c. Exactly one of these 10 guys SLAP punished his son.
=> exactly one of these 10 guys punished his son, and he did so by pulling slapping him

As was mentioned at the outset, such examples illustrate a part of the initial generalizations in (6), namely (6)b(i): *Exactly one of these 10 guys helped his son* entails that there is exactly one pair $\langle x, x\text{'s son} \rangle$ that satisfies *help*, and the co-speech gesture triggers the inference that slapping was involved in that pair's interaction.

Under negative quantifiers such as *never* or *none of these 5 philosophers*, co-speech gestures give rise to conditional inferences, as is expected given our initial generalization in (6)b(ii).

- (17) a. On Mondays, John never brought LARGE [a bottle of beer].
=> On Mondays, John never brought a bottle of beer, but if he had brought one, it would have been a large bottle
b. On Mondays, John never UP helped his son.
=> On Mondays, John never helped his son, but if had done so, it would have been by pushing him upwards
c. On Mondays, John never SLAP punished his son.
=> On Mondays, John never punished his son, but if had done so, it would have been by slapping him
- (18) a. None of these 5 philosophers found LARGE [a bottle he liked].
=> none of these 5 philosophers found a bottle he liked; but for each of them, if he had found a bottle he liked, it would have been a large one
b. None of these 10 guys UP helped his son.
=> none of these 10 guys helped his son; but for each of them, if he had helped his son, it would have been by pushing him upwards
c. None of these 10 guys SLAP punished his son.
=> none of these 10 guys punished his son; but for each of them, if he had punished his son, it would have been by slapping him

We note that our initial generalization in (6)b(ii) is not precise enough to do justice to the facts. Consider for instance (18)c. *None of these 10 guys punished his son* fails to entail that at least one pair $\langle x, y \rangle$ satisfies *help*, and thus our statement of (6)b(ii) would lead us to expect a broad inference to the effect that if there were pairs $\langle x, y \rangle$ satisfying *help*, their interaction would involve some slapping. But of course the inference we obtain is much more precise: it is only for the 10 guys involved, and their sons, that this counterfactual inference holds. The correct prediction will be made by the analysis of Section 3, but it will hinge on the details of a presuppositional analysis based on local contexts.

For future reference, we note that the non-monotonic quantifier *between 3 and 5* gives rise to the inferences we expect in view of the generalization in (6)b(i).

- (19) a. Between 3 and 5 of these 10 philosophers found LARGE [a bottle they liked].
=> Between 3 and 5 of these 10 philosophers found a bottle they liked, and in each case it was a large bottle
b. Between 3 and 5 of these guys UP helped their son.
=> Between 3 and 5 of these guys helped their son, and in each case it was by pushing him upwards
c. Between 3 and 5 of these guys SLAP punished their son.
=> Between 3 and 5 of these guys punished their son, and in each case it was by slapping him

2.4 Attitudinal examples

We turn to gesture projection under attitude verbs. Since in possible worlds semantics *x believes that F* is given an analysis in terms of universal quantification over worlds compatible with what *x* believes, we expect to find the same kind of patterns as under universal quantifiers. We note that projection patterns are harder to assess in questions, as in (21); we leave a closer investigation of the facts for future research.

- (20) a. Sam believes that John brought LARGE [a bottle of beer].
=> Sam believes that John brought a large bottle of beer
b. Sam believes that John UP helped his son.
=> Sam believes that John helped his son and did so by pushing him upwards
c. Sam believes that John SLAP punished his son.
=> Sam believes that John punished his son and did so by pulling him upwards
- (21) a. Does Sam believe that John brought LARGE [a bottle of beer]?
=>?? Sam/the speaker believes that if John brought a bottle of beer, it was a large one

- b. Does Sam believe that John UP helped his son?
=>?? Sam/the speaker believes that if John helped his son, it was by pushing him upwards
- c. Does Sam believe that John SLAP punished his son?
=>?? Sam/the speaker believes that if John punished his son, it was by slapping him.

More striking are the patterns we find under *be unaware that. x is unaware that F* is a particularly interesting construction because it has a negative component – *x doesn't believe that F* – and also a positive one, which is presupposed – namely that *F* is in fact the case.⁴ In accordance with generalization (6)(ii)a, the gestural inference makes itself felt in the positive but not (or not clearly) in the negative component of the meaning, as is illustrated in (22)-(23).

- (22) a. Sam is unaware that John brought LARGE [a bottle of beer].
=> Sam doesn't believe that John brought a bottle of beer
=> John brought a large bottle of beer
- b. Sam is unaware that John UP helped his son.
=> Sam doesn't believe that John helped his son
=> John helped his son by pushing him upwards
- c. Sam is unaware that John SLAP punished his son.
=> Sam doesn't believe that John punished his son
=> John punished his son by lapping him
- (23) a. Is Sam unaware that John brought LARGE [a bottle of beer]?
=> John brought a large bottle of beer
- b. Is Sam unaware that John UP helped his son?
=> John helped his son by pushing him upwards
- c. Is Sam unaware that John SALP punished his son?
=> John punished his son by slapping him

Similar data can be obtained with *x doesn't realize that F* as with *x is unaware that F*, as can be seen in (24).

- (24) a. Sam doesn't realize that John brought LARGE [a bottle of beer].
=> Sam doesn't believe that John brought a bottle of beer
=> John brought a large bottle of beer
- b. Sam doesn't realize that John UP helped his son.
=> Sam doesn't believe that John helped his son
=> John helped his son by pushing him upwards
- c. Sam doesn't realize that John SLAP punished his son.
=> Sam doesn't believe that John punished his son
=> John punished his son by slapping him

2.5 Facial Expressions

We believe that some of the same generalizations hold of some co-speech facial expressions. The advantage of facial expressions is that they seem rather natural over long stretches over which it would be unnatural to produce a single co-speech gesture. In simple cases, we replicate the inferences obtained with co-speech gestures. Here :- (stands for an unhappy or disgusted face, which we illustrate in (25)a.

(25) Propositional examples



- a. Sam went :- ([skiing with his parents].
=> for Sam to go skiing with Sam's parents wasn't fun
- b. Sam won't go :- ([skiing with his parents].
=> for Sam to go skiing with his parents wouldn't be fun
- c. Did Sam go :- ([skiing with his parents]?
=> for Sam to go skiing with his parents wouldn't have been fun
- d. If Sam goes :- ([skiing with his parents], I'll hear about it.
=> for Sam to go skiing with his parents wouldn't be fun

⁴ See Spector and Sudo, to appear, for other theoretical uses of this construction (in the analysis of the interaction between implicatures and presuppositions).

In more complex examples, we can see the effect of a facial modifier co-occurring with an expression that contains a bound variable.

(26) **Quantificational examples**

- a. Each of my friends goes :-([skiing with his parents].
=> for each of my friends, skiing with his parents isn't fun
- b. Does each of your friends go :-([skiing with his parents]?
=> for each of my friends, skiing with his parents wouldn't be fun
- c. None of my friends goes :-([skiing with his parents]
=> for each of my friends, skiing with his parents wouldn't be fun
- d. Exactly four of my friends went :-([skiing with their parents]
=> exactly four of my friends went skiing with their parents, and
 - (i) for each of these four friends, skiing with his parents wasn't fun, or possibly
 - (ii) for my friends in general, skiing with their parents isn't fun.
- d. An odd number of of my friends went :-([skiing with their parents]
=> an odd number of my friends went skiing with their parents, and
 - (i) for each of these friends, skiing with his parents wasn't fun, or possibly
 - (ii) for my friends in general, skiing with their parents isn't fun.

As mentioned at the outset, timing matters. Thus we believe that when a disgusted or sad facial expression *follows* a quantified statement, as in (27), one can obtain very different readings – similar to ones that could be obtained with an appositive relative clause modifying an entire proposition (as in (27)(i)) or a VP (as in (27)(ii)).

- (27) None of my friends goes skiing with his parents – :-(.
=> None of my friends goes skiing with his parents,
(i) which is sad [i.e. it is sad that none of my friends goes skiing with his parents];
(ii) which is unpleasant [i.e. it is generally unpleasant to go skiing with one's parents].

We come back to appositive readings of co-speech gestures and facial expressions in Section 4.2.2.

Additional complexities are involved with facial expressions in attitude environments. It seems to us that their content may be attributed to the agent, or maybe to the speaker, and possibly even to both, as illustrated in (28); we leave this issue for future research.⁵

- (28) John thinks that his son goes :-([skiing with his parents].
=> the speaker and possibly John finds that for his son to go with her parents wouldn't be fun

3 A Cosuppositional Analysis

3.1 Main idea

Let us turn to the analysis. The main intuition is straightforward: we take a co-speech gesture to come with a requirement that the property it contributes should follow from the contextual meaning of the constituent it co-occurs with. In order to account for the interaction of gestural enrichments with the compositional semantics of the sentences they interact with, we take the relevant notion of 'contextual meaning' to be: meaning *relative to the local context* of the expression. Dynamic semantics has offered an articulated account of local contexts and dynamic updates. But because that account is intrinsically lexicalist (the update behavior of connectives and operators is stipulated on a case-by-case basis), we will occasionally follow the framework of Schlenker 2009, which provides a general recipe to compute local contexts once the bivalent semantics and syntax of a sentence have been specified.

Our analysis can be stated in presuppositional terms, as in (29): a co-speech gesture triggers a presupposition that its content is entailed by that of the expression it modifies.

(29) **Cosuppositions triggered by co-speech gestures**

Let G a co-speech gesture co-occurring with an expression d , and let g be the content of G . Then G

⁵ See Schlenker to appear b, 2015a for a discussion of happy faces in attitude reports in sign language (in that piece, the focus is on happy faces co-occurring with 'role-shifted clauses', which are context-shifting constructions. But the controls involve happy faces on standard indirect discourse, which is comparable to English indirect discourse.

triggers a presupposition $d \Rightarrow g$, where \Rightarrow is generalized entailment (among expressions whose type 'ends in t').⁶

To put things differently, co-speech gestures trigger presuppositions of a particular sort, namely ones that are conditionalized on the assertive content of the expression they modify, as specified in (30).

(30) **Cosuppositions as conditionalized presuppositions**

An expression E triggers a cosupposition if E triggers a presupposition of the form $a \Rightarrow e$, where a is the assertive content of an expression E co-occurs with.

It should be kept in mind that cosuppositions are presuppositions of a particular sort, and thus that we can rely on the theory of presupposition to make predictions about cosuppositions. The general insight and is that the presupposition triggered an expression must be entailed by the 'local context' of that expression (this insight is incorporated in the dynamic update rules posited by Heim 1983; and it plays a direct role in the reconstruction of local contexts of Schlenker 2009). The general case is stated in (31)a, and the special case of presuppositions in (31)b.

(31) **Presupposition Satisfaction**

Let E be an expression with an assertive component a and a presupposition π occurring in a sentence ... E ... uttered relative to a Context Set C .

a. General case

Then E is licensed in ... E ... relative to C only in case its local context given C entails π .

b. Special case of cosuppositions

Suppose that π is of the form $a \Rightarrow e$. Then E is licensed in ... E ... relative to C only in case its local context entails $a \Rightarrow e$.

Notation: in simple cases, local contexts may be of propositional or predicative type. If lc is a local context and F is a formula of the appropriate type, we write $lc \models F$ in case lc entails F by generalized entailment.⁷

Let us immediately illustrate the main intuition with a particularly simple example, involving an expression of disgust co-occurring with the consequent of a conditional.

(32) Context: it is hot and humid.

a. If you go running, :- (you'll sweat).

b. if r , G s

We analyze this sentence as having the Logical Form in (32)b, where r stands for *you go running*, s for *you'll sweat*, and G for the speaker's disgusted expression. We now apply the satisfaction condition in (31) to (32)b.

• First, we need to compute the local context of s in *if r , s* given C ; we call this local context $lc(s)$. Standard dynamic analyses as well as the reconstruction of local contexts of Schlenker 2009 take $lc(s)$ to be the set of worlds in C that also satisfy r , as is written in (33).

(33) Local context of s in (32)b given a Context Set C

$lc(s) = \lambda w_s$ w is in C and w satisfies r

• Second, the licensing condition in (31)b specifies that relative to $lc(s)$, s should entail the content g of G . Since in this case G is a disgusted facial expression, its content is something like: 'disgust is licensed'. We obtain the result in (34).

(34) **Licensing condition on G in (32)b given a Context Set C**

a. $lc(s)$ as computed in (33) should guarantee that $lc(s) \models s \Rightarrow g$, where g is the content of G

b. $[\lambda w_s$ w is in C and w satisfies r] $\models [s \Rightarrow g]$

⁶ If x and x' are two objects of a type τ that 'ends in t', and can take at most n arguments, $x \Rightarrow x'$ just in case whenever y_1, \dots, y_n are objects of the appropriate type, if $x(y_1) \dots (y_n) = 1$, then $x'(y_1) \dots (y_n) = 1$

⁷ More precisely, we can define the relevant notion of generalized entailment (between a set-theoretic object and formula) as follows:

If o is a type-theoretic object of type t or such that, for some objects x_1, \dots, x_n of types τ_1, \dots, τ_n , $o(x_1) \dots (\tau_n)$ is of type t , and if F is a formula of the same type as o with meaning \mathbf{F} , then $o \models F$ just in case for all objects x_1, \dots, x_n of types τ_1, \dots, τ_n , if $o(x_1) \dots (\tau_n) = 1$, then $\mathbf{F}(x_1) \dots (\tau_n) = 1$.

and thus we get the result that every world w in C that satisfies r should satisfy $s \Rightarrow g$.
 In words, every world in C in which the addressee goes running guarantees in one in which the addressee's sweating would be disgusting

This seems appropriate: we obtain an inference that, given the hot and humid weather, if the addressee runs, then sweating will be disgusting.

Having illustrated the licensing condition in (31)b, it is worth noting that it is equivalent to a treatment of the co-speech gesture as a separate and purely presuppositional conjunct *following* the expression it modifies. The reason is that in a sentence of the form ... [a & e] ..., where the second embedded conjunct e is purely presuppositional, standard rules of presupposition projection require that, relative to its local context, a should materially entail e – which is precisely the result we obtain for a sentences ... E ... where E contributes an assertive component a and a cosupposition $a \Rightarrow e$. This small result is stated more precisely in (35), with a brief argument within dynamic semantics (an argument within the framework of Schlenker 2009 is given in Appendix I).

(35) Two equivalent conditions

Let E be an expression with an assertive component a and a presupposition π occurring in a sentence ... E ... uttered relative to a Context Set C . Suppose that π is of the form $a \Rightarrow e$, where a is the assertive content of E .

a. Licensing condition on cosuppositions, repeated from (31)b

Then E is licensed in ... E ... relative to C only in case its local context entails $a \Rightarrow e$.

b. Equivalent condition

Then E is licensed in ... E ... relative to C only in case relative to C a & e is licensed in the sentence ... a & e ... obtained by replacing E with a & e .

c. Brief reasoning within dynamic semantics (propositional case only)⁸

Let C' be any Context Set. Writing $C'[\bullet]$ for the update of C' with a formula \bullet , we have:

$C'[a \& e] = C'[a][e]$

$= \#$ iff $C' = \#$ or some world w in C' satisfies a and doesn't satisfy e ;

$= C'[a]$ otherwise

$C'[E] = \#$ iff $C' = \#$ or some world in C' doesn't satisfy $a \Rightarrow e$, iff $C' = \#$ or some world in C' satisfies a and doesn't satisfy e ;

$= C'[a]$ otherwise

It is clear that the two conditions are equivalent.

Finally, we must make standard provisions for local accommodation of the presupposition (Heim 1983). The condition is standard; one version is defined in (36)a for the general case, and applied in (36)b to the case of cosuppositions (in either one of versions defined in (35)).

(36) Local Accommodation

a. General Case

A presupposition which is 'locally accommodated' is treated as part of the assertive component of the expression it belongs to.

b. Application to cosuppositions

A cosuppositional expression with assertive component a and presupposition $a \Rightarrow e$ acquires an assertive component (a & ($a \Rightarrow e$)), i.e. a & e , after local accommodation (the same result is obtained for the 'official' condition in (35)a and for the equivalent condition in (35)b: it is immediate in the latter case that a & e gets locally accommodated to a & e).

In the case of gestural enrichments, the effect of local accommodation will simply be to turn the gestural enrichment into part of the assertive component.

3.2 Simple cases

We turn to a treatment of a few sample cases. As before, co-speech gestures appear in capital letters, e.g. S (for *SLAP*); and their content appears in normal letters, e.g. s . We write the global context set as C . In each case, the a. (a', a'') examples include an English sentence with co-speech gestures, and a simplified representation of its Logical Form; b. gives the licensing condition and the inferences that can be drawn from it. When needed, we write as $|F|$ the semantic value of an expression F .

⁸ The result carries over to the framework of Schlenker 2009, which is equivalent in the propositional case (and near-equivalent in the quantificational case).

3.2.1 Propositional examples

We start with simple propositional examples. In (37), we consider cases in which the local context of the enriched expression is identical to the global context of the conversation.⁹ In case the sentence is a question, as in (37)a, we obtain the desired cosupposition: if John punished his son, this involved some slapping, or with our notation: $C \models p \Rightarrow s$. If instead of a question we have an assertion, as in (37)a', it interacts with the cosupposition to yield the inference that the content of the gesture was in fact instantiated – in our example, this is the inference that John in fact punished his son by slapping him.

- (37) a. John SLAP punished his son.
 a'. Did John SLAP punish his son?
 a". Sp (?)
 b. Licensing condition:
 $lc(Sp) \models p \Rightarrow s$
 hence
 $C \models p \Rightarrow s$
 and when a is asserted, we thus infer: p and s.

In (38), we obtain a further relativization to the antecedent of the conditional, as is desired as well. And in (39), the same cosupposition is obtained as in (37), simply because negation does not affect the computation of local contexts.

- (38) a. If John is present at the right time, will he SLAP punish his son?
 a'. if r, Sp?
 b. Licensing condition:
 $lc(Sp) \models p \Rightarrow s$
 hence
 $C \models r \Rightarrow (p \Rightarrow s)$
 hence an inference (which survives under questions) that
 if John is present at the right time, then if he punishes his son, he will do so by slapping him.

- (39) a. John didn't SLAP punish his son.
 a'. not Sp
 b. Licensing condition:
 $lc(Sp) \models p \Rightarrow s$
 But in a'. $lc(Sp) = C$, the global context,
 hence
 $C \models p \Rightarrow s$
 In the end, we have two inferences:
 relative to the Context Set, if John punishes his son, he does so by slapping him;
 relative to the (more specific) speaker's beliefs, John didn't punish his son.

3.2.2 Quantified examples

Let us turn to quantified examples.

□ Basic results

As desired, we derive a universal conditional presupposition in the case of embedding under *none of these 10 guys*. The key is that this quantifier gives rise to universal presuppositions – for instance, *None of these 10 guys takes good care of his computer* presupposes that *each of these 10 guys has a computer*, a result confirmed with experimental means in Chemla 2009. On a theoretical level, Heim 1983 incorporates this pattern of 'universal projection' in her analysis of all generalized quantifiers; and Schlenker 2009 shows that the local context of the Verb Phrase in such cases is the property of being one of these 10 guys (relative to the Context Set) – which derives the very same result (since the property of being one of these 10 guys must entail the relevant presupposition).

- (40) a. None of these 10 guys SLAP punished his son.
 a'. [No g] Sp
 b. Licensing condition:

⁹ This is true to the extent that *John punished his son* is treated as an atomic propositional expression. In a less simplified system, *punished his son* would have a predicative meaning enriched by the co-speech gesture, and the enriched predicative meaning would then be applied to the subject.

$lc(Sp) \models p \Rightarrow s$

Given standard results about the local contexts of the restrictor in quantified statements, this derives the following result:

$C \models \forall x (g(x) \Rightarrow (p(x) \Rightarrow s(x)))$

In the end, we have two inferences:

relative to the Context Set, if any of the 10 guys punishes his son, he does so by slapping him;

relative to the (more specific) speaker's beliefs, None of these 10 guys punished his son.

In Heim's analysis of presupposition projection (Heim 1983), all generalized quantifiers trigger universal presuppositions in their nuclear scope. This result is derived, *modulo* some technical assumptions, in the reconstruction of local contexts developed in Schlenker 2009 (see Appendix I for a summary). This makes it possible to derive the desired inferences about the 'positive' part of the sentences in (41)-(42). The key is that in each case we derive a universal presupposition that for each of the relevant guys, if he punished his son, he did so by slapping him. This universal conditional presupposition then interacts with the assertive component to yield the inference that those guys that did in fact punish their sons did so by slapping them.

- (41) a. Exactly one of these 10 guys SLAP punished his son.

a'. $[=1 g] Sp$

b. Licensing condition:

$lc(Sp) \models p \Rightarrow s$

Given standard results about the local contexts of the restrictor in quantified statements, this derives the following result:

$C \models \forall x (g(x) \Rightarrow (p(x) \Rightarrow s(x)))$

In the end, we have two inferences:

relative to the Context Set, if any of the 10 guys punishes his son, he does so by slapping him.

relative to the (more specific) speaker's beliefs, exactly one of the 10 guys punished his son; by the first inference, he did so by slapping him.

- (42) a. Between 3 and 5 of these 10 guys SLAP punished their sons.

a'. $[3 \leq \bullet \leq 5 g] Sp$

b. Licensing condition:

$lc(Sp) \leq p \Rightarrow s$

Given standard results about the local contexts of the restrictor in quantified statements, this derives the following result:

$C \models [\forall x: g(x)](p(x) \Rightarrow s(x))$

In the end, we have two inferences:

relative to the Context Set, if any of the 10 guys punished his son, he did so by slapping him;

relative to the (more specific) speaker's beliefs, between 3 and 5 of these 10 guys punished their sons; by the first inference, they did so by slapping them.

□ *A problem and some potential solutions*

Still, in addition to the desirable inferences we derive, we also predict a universal conditional inference which is more dubious, namely that it is presupposed that *for each of these 10 guys, if he punishes his son, he will do so by slapping him*. While this inference seems appropriate when the target sentence is *None of these 10 guys will SLAP his son*, it is more dubious in (41) and (42). If this empirical difference is confirmed, various strategies could be explored to solve it.¹⁰

¹⁰ We are not certain of the data. In (i)a, the universal inference (= *for each of these 10 guys, if he had punished his son, he would have done so by slapping him*) might be too strong. But in (i)b, we do get the inference that each of the guys that *did* slap his son did so by slapping him.

- (i) a. Exactly one of these 10 guys SLAP punished his son.
 b. Exactly one of these 10 guys didn't SLAP punish his son.
 c. Maybe exactly one of the 10 guys didn't SLAP punish his son.
 d. Did exactly one of the 10 guys not SLAP punish his son?

Now the observed inference in (i)b could be derived simply on the basis of the assertive component of (i)b, on the assumption that the presupposition is *also* part of the assertive component (see Sudo 2012, 2014 for a discussion of such a hypothesis): from *exactly one of these 10 guys didn't punish his son by slapping him*, we can infer that *each of the other nine guys punished his son by slapping him*. In order to observe the specific contribution of the presupposition, we must thus consider cases such as (i)c,d, where the inference that *each of*

(i) First, this difficulty might be related to an independent problem with the theory of presupposition projection: as shown with experimental means by Chemla 2009, universal presupposition projection is much stronger under *No* than under modified numerals. This does not solve our problem but might reduce it to an independent one.

(ii) Second, one could posit that gestural cosuppositions are easily accommodated globally, in the sense that they can be assessed relative to the speaker's beliefs rather than to the Common Ground of the conversation – unless doing so makes the co-speech gesture vacuous (in the sense that it leads to the very same inferences as the sentence without the co-speech gesture). This (43) would make the gestural contribution vacuous in the case of embedding under *No*.

(43) **Epistemic status** (tentative)

a. A gestural cosupposition can be assessed with respect to the speaker's beliefs (rather than with respect to the Common Ground)...

b. ... unless doing so would make the co-speech gesture vacuous, in the sense that the sentence with the co-speech gesture triggers the very same inferences as the sentence without it (if so, the gestural cosupposition must be evaluated with respect to the Common Ground).

Let us see how our proposed modification could solve our problem.

- Since the assertive component of (40)a is that none of the relevant individuals punished his son, the additional conditional inference that *for each of these guys, if he punished he son, he did so by slapping him* is vacuous when evaluated with respect to the speaker's belief state. The reason is that the conditional which is derived is a material implication evaluated with respect to the speaker's belief state B (formally: $B \models \forall x (g(x) \Rightarrow (p(x) \Rightarrow s(x)))$). It is clear that if the speaker believes that none of the relevant individuals punished his son, the (rightmost) material implication will be vacuously satisfied (since $B \models \forall x (g(x) \Rightarrow \text{not } p(x))$). Thus the strategy outlined in (43)a fails because it makes the gestural contribution vacuous, so one must resort to the strategy in (43)b, which yields a stronger conditional presupposition – one that must hold throughout the Context Set C rather than just throughout the set of worlds B compatible with what the speaker believes.

- Things are different in the case of quantifiers that have a positive component, for instance those in (41) and (42): in such cases, the universal conditional presupposition yields inferences that the gesture-less sentence wouldn't. As a result, we can assess (41) and (42) relative to the beliefs B of the speaker, yielding the (assertive-like) inference that *those guys who punished their sons did so by slapping them*.

(iii) Third, one could posit that in all cases gestural presuppositions are evaluated with respect to the speaker's belief state, but that when the result is to make the gestural contribution vacuous, the rightmost material implication in formulas such as $\forall x (g(x) \Rightarrow (p(x) \Rightarrow s(x)))$ is reinterpreted as a stronger conditional – e.g. a counterfactual conditional. This would yield for (40)a an inference that for each of the relevant individuals, *if he had punished his son, he would have done so by slapping him*. This seems adequate as well, although it is not trivial to see how this stronger conditional interpretation could be derived in the case at hand.

For present purposes, the solution in (ii) might be preferable, as it relies on principles that are independently plausible: first, the fact that global accommodation might be 'easy'; second, the fact that one probably doesn't want to use a co-speech gesture that won't have any inferential effect whatsoever.

3.2.3 Simple attitudinal examples

Let us turn to co-speech gestures that are embedded within belief reports. Since belief reports are standardly analyzed in terms of universal quantification over possible worlds, we obtain the same kinds of predictions as we did under universal quantifiers. The simplest case involves embedding of a co-speech gesture under *believe*, as in (44); the prediction seems roughly accurate.

(44) a. Sam believes that John SLAP punished his son.

a'. $\text{Bel}_s \text{ Sp}$

b. Licensing condition:

$C \models \text{Bel}_s p \Rightarrow s$

the guys who punished his son did so by slapping him is not licensed by the assertive component. Unfortunately, judgments are subtle in such cases.

Heim 1983, 1992 and Schlenker 2009 derive the result that for a presupposition trigger qq' with presupposition p , $Bel_s qq'$ presupposes $Bel_s q$. In a' , the presupposition is of the form $p \Rightarrow s$, hence the result.¹¹

The assertive component yields $Bel_s p$, hence also an inference that $Bel_s s$: Sam believes that John punished his son and slapped him.

The data are more subtle and harder to assess in the negative example in (45). The predicted cosupposition is the same as in (44), but given the negative nature of the assertive component, the overall inferences obtained are of course different.

(45) a. Sam doesn't believe that John SLAP punished his son.

a'. not $Bel_s Sp$

b. Licensing condition:

$C \models Bel_s p \Rightarrow s$

The presuppositional result is the same as in (44), but the assertive component is negative and thus we do not obtain the inference that Sam believes John did in fact punish his son and slapped him.

3.3 The case of be unaware and not realize

Let us turn to the case of *be unaware* and *not realize*, which yield the interesting inferences we discussed in Section 2.4. While one is typically interested in the local context of the entire constituent [*be unaware that F*] or [*realize that F*] (as these are presupposition triggers), we need to compute the local context of the embedded clause F in order to be able to assess the effects of a gesture that co-occurs with it. Dynamic accounts do not provide a straightforward (or a predictive) way to assess what the value of that local context is, and thus it is particularly useful to resort to the reconstruction of local contexts in Schlenker 2009, 2010.

By way of motivation, let us explain what would go wrong if we simply assumed that the presupposition of the embedded clause is treated as part of the assertive component of the latter, so that *John SLAP punished his son* means $p \ \& \ (p \Rightarrow s)$: *John punished his son, and if he punished him, he did so by slapping him*. This is immediately equivalent to: $p \ \& \ s$, i.e. *John punished his son and slapped him*. Under *be aware*, *be unaware*, *realize* and *not realize*, both components become presupposed, which is intuitively right. But under *be unaware* and *not realize*, the assertive component of the sentences is now too weak: we infer that *Sam doesn't believe that John punished his son by slapping him*, whereas the correct result is arguably that *Sam doesn't believe that John punished his son*. We will now show that this stronger result does follow from the mechanism of local context computation of Schlenker 2009 (though we might also obtain a result which is in some respects too strong).

We start with the case of *be aware* and *be unaware*, where we can assume that the bivalent meaning of a formula $Unaware_s F$ is in essence that (i) F is true, and (ii) individual s doesn't believe that F ; and that the bivalent meaning of a formula $Aware_s F$ is that (i) F is true, and (ii) individual s believes that F . With these assumptions, it can be shown that, *modulo* a technical assumption, the local context of F in both constructions is the function that assigns to each world w^* the null set if w^* is not in the Context Set C , and otherwise the set of worlds compatible with what individual s believes in w^* (as was the case for the local context of a clause embedded under *believe*), augmented with the world w^* itself (due to the factive entailment of *be unaware*). In technical terms: $lc(F) = \lambda w^* \lambda w (w^* \in C \text{ and } (w \in Dox_s(w^*) \text{ or } w = w^*))$. This result is stated in (46) and proven in Appendix II. But a word of explanation might be useful. In the simple framework of Schlenker 2009, one should think as the first world argument w^* as the context parameter. Thus if $(Un)aware_j F$ is evaluated within a

¹¹ More precisely, the condition we derive is that the local context of the embedded clause entails its cosupposition, as stated in (i):

(i) $lc(Sp) \models p \Rightarrow s$

Schlenker 2009 shows that the value of the local context of the embedded clause in a formula $Bel_s F$ is $lc(F) = \lambda w^* \lambda w (w^* \in C \text{ and } w \in Dox_s(w^*))$, where abstraction over w^* corresponds to the context parameter and abstraction over w corresponds to the world parameter, while $Dox_s(w^*)$ is the set of worlds compatible with what individual s believes in world w^* . As a result, we obtain the presupposition in (ii), which is also what Heim 1992 posits:

(ii) $C \models Bel_s p \Rightarrow s$

context c of the context set C , the local context of F is $lc(F)(c) = \lambda w (w \in \text{Dox}_s(c) \text{ or } w = c)$, which can be thought of as the set $\text{Dox}_s(c) \cup \{c\}$; this, in turn, captures the intuition that when evaluating the embedded clause one can restrict attention to the worlds compatible with the agent's beliefs (given c) and to c itself, as the embedded clause will only provide information about such worlds.

(46) **Local context of F in $(Un)aware_j F$**

Assume that $(Un)aware_j F$ is equivalent to $F \ \& \ (\text{not}) \ Bel_j F$.

In the framework of Schlenker 2009, *modulo* some technical assumptions stated in Appendix II, the local context of the embedded clause F is:

$$lc(F) = \lambda w^* \lambda w (w^* \in C \text{ and } (w \in \text{Dox}_s(w^*) \text{ or } w = w^*))$$

We are now in a position to consider the case of *Sam is aware that John SLAP punished his son*, analyzed in (47).

(47) a. Sam is aware that John SLAP punished his son.

a'. $Aware_s Sp$

b. Licensing condition:

$$lc(Sp) \models p \Rightarrow s$$

By (46), $lc(Sp) = \lambda w^* \lambda w (w^* \in C \text{ and } (w \in \text{Dox}_s(w^*) \text{ or } w = w^*))$, where $\text{Dox}_s(w^*)$ is the set of worlds compatible with what individual s believes in world w^* . As a result, we obtain the presupposition that

$$(i) C \models p \Rightarrow e \ \& \ Bel_s p \Rightarrow s$$

In addition, we can derive a factive presupposition triggered by *aware* (we do not provide the derivation here, as it is straightforward in dynamic semantics or in the reconstruction of local contexts of Schlenker 2009). Thus we also have:

$$(ii) C \models p \ \& \ (p \Rightarrow s), \text{ hence } C \models p \ \& \ s$$

Since $Aware_s Sp$ is asserted, if the speaker is sincere we obtain the result in (iii), with B = the set of worlds compatible with the speaker's beliefs:

$$(iii) B \models Bel_s(p \ \& \ s)$$

The presuppositions are the same if *aware* is replaced with *unaware*, but we lose the inference in (iii):

(48) a. Sam is unaware that John SLAP punished his son.

a'. $Unaware_s Sp$

b. Since the local context of the embedded clause is the same under *Unaware* and under *Aware*, and since the factive presupposition is the same for both constructions, we obtain exactly the same presuppositions in the two cases, hence:

$$(i) C \models (p \Rightarrow s) \ \& \ Bel_s (p \Rightarrow s)$$

$$(ii) C \models p \ \& \ s$$

The assertive component gives us in the present case:

$$(iii) B \models \text{not } Bel_s(p \ \& \ s)$$

and by (i) and (iii)

$$(iv) B \models \text{not } Bel_s p$$

since for $w \in B$, if $w \models Bel_s p$, by (i) $w \models Bel_s (p \Rightarrow s)$ [because $C \models F$ implies $B \models F$], hence $w \models Bel_s (p \ \& \ s)$, which contradicts (iii).

Now it might seem that $Bel_s (p \Rightarrow s)$ in (i) is still too strong (this is related to the problem we already encountered in (41)-(42)). We could assume that the gestural cosupposition is globally accommodated and thus assessed with respect to the beliefs B of the speaker. If in addition (iv) is strengthened to $B \models Bel_s \text{ not } p$, the conditional embedded under the belief operator will be vacuously satisfied, and thus the effect of $Bel_s (p \Rightarrow s)$ will be hard to detect. Whether this is the right line of analysis remains to be seen, however – a point we leave for future research.

Since the local context of the embedded clause is the same under *aware* and *unaware*, we also get the result that *Sam is not aware that John SLAP punished his son* is predicted to behave like (48)a.

- (49) a. Sam is not aware that John SLAP punished his son.
 a'. not Aware_s Sp
 b. Since the local context of the embedded clause is the same under *Unaware* and under *Aware*, and since the factive presupposition is the same for both constructions, we obtain exactly the same presuppositions in the two chases, hence:

(i) $C \models p \Rightarrow e \ \& \ Bel_s p \Rightarrow s$

(ii) $C \models p \ \& \ s$

The assertive component gives us in the present case:

(iii) $B \models \text{not } Bel_s(p \ \& \ s)$

and by (i) and (iii)

(iv) $B \models \text{not } Bel_s p$

since if $B \models Bel_s p$, by (i) $B \models Bel_s p \Rightarrow s$ [because $C \models F$ implies $B \models F$], hence $B \models Bel_s p \ \& \ s$, a contradiction.

3.4 Summary

In this section, we have implemented in some detail the idea that *the contextual meaning of an expression should entail the content of a co-speech gesture that modifies it*. By taking the contextual meaning of an expression to be the meaning it has *relative to its local context*, we were able to predict a subtle interaction between co-speech gestures and the logical operators they are in the scope of. For a (presupposition-less) expression p modified by a co-speech gesture G in a sentence ... $G p$... uttered in a Context Set C , they key condition was that $lc \models p \Rightarrow g$, where lc is the local context of $G p$ and g is the content of the gesture G . We were able to derive several instances of the initial generalization in (6). First, barring local accommodation, co-speech gestures strengthen the meaning of sentences because their contribution is presuppositional (in the special, cosuppositional form they take). Second, in cases in which the cosuppositional inference $p \Rightarrow g$ projects universally, we get the inference that those tuples that satisfy p must satisfy g . Third, we derived a conditional inference that those objects that don't satisfy p would satisfy g if they satisfied p . This was a good inference in some cases (e.g. under the quantifier *no*), and possibly an overly strong inference in some other cases.

4 Alternative Analyses

In this section, we consider two alternative analyses. One treats co-speech gestures as having a supervaluationist account – the intuition being that one wishes to guarantee that the same semantic result is obtained whether the content of the co-speech gesture is taken into account or not. This makes interesting predictions, but ones which, in the end, are flawed. The second alternative treats co-speech gestures as having a 'supplementary' contribution, or in other words as having the same kind of contribution as appositive relative clauses, as argued in Ebert and Ebert 2014. We argue that this analysis is incorrect for co-speech gestures but works well for post-speech gestures, which come after they expressions they modify and have their own time slot.

4.1 Supervaluationist accounts

As noted, our initial cosuppositional analysis predicted in some cases inferences that are somewhat too strong. This was in particular the case in (41)a (*Exactly one of these 10 guys SLAP punished his son.*), for which we initially predicted a universal conditional presupposition to the effect that for each of these 10 guys, if he punished his son, he did so by slapping him. The same potential problem arose in (48)a (*Sam is unaware that John SLAP punished his son.*), for which we initially predicted a presupposition that Sam believes that either John didn't punish his son, or he did so by slapping him. We explored various possible solutions in Section 3.2.2. But these difficulties motivate the exploration of a different theory, which goes like this:

when we perceive a co-speech gesture, we interpret it as enriching the meaning of the sentence, but instead of doing so in a cosuppositional fashion, we simply take the conjunction of the sentence without the co-speech gesture, and of the sentence with the co-speech gesture.

In effect, this amounts to treating the sentence in a supervaluationist fashion: we take the presence of the co-speech gesture to be somehow 'vague', and we require that the sentence should be true both with and without the co-speech gesture (this could be motivated by the fact that we wish to interpret co-speech gesture, but without any risk of somehow 'losing' the literal meaning; a similar idea was explored in unpublished work by Fox and Spector on the interpretation of the covert exhaustivity operator that accounts for embedded implicatures). This leads us to an initial statement of a basic supervaluationist account, as in (50).

(50) **Basic Supervaluationist Account**

A sentence for the form ... Ge ... with a co-speech gesture *G* with a content that can be expressed by *g* modifying an expression *e* of the same (conjoinable) type is true (resp. false) just in case the two sentences ... e ... and ... [e & g] ... are both true (resp. false).

Consider how this mechanism can be applied to the sentence *Sam is unaware that John SLAP punished his son*. As shown in (51), this ends up yielding the strongest of the presuppositions of *Unaware_s p* and *Unaware_s (p & s)*, hence a presupposition that John punished his son by slapping him; while this also yields the strongest of the two assertive components, and hence (since the assertive component is negative) an assertion that Sam doesn't believe that John punished his son – irrespective of whether this was by slapping him or by other means.

- (51) a. Sam is unaware that John SLAP punished his son.
 a'. *Unaware_s Sp*
 b. (a') is true iff *Unaware_s p* and *Unaware_s (p & s)* are both true,
 iff it is presupposed that *p*, and *s* doesn't believe that *p*, and it is presupposed that (*p & s*), and *s* doesn't believe that (*p & s*)
 iff it is presupposed that *p* and *e*, and *s* doesn't believe that *p*.

Importantly, we do not derive in this way the somewhat questionable inference that Sam believes that if John punished his son, he did so by slapping him (or more precisely: Sam believes that John didn't punish his son, or that he did so by slapping him).

This initial success of the Basic Supervaluationist Account extends to other non-monotonic cases such as *exactly one*, as shown in (52).

- (52) a. Exactly one of these 10 guys SLAP punished his son.
 a'. [=1 g] Sp
 b. (a') is true and felicitous iff [=1 g] *p* and [=1 g] (*p & s*) are both true and felicitous. It follows that exactly one guy punished his son, and furthermore he did so by pulling his ear.

This result can be compared to the stronger inference we obtained in our presuppositional approach in (41). As will be recalled, we had derived two relevant inferences:

–relative to the Context Set, if any of the 10 guys punishes his son, he does so by slapping him.
 –relative to the (more specific) speaker's beliefs, exactly one of the 10 guys punished his son; by the first inference, he did so by slapping him.

The supervaluationist approach has the apparent advantage of not generating a conditional presupposition about the 9 guys who didn't punish their sons.

Several supervaluationist treatments are discussed in greater detail in Appendix III. Despite its initial successes, the Basic Supervaluationist Account is seen to suffer from two deficiencies.

(i) First, it predicts no gestural inference at all in downward-monotonic environments. The reason is that in such cases the version of the sentence with the gestural enrichment is *weaker* than the one without it, and thus conjoining the two yields the same result as if there had been no gestural enrichment in the first place. For instance, for *None of these 10 guys SLAP punished his son*, we obtain in essence the conjunction of *None of these 10 guys punished his son*, and *None of these 10 guys punished his son by slapping him* – which is equivalent to the first conjunct.

In Appendix III, we consider a more complex Mixed Supervaluationist Account, which combines mechanisms of the Basic Supervaluationist Account of our cosuppositional analysis, as stated in (53).

(53) **Mixed Supervaluationist Account**

A co-speech gesture

(i) is treated in terms of the Basic Supervaluationist Account in (50)...

(b) ... unless this fails to strengthen the meaning, in which case it is treated as a cosupposition (as outlined in Section 3).

The resulting theory combines many of the advantages of the supervaluationist and of the cosuppositional theory.


(ii) Still, there are further problems with both versions of the Supervaluationist Account. As we discuss in Appendix III, it predicts odd inferences under expressions such as *an odd number of*, or *between 3 and 5*. Consider for instance the sentence *Between 3 and 5 guys SLAP punished their son*. In essence, the Basic Supervaluationist Account treats this as the conjunction of *Between 3 and 5 guys punished their son*, and *Between 3 and 5 guys punished their son by slapping him*. This turns out to be true in case exactly 5 guys punished their son, and exactly 3 guys punished their son by slapping him – which seems to us to be a counterintuitive result. We believe the desired inference is more something like: *Between 3 and 5 punished their son, and those that did slapped him*. The latter inference is correctly obtained on the cosuppositional approach of Section 3.


We also discuss in Appendix III a stronger supervaluationist account, which solves some of these problems (with *an odd number*) but not others (with *between 3 and 5*). It still has the drawback of predicting no gestural enrichment in downward-monotonic environment, and thus it too must be transformed into a 'mixed theory' in which in these cases the cosuppositional approach is adopted. Further variants are discussed in Appendix III.

4.2 Gestures as Supplements?


4.2.1 Supplementary analyses of co-speech gestures

Ebert and Ebert 2014 have suggested that co-speech gestures should be analyzed as supplements. As we will see in Section 4.2.2, some speech-accompanying gestures do indeed display a supplementary behavior, but in the cases we have studied this happens if they are *post-speech* gestures, i.e. if they come *after* the expressions they modify. If they co-occur *with* them, we obtain the behavior that was described above. And *that* behavior cannot be easily accounted for in terms of supplements. The point is made in Schlenker 2014 by focusing on negative environments in which supplements are degraded – but in which co-speech gestures appear rather freely:

(54) a. It's unlikely that the next speaker will bring  a **bottle of beer** to his talk.
=>? if the session chairman brings a bottle of beer, it will be a large one

b. #It's unlikely that the next speaker will bring a bottle of beer, which is **[this]**  large.
(Schlenker 2014)





(55) a. No philosopher brought  **[a bottle of beer]** to the workshop.
=>? when a philosopher brings a bottle of beer, it is usually a large one

b. #No philosopher brought a bottle of beer, which is  **[this]** large.
(Schlenker 2014)

As mentioned in Schlenker 2014, a supplementary approach could deal with (54)-(55) by taking the gestures to behave like the appositives in b., but with *which would be* replacing *which is*. On the assumption that the resulting sentence is more acceptable, one would still need to ask why such an option should be available – it would amount to positing a covert counterfactual mood to achieve the desired result.

4.2.2 Post-speech gestures as supplements

Importantly, however, some gestural enrichments seem to have the status of supplements, but they are ones that come after the expressions they modify – 'post-speech' gestures, in our terminology. Thus to our ear (and eye), there is a sharp acceptability contrast between (56)a and (56)b, which mirrors that between (57)a and (57)b.

- (56) a. A philosopher brought a bottle of beer – 
=> a philosopher brought a bottle of beer, and it was a large bottle.
- b. ??No philosopher brought a bottle of beer – 
- (57) a. A philosopher brought a bottle of beer, which was  this large.
=> a philosopher brought a bottle of beer, and it was a large bottle.
- b. ??No philosopher brought a bottle of beer, which was  this large.

We submit that gestures can receive a supplementary reading when they are post-posed rather than simultaneous with the expressions they modify (see Pasternak 2014 for further discussion).

A similar conclusion can arguably be obtained on the basis of facial expressions. (58)a is an example we already discussed to highlight the presuppositional effect of facial expressions co-occurring with a predicate under the quantifier *no*. In (58)b, the same facial expression is post-posed. We believe this yields several readings, which can be paraphrased with different supplements, depending on the size of the constituent that the post-speech gesture modifies.

- (58) a. None of my friends goes :- (skiing with his parents)
=> for each of my friends, skiing with his parents wouldn't be fun
- b. None of my friends goes skiing with his parents – :- (.
- c. None of my friends goes skiing with his parents,
(i) which is sad [i.e. it is sad that none of my friends goes skiing with his parents];
(ii) which is unpleasant [i.e. it is generally unpleasant to go skiing with one's parents].

4.2.3 Co- vs. post-speech gestures

If these observations are on the right track, it remains to ask gestural enrichments could have a cosuppositional or a supplementary status depending on simple timing considerations. This observation is a bit surprising if supplements and presuppositions in general (and cosuppositions in particular) are two entirely different phenomena. To the extent that supplements are taken to be introduced by a 'comma intonation' (Potts 2005), one could reduce the special status of post-speech gestures to the presence of a pause separating them from the expressions they modify. But one would still have to explain the special status of co-speech gestures.

We believe that it is easier to make sense of these data within a framework in which presuppositions and supplements have a common core, as argued in Schlenker 2010b, 2013. In that framework, presuppositions are (as in dynamic semantics) expressions that must be trivial or 'transparent' in their local context. Supplements are expressions that should not be trivial (as emphasized by Potts 2005), but which are 'translucent' in the sense that it should be easy to add to the context of the conversation uncontroversial assumptions that would make the supplements trivial. The underlying intuition was that appositives are syntactically autonomous and thus non-trivial, but still parasitic (easily omissible) and thus semantically non-essential. Now these intuitions can be adapted to the case at hand.

- Consider first co-speech gestures. They are elements that can be taken to be parasitic on the content of the expressions they modify, in the sense that they come from a different modality and could thus be omitted without syntactic loss. Furthermore, nothing in their timing requires that they should have a non-trivial contribution (since they do not occupy a separate time slot), and thus nothing prevents them from being entirely trivial or 'transparent' in their local context.
- The case of post-speech gestures are different. While also parasitic, they occupy a separate time slot, and thus they should make an informational contribution, which means they could not be trivial or 'transparent' in their local context. On the other hand, they can be 'translucent' or supplementary, which will guarantee that they are 'semantically parasitic'.

These speculations leave several questions open.

- First, do post-speech gestures have the same conditional semantics as co-speech gestures? In simple cases, an assertion-dependent semantics won't hurt. For instance, in (58)b, a conditional contribution for the post-posed expression might have the form: $\lambda p. (p \Rightarrow \text{the agent of } c \text{ is sad})$, where *c* is that context of evaluation, but with a supplementary status (achieved by way of a comma intonation in the framework of Potts 2005, and by pragmatic means in the framework of Schlenker 2010b, 2013). In

this case, the post-speech gesture is attached to an unembedded clause, and thus the overall contribution of the post-speech gesture will turn out to be unconditional. The question we leave for future research, however, is what happens when a post-speech gesture is attached to a clause which is embedded under other operators. (Scopal interaction with operators is precluded by the theory of Potts 2005, but it has been argued by several authors to be possible; see Schlenker 2015 for a survey.)

- Second, and relatedly, what are the attachment possibilities of post-speech gestures? This issue too is controversial in the research on appositives: some researchers claim that they are attached to their surface position, and that their apparent 'wide-scope behavior' is due to a non-standard semantics (Potts 2005); while others claim that appositives can be attached to any propositional node that c-commands their surface position (Schlenker 2010b, 2013). Systematic work would be needed to compare the attachment possibilities of standard appositives and of post-speech gestures.

5 Conclusion

We have suggested that a very simple analysis can account for complex patterns of 'gesture projection'. There were two parts to our proposal. Our main hypothesis was that the content of a co-speech gesture should be entailed by the 'contextual meaning' of the expression it modifies – in effect, the intuition was that a co-speech gesture serves to illustrate (and thus make more precise) the meaning of the expression it modifies. The second part consisted in a completely standard explication of 'contextual meaning' as the meaning an expression has relative to its local context, as defined in dynamic semantics (Heim 1983) or in reconstructions of it (Schlenker 2009). Putting both parts together, we arrived at a cosuppositional analysis in which the content of a co-speech gesture is presupposed to follow from the content of the expression it modifies, and interacts with the logical structure of a sentence by way of local context computation.

We argued against a supervaluationist alternative to our analysis. On the other hand, we did not so much *refute* the supplemental analysis of Ebert and Ebert 2014 as *displace* it: we argued that it is not correct for co-speech gestures, but does capture the properties of post-speech gestures. If this typology is on the right track, one should of course ask how the same primitive objects – speech-accompanying gestures – can have a cosuppositional behavior in some cases and a cosuppositional behavior in others. Thus the analysis of gestures leads to new questions about some foundational issues in semantics – in particular on the connection between presuppositions and supplements.

Finally, the analysis of the semantic behavior of co-speech gestures matters for a broader debate about the expressive power of spoken and sign language. Due to the presence of a rich iconic component at the logical core of sign language, some researchers have argued that the latter is in some respects more expressive than spoken language (e.g. Schlenker et al. 2013, Schlenker 2015b). Others have countered that both sign and spoken language have a gestural component, and that some instances of iconicity in sign language should really be compared to co-speech gestures in sign language (Davidson, to appear; Goldin-Meadow and Brentari 2015). The present piece indirectly contributes to this debate by sharpening our understanding of the 'co-speech gesture' side of the comparison – and by suggesting that co-speech gestures often have a presuppositional/cosuppositional status. Importantly, although the iconic effects in sign language discussed in Schlenker et al. 2013 were presuppositional in nature, many others are assertive in nature (Schlenker 2015b). This state of affairs raises two questions: first, can co-speech gestures have assertive rather than presuppositional/co-suppositional status, as argued here? The question is delicate because presuppositions can be locally accommodated, and the cosuppositions triggered by co-speech gestures are no exception. So the question to be asked is whether some co-speech gestures have a 'purely' assertive contribution – a subtle empirical question, but an important one if the comparison with iconic effects in sign language is to be systematized. Second, are there genuinely cosuppositional inferences in sign language? The presuppositional data discussed in Schlenker et al. 2013 pertained to high and low loci used to realize pronouns, and their semantic contribution was likened to that of gender features in spoken languages. Thus it is not yet clear whether these or other iconic effects might have the cosuppositional status discussed in this piece.

Appendix I. Local Contexts in Schlenker 2009, 2010¹²

The reconstruction of local contexts proposed in Schlenker 2009, 2010 works is defined for a fragment that includes propositional connections, unary predicates and generalized quantifiers.

(59) Syntax

- a. Predicates: $P ::= P_i \mid \underline{P}_i P_k$
- b. Propositions: $p ::= p_i \mid \underline{p}_i p_k$
- c. Formulas: $F ::= p \mid (\text{not } F) \mid (F \text{ and } F) \mid (F \text{ or } F) \mid (\text{if } F, F) \mid (\text{Each } P . P) \mid (\text{No } P . P) \mid (\text{Most } P . P)$

(60) Semantics

We take as given a domain D of individuals and a domain W of possible worlds.

The initial valuation assigns to each elementary predicate P_i a value $\mathbf{P}_i^w \subseteq W$ and to each elementary proposition p_i a value $\mathbf{p}_i^w \in \{0, 1\}$. For any world w of W :

- a. $(\underline{P}_i P_k)^w = 1$ iff $\mathbf{p}_i^w = \mathbf{p}_k^w = 1$; $(\underline{P}_i \underline{P}_k)^w = \mathbf{P}_i^w \cap \mathbf{P}_k^w$;
- b. $(\text{not } F)^w = 1$ iff $F^w = 0$; $(F \text{ and } F')^w = 1$ iff $F^w = F'^w = 1$; $(F \text{ or } F')^w = 1$ iff $F^w = 1$ or $F'^w = 1$; $(\text{if } F . F')^w = 1$ iff $F^w = 0$ or $F'^w = 1$; $(\text{Each } P . P')^w = 1$ iff each object $d \in D$ such that $d \in \mathbf{P}^w$ satisfies $d \in \mathbf{P}'^w$; $(\text{No } P . P')^w = 1$ iff no object $d \in D$ such that $d \in \mathbf{P}^w$ satisfies $d \in \mathbf{P}'^w$; $(\text{Most } P . P')^w = 1$ iff more than half of the objects $d \in D$ such that $d \in \mathbf{P}^w$ satisfy $d \in \mathbf{P}'^w$

(61) Local Contexts

The local context of an expression d of propositional or predicative type which occurs in a syntactic environment $a _ b$ in a context C is the strongest proposition or property x which guarantees that for any expression d' of the same type as d , for all strings b' for which $a \ d' \ b'$ is a well-formed sentence,

$$C \models^{c-x} a \ (c' \text{ and } d') \ b' \Leftrightarrow a \ d' \ b'$$

(If no strongest proposition or property x with the desired characteristics exists, the local context of d does not exist¹³).

(62) Presupposition Satisfaction

An elementary presuppositional expression E is acceptable in a sentence S uttered in a context C just in case the presupposition of E is entailed by the local context of E (if it exists).

Schlenker 2009 proves some general results about the relation between this reconstruction of local contexts and a Heimian dynamic semantics in which generalized quantifiers trigger universal presuppositions (see Section 2.4. and Appendix A)

In the propositional case, we obtain full equivalence with the system outlined in Heim 1983, enriched with the asymmetric dynamic disjunction of Beaver 2001. Specifically, it can be shown that for any propositional formula F and for any context set C , the local contexts as we have defined them always exist. Furthermore, if we write as $C[F]$ the Heimian update of C with F , $C[F] \neq \#$ just in case for each presupposition trigger of the form $\underline{d}d'$ that occurs in F , d is entailed by its local context as reconstructed here (using our earlier notation, we write this as $\text{Sat}(C, F)$). This result is summarized in (63).

(63) Let $C \subseteq W$ be a context set and let F be a propositional formula. Then:

- (i) for all expressions $a, b, \underline{d}d'$, if $F = a \ \underline{d}d' \ b$, $\text{lc}(C, \underline{d}d', a_b) \neq \#$. Furthermore,
- (ii) $\text{Sat}(C, F)$ iff $C[F] \neq \#$.

In the quantificational case, things are more complicated. In a nutshell, when all the relevant local contexts exist, (63) also holds for all generalized quantifiers that can be defined by way of the 'tree of numbers', but only when two technical conditions are met:

-*Non-Triviality*: quantificational clauses should not be 'trivial' (i.e. replaceable with a tautology or a contradiction).

¹² This Appendix borrows from Schlenker 2009 and Schlenker 2010 (especially Section 2.4. of Schlenker 2009).

¹³ See Schlenker 2009 for a discussion of the case in which local contexts do not exist.

-*Constancy*: the domain of individuals should be finite, and in addition restrictors should hold true of a constant number of individuals throughout the context set.¹⁴

In case local contexts fail to exist, a modified version of the present theory guarantees full equivalence with Heim's result when Non-Triviality and Constancy are satisfied. Details can be found in Schlenker 2009, which in turn builds on some technical results of Schlenker 2007.

Note that an extension of the theory must be countenanced to compute the local context of expressions embedded under attitude reports, as is discussed in Appendix II.

¹⁴ While Non-Triviality is quite natural (why would one use a quantificational clause if it is trivial?), I cannot think of any good justification for Constancy. Without the latter, we obtain presuppositions that are sometimes weaker than those predicted by Heim 1983.

Appendix II. Computing the local context of the embedded clause under factive verbs

We assume the general framework of Schlenker 2009, Section 3.1.2 ('Adding Belief Reports'), and in particular the device of double indexing (hence the double abstraction over w^* and over w), which is necessary to compute local contexts in intensional cases. For comparison, Schlenker 2009 shows that for non-factive formulas such as *Believe_j F*, the value of the local context of the embedded clause is $lc(F) = \lambda w^* \lambda w (w^* \in C \text{ and } w \in \text{Dox}_s(w^*))$ – hence the same result as in the factive case, *except* for the disjunct *or* $w = w^*$.

(64) **Local context of *F* in *(Un)aware_j F***

Assume that *(Un)aware_j F* is equivalent to *F & (not) Bel_j F*.

Claim: $lc(F) = \lambda w^* \lambda w (w^* \in C \text{ and } (w \in \text{Dox}_s(w^*) \text{ or } w = w^*))$

By definition, $lc(F)$ is the most restrictive c' of type $\langle s, \langle s, t \rangle \rangle$ such that for all d'

$C \models (\text{Un)aware}_j (c' \ \& \ d') \iff (\text{Un)aware}_j d'$, or in other words

$C \models [(c' \ \& \ d') \ \& \ (\text{not}) \text{Bel}_j (c' \ \& \ d')] \iff [d' \ \& \ (\text{not}) \text{Bel}_j d']$

(i) Clearly, the equivalence holds if c' denotes $lc(F)$ as defined above.

(ii) Now suppose that for w^* and w that fail to satisfy $(w^* \in C \text{ and } (w \in \text{Dox}_s(w^*) \text{ or } w = w^*))$, $lc'(w^*)(w) = 1$.

a. If $w^* \notin C$, it is possible to find a more restrictive c'' by setting:

for every $w^{*'}, w'$,

$lc''(w^{*'})(w') = 0$

if $w^{*'} \neq w^*$, $lc''(w^{*'})(w') = lc'(w^*)(w')$

b. Suppose that $w^* \in C$, and that $lc'(c)$ denotes a proper subset of $\{c\} \cup \text{Dox}_s(c)$, with $w \in \{c\} \cup \text{Dox}_s(c)$ and $w \notin lc'(c)$.

Unaware

Assumption: for each $c \in C$, $\text{Dox}_s(c)$ excludes at least one world w_c .

Case 1. $lc'(c)$ excludes c

Take $ld'(c) = \{c, w_c\}$

$c \not\models (c' \ \& \ \dots)$

since by assumption $c \notin lc'(c)$

But

$c \models d' \ \& \ \text{not} \ \text{Bel}_j d'$

since $ld'(c) = \{c, w_c\}$ and $\text{Dox}_s(c)$ excludes w_c .

Case 2. $lc'(c)$ doesn't exclude c ; so $lc'(c)$ excludes some $w \in \text{Dox}_s(c)$.

Take $d' = \text{tautology}$. Then:

$c \models (c' \ \& \ d') \ \& \ \text{not} \ \text{Bel}_j (c' \ \& \ d')$ because $lc'(c)$ doesn't exclude c , d' is a tautology, and $lc'(c)$ excludes some $w \in \text{Dox}_s(c)$

$c \not\models [d' \ \& \ \text{not} \ \text{Bel}_j d']$

because d' is a tautology, hence $c \not\models \text{not} \ \text{Bel}_j d'$.

Aware

Case 1. $lc'(c)$ excludes c

Take $d' = \text{tautology}$. Then:

$c \not\models (c' \ \& \ \dots)$

since by assumption $c \notin lc'(c)$.

But

$c \models d' \ \& \ Bel_j \ d'$

because d' is a tautology.

Case 2. $lc'(c)$ doesn't exclude c ; so $lc'(c)$ excludes some $w \in Dox_s(c)$.

Here too, take $d' = \text{tautology}$. Then:

$c \not\models (c' \ \& \ d') \ \& \ Bel_j \ (c' \ \& \ d')$ because $lc'(c)$ doesn't exclude c , d' is a tautology, and $lc'(c)$ excludes some $w \in Dox_s(c)$.

$c \models [d' \ \& \ Bel_j \ d']$

because d' is a tautology.

Appendix III. More on Supervaluationist Accounts

This Appendix discusses in greater detail the supervaluationist accounts of co-speech gestures that were sketched in Section 4.1.

□ *The cosuppositional approach predicts stronger inferences than the Basic Supervaluationist Account*

As we saw in Section 4.1, both in the case of embedding under *unaware* and under *exactly one*, we saw that the supervaluationist approach generates presuppositions that are weaker than those of the presuppositional approach. This is an entirely general fact: the supervaluationist approach predicts inferences that are entailed – sometimes asymmetrically entailed – by those derived by the presuppositional approach. To put it informally, the key is that the presuppositional approach derives conditions on the Context Set C that are so strong that they guarantee that, relative to C , the sentence with the co-speech gesture is *equivalent* to the bare sentence without the co-speech gesture; as a result, if one of them is true, *both* are – and thus whenever the presuppositional condition is satisfied, so is the supervaluationist condition. This is explained in greater detail in (65).

- (65) The presuppositional account predicts stronger inferences than the supervaluationist account
 a. Suppose that a sentence ... Ge ... with a gesture G modifying an expression e is true and felicitous according to the presuppositional approach. Then it is true and felicitous according to the supervaluationist approach.

Proof: If ... Ge ... is felicitous in a Context Set C , then relative to the local context of e , $e \Rightarrow g$ (if g is an expression with the same content as G). This guarantees that

$C \models \dots e \dots \Leftrightarrow \dots (e \& g) \dots$

If ... Ge ... is felicitous and true, then both ... e ... and ... $(e \& g)$... are felicitous and true, which shows that ... Ge ... is felicitous and true on the supervaluationist account.

- b. There are cases in which the presuppositional account predicts stronger inferences than the supervaluationist account.

A case in point, pertaining to gestures embedded under *exactly one*, is given in (41) and (52)

An additional remark will be useful below. In our Basic Supervaluationist Account, we only required that two versions of the sentence be true, one with and one without the gestural enrichment. But one could investigate a supervaluationist approach that makes stronger predictions, and requires that a sentence with ... Ge ... with a predicate e enriched with a gesture G with content g should be true on all the possible *point-wise* resolutions of the uncertainty about the presence of the enrichment g . In other words, we consider all the values of the predicate e obtained by requiring that arbitrary objects satisfying e also satisfy the gestural enrichment g . Technically, this amounts to a requirement that the original sentence ... Ge ... should be true just in case ... e ... is true on all interpretation on which e takes values within the set $R(e, g)$ defined as in **Erreur ! Source du renvoi introuvable.**:

$$(66) R(e, g) = \{ \{ \lambda w \lambda x . |e|(w)(x) = 1 \ \& \ \langle x, w \rangle \in F \Rightarrow |g|(w)(x) = 1 \} : F \subseteq D \times W \}$$

This yields a stronger requirement than our 'official' supervaluationist approach, since on the latter which we only require that ... e ... should be true when e takes the two values $\{ \lambda w \lambda x . |e|(w)(x) = 1, \lambda w \lambda x . |e|(w)(x) = 1 \ \& \ |g|(w)(x) = 1 \}$; by contrast, the point-wise definition using **Erreur ! Source du renvoi introuvable.** requires that the sentence should be true for many other values of the predicate.

□ *The problem of downward-monotonic environments*

At this point, it might seem that the supervaluationist analysis has an advantage over the presuppositional one. But when we consider downward-monotonic environments, the situation changes. We saw that under the quantifier *no* one intuitively obtains a non-trivial conditional inference – e.g. for *None of these 10 guys SLAP punished his son*, we get the inference that for each of these 10 guys, if he had punished his son, he would have done so by slapping him. The presuppositional approach derives something close to this. But the supervaluationist approach derives nothing at all: the problem is that in downward-monotonic environments, the enriched version of the sentence is *weaker* than the original version, with the result that only the latter makes its truth-conditional effects felt.

- (67) a. None of these 10 guys SLAP punished his son.
 a'. [No g] Sp
 b. (a') is true and felicitous iff [No g] p and [No g](p and e) are both true and felicitous, iff [No g] p is true and felicitous.

Importantly, the very same problem extends to the stronger supervaluationist account we considered at the end of the preceding section. In that approach, the sentence [No g] Sp is true just in case [No g] p under all interpretations on which p takes values within the set $R(p, s) = \{[\lambda w \lambda x . |p|(w)(x) = 1 \ \& \ \langle x, w \rangle \in F \Rightarrow |s|(w)(x) = 1]: F \subseteq D \times W\}$, since each of these values is at least as restrictive as the original value |p|; as a result, the truth conditions of [No g] Sp end up being the same as those of the bare sentence [No g] p.

□ *A mixed approach*

We could try to fix the problem by combining the advantages of the supervaluationist and of the presuppositional approach, along the following lines:

- (68) Mixed Supervaluationist Account: supervaluations and presuppositions
 (i) One first tries to interpret a co-speech gesture according to a supervaluationist analysis.
 (ii) However, if the result makes the co-speech gesture vacuous, we interpret it according to the presuppositional approach.

While this approach is obviously complicated, it has the possible advantage of predicting conditional presuppositions under the quantifier *no*, but not under non-monotonic quantifiers such as *exactly one*. Still, as we will now see, it suffers from further deficiencies.

□ *Remaining problems for the supervaluationist accounts*

Even the mixed solution envisaged in (68) ('first try a supervaluationist analysis; and if this still makes the gesture vacuous, go for a presuppositional treatment') encounters problems. Consider the sentences in (69):

- (69) a. An odd number of guys SLAP punished their son.
 => an odd number of guys punished their son, and they did so by slapping them
 b. Three or five guys SLAP punished their son.
 => three or five guys punished their son, and they did so by slapping them

I believe we get a fairly strong inference that all the guys who punished their son did so by slapping him. But this is not predicted by the supervaluationist account, even in its mixed version. The predictions of our basic supervaluationist theory are laid out in (70) and (71).

- (70) a. An odd number of guys SLAP punished their son.
 a'. [Odd g] Sp
 b. (a') is true iff [Odd g] p and [Odd g] (p and s) are both true.
 Thus (a') is true in particular if exactly five guys punished their son, and exactly three guys punished their son by slapping him.

- (71) a. Three or five guys SLAP punished their son.
 a'. [=3 or =5 g] Sp
 b. (a') is true iff [=3 or =5 g] p and [=3 or =5 g] (p and s) are both true.
 Thus (a') is true in particular if exactly five guys punished their son, and exactly three guys punished their son by slapping him.

In both cases, the supervaluationist analysis that the gesture is non-vacuous. To see this, notice that without the gesture, the sentences would end up meaning that *an odd number of guys punished their son* or that *three or five guys punished their son*. With the gesture, the supervaluationist approach imposes the additional requirement that *an odd number of guys punished their son by slapping them*, or that *three or five guys punished their son by slapping them* – a non-trivial condition that doesn't follow from the meaning of the bare sentence. So even the mixed theory predicts that we should stick to the supervaluationist analysis of these sentences. However the readings we obtain are too weak: we do not get the intuitively desirable inference that *all* the guys who punished their son did so by slapping them, as the supervaluationist truth conditions allow for the possibility that only a strict subset of the guys who punished their son did so by slapping them. In particular, as mentioned in (70)

and (71), both sentences are predicted to be true if exactly five guys punished their son, and exactly three guys punished their son by slapping them – a dubious result.

□ *Solution within the Point-wise Supervaluationist Account*

On our point-wise supervaluationist analysis, the initial problem gets solved. Recall that on this approach $[Odd\ g] Sp$ is true just in case $[Odd\ g] p$ is true on all interpretations on which p takes values within the set: $R(e, g) = \{[\lambda w \lambda x . |e|(w)(x) = 1 \ \& \ \langle x, w \rangle \in F \Rightarrow |g|(w)(x) = 1] : F \subseteq D \times W\}$. Clearly, this interpretation is stronger than the bare sentence without a co-speech gesture, and thus it entails that an odd number of guys punished their son. Furthermore, relative to the Context Set it is equivalent to: *An odd number of guys punished their son, and all these guys did so by slapping them.* The argument is in two steps.

(i) Clearly, if an odd number of guys punished their son, and all did so by slapping him, the sentence will be true on the relevant supervaluationist analysis, because relative to the Context Set the underlined condition will be vacuous for all w, x for which $|p|(w)(x) = 1$.

(ii) To establish the converse, we show that if an odd number of guys punished their son in a world w^* of the Context Set, and if at least one guy g^* didn't do so by slapping him, then the target sentence will not be true on the relevant supervaluationist interpretation. To see this, take $W' = \{w^*\}$ and $D' = \{g^*\}$. It is clear that if for $w = w^*$ an odd number of guys punished their son in w , then in w an even number of guys x satisfy the condition $|p|(w)(x) = 1 \ \& \ \langle x, w \rangle \in F \Rightarrow |s|(w)(x) = 1$ – since the underlined condition is vacuously satisfied by all guys except g^* , and g^* falsifies it.

Furthermore, on the point-wise supervaluationist approach, we still preserve the positive features of our analysis of embedding under *exactly one*, as shown in (72).

(72) a. Exactly one of these 10 guys SLAP punished his son.

a'. $[=1\ g] Sp$

b. Basic supervaluationist account

(a') is true and felicitous iff $[=1\ g] p$ and $[=1\ g] (p \ \& \ s)$ are both true and felicitous. It follows that exactly one guy punished his son, and furthermore he did so by slapping them.

b'. Pointwise supervaluationist account

(a') is true iff $[=1\ g] p$ is true at a certain world w^* on all interpretations on which p takes values within the set: $R(p, s) = \{[\lambda w \lambda x . |p|(w)(x) = 1 \ \& \ \langle x, w \rangle \in F \Rightarrow |s|(w)(x) = 1] : F \subseteq D \times W\}$.

As in b., at a world w^* this requires that exactly one guy g^* punished his son, and g^* did so by slapping him (if g^* didn't punish his son by slapping him, by taking $F = \{w^*\} \times \{g^*\}$, we could find an interpretation of p on which zero guy satisfies the interpretation of p).

Still, in other types of examples, this modified supervaluationist analysis yields potentially unintuitive results – although judgments are somewhat subtle. In particular, in the case of the quantifier *between 3 and 5 guys*, we fail to derive the inference that all the guys who punished their son did so by slapping him. This is shown for both versions of the supervaluationist account in (73).

(73) a. Between 3 and 5 guys SLAP punished their son.

a'. $[3 \leq \bullet \leq 5\ g] Sp$

b. Standard supervaluationist account

(a') is true iff $[3 \leq \bullet \leq 5\ g] p$ and $[3 \leq \bullet \leq 5\ g] (p \ \& \ s)$ are both true.

Thus (a') is true in particular if exactly 5 guys punished their son, and exactly 3 guys punished their son by slapping him.

b'. Revised supervaluationist account

(a') is true iff $[3 \leq \bullet \leq 5\ g] p$ is true at a certain world w^* on all interpretation on which p takes values within the set: $R(p, s) = \{[\lambda w \lambda x . |p|(w)(x) = 1 \ \& \ \langle x, w \rangle \in F \Rightarrow |s|(w)(x) = 1] : F \subseteq D \times W\}$.

We still have the result that (a') is true in particular if exactly 5 guys punished their son, and exactly 3 guys punished their son by slapping him.

□ *Tentative conclusions*

Our current conclusions are as follows.

(i) In upward-monotonic and some non-monotonic environment (e.g. embedding under *be unaware*, *exactly one*), supervaluationist accounts make good predictions.

(ii) In downward-monotonic environments, the supervaluationist accounts we have discussed make the incorrect prediction that the gestural enrichment doesn't affect the meaning. They could be supplemented with a presuppositional treatment for such cases, but at the cost of making the analysis quite a bit more complicated.

(iii) On the most intuitive supervaluationist treatment (which takes the presence of the gesture to be somehow vague), incorrect predictions are made even with the modification in (ii), especially for the case of embedding under *an odd number* and *between 3 and 4*. The less natural point-wise supervaluationist account makes better predictions for the first case, but still makes somewhat unintuitive predictions for the second – although these would need to be tested.

□ *A cautionary note: supervaluationist accounts of gestures vs. supervaluationist accounts of presuppositions*

In the foregoing, our use of supervaluations was motivated by the idea that the presence of a gestural enrichment makes it ‘vague’ whether it should be taken into account or not. In our basic account, we thus provided truth conditions on which a sentence with a co-speech gesture is true just in case it is true both in its bare form and in its enriched version. In our stronger (and less natural) account, we extended this approach ‘point-wise’ to predicates. These vagueness-inspired approaches inspired should be compared to supervaluation-based analyses of presuppositions, which have been discussed (and sometimes advocated) in George 2008a, b, Fox 2008, and Schlenker 2008, among others (see also Beaver and Krahmer 2001 for related approaches using modified versions of Strong Kleene logic). These analyses work in two steps.

(i) They are initially based on the idea that an expression *dd* whose presupposition *d* is not satisfied by objects or parameters o_1, \dots, o_n is neither true nor false of these objects – hence a value # when evaluated with respect to these objects.

(ii) However, if no matter how the value # is resolved the entire sentence has one and the same value, it can ‘recover’ from the failure and be evaluated as true or false, as the case may be.

There are some similarities and some differences between the supervaluationist approach to presuppositions and that based on local contexts.

(a) In upward-monotonic environments, the supervaluationist approach often predicts weaker and possibly more adequate inferences than the approach based on local contexts. For instance, for *At least one of these 10 students knows that he is incompetent*, the supervaluationist approach just predicts an inference that at least one of these 10 students is incompetent (and knows it), whereas approaches based on local contexts typically predict an inference that each of these ten students is incompetent.

(b) For *None of these 10 students knows that he is incompetent*, the supervaluationist approach to presuppositions yields an entailment that each of these 10 students is incompetent (because if at least one isn’t, the value of the predicate evaluated at that student would yield the value #, and if this value is resolved as ‘true’, this would falsify the statement). In this simple case, the inference is comparable (though not quite of the same nature) as the one we derive from an approach based on local contexts, which yields a presupposition that each of these 10 students is incompetent.

(c) For *Exactly 1 of these 10 students knows that he is incompetent*, the supervaluationist approach to presuppositions also predicts a universal entailment that each of these 10 students is incompetent. This is again because if any one of them isn’t, the predicate evaluated at that student would yield the value #; and resolving this value as ‘true’ or as ‘false’ would affect the number of students that satisfy the predicate, hence the impossibility of getting a true ‘exactly one’ statement no matter how the value is resolved.

In two of these three cases, we would get different results from our vagueness-inspired account if we treated gestures as simple presupposition triggers within a supervaluationist treatment of presuppositions:

(a’) For *At least one of these 10 guys SLAP punished his son*, we get the same result, namely an inference that at least one of these 10 guys punished his son by slapping him.

(b’) For *None of these 10 guys SLAP punished his son*, the two accounts make entirely different predictions, since we saw that vagueness-inspired accounts predict that the co-speech gesture should be vacuous (which in turn argues for an enrichment of the theory), whereas if *SLAP* introduces a simple presupposition it will yield an inference that each of these 10 guys did something (presumably punish his son) by slapping him.

(c’) For *Exactly one of these 10 guys SLAP punished his son*, the two accounts make again entirely different predictions. Vagueness-inspired accounts predict that exactly one of these 10 guys punished his son, and he did so by slapping him; whereas if we treat *SLAP* as a simple presupposition resolved by supervaluationist means, we obtain an inference that each of these ten guys did something (presumably punish his son) by slapping him.

Interestingly, the vagueness-inspired supervaluationist account turns out to converge with the point-wise supervaluationist account of presuppositions if within the latter we treat the gestural contribution as a cosupposition rather than as a simple presupposition. This result is sketched in a special case in (74).

(74) **Equivalence between the vagueness-inspired pointwise supervaluationist account and a supervaluationist account of cosuppositions (special case)**

Consider a sentence ... Ge ... which only contains bivalent material except for Ge , where G is a co-speech gesture and e is a predicative expression.

a. Vagueness-inspired pointwise supervaluationist account

The sentence is true (resp. false) just in case ... e ... is true (resp. false) on all interpretations on which e takes values within the set $R(e, g)$ defined as:

$$R(e, g) = \{[\lambda w \lambda x . |el(w)(x) = 1 \ \& \ \langle x, w \rangle \in F \Rightarrow |gl(w)(x) = 1] : F \subseteq D \times W\}$$

(In effect, interpretations for Ge are obtained by selecting arbitrary members of $D \times W$ that satisfy e , and asking that they also satisfy g .)

b. Supervaluationist account of presuppositions with a cosuppositional treatment of gestural enrichments

If Ge is analyzed as $e \ \& \ g$ within a supervaluationist account of presupposition,

$$\begin{aligned} \text{for all } w \in W, x \in D, |Gel(w)(x) &= 1 \text{ iff } |el(w)(x) = |gl(w)(x) = 1 \\ &= 0 \text{ iff } |el(w)(x) = 0 \text{ and} \\ &= \# \text{ iff } |el(w)(x) = 1 \text{ and } |gl(w)(x) = 0 \end{aligned}$$

Given standard supervaluationist rules,

... Ge ... is true (resp. false) just in case it is true (resp. false) for all possible ways of resolving the values # yielded by Ge , i.e. just in case ... e ... is true (resp. false) on all interpretation on which e takes values within the set $R'(e, g)$ defined as:

$$R'(e, g) = \{P \in D_{\langle e, \langle s, t \rangle \rangle} : P \text{ is bivalent and for every } \langle x, w \rangle \in D \times W, \text{ (a) if } |el(w)(x) = |gl(w)(x) = 1, P(w)(x) = 1, \text{ and (b) if } |el(w)(x) = 0, P(w)(x) = 0\}$$

Thus if $P \in R'(e, g)$, P is fully defined by

(c) its behavior on $\{\langle x, w \rangle \in D \times W : |el(w)(x) = 1 \text{ and } |gl(w)(x) = 0\}$.

c. With these definitions, $R(e, g) = R'(e, g)$, and as a result a. and b. make the same predictions.

(i) First, we show that if $P \in R'(e, g)$, $P \in R(e, g)$. We define $F' = \{\langle x, w \rangle : |el(w)(x) = 1 \text{ and } |gl(w)(x) = 0 \text{ and } P(w)(x) = 0\}$. And we note that $P = \lambda w \lambda x . [|el(w)(x) = 1 \ \& \ \langle x, w \rangle \in F' \Rightarrow |gl(w)(x) = 1]$, which shows that $P \in R(p, s)$.¹⁵

(ii) Second, we show that $P \in R(e, g)$, $P \in R'(e, g)$. So let $F \subseteq D \times W$ and let $P = \lambda w \lambda x . |el(w)(x) = 1 \ \& \ \langle x, w \rangle \in F \Rightarrow |gl(w)(x) = 1$. Then:

if $|el(w)(x) = |gl(w)(x) = 1$, then $P(w)(x) = 1$ (since the consequent of $\langle x, w \rangle \in F \Rightarrow |gl(w)(x) = 1$ will be true);

if $|el(w)(x) = 0$, $P(w)(x) = 0$.

We leave a more detailed discussion of supervaluationist approaches for future research.

¹⁵ In greater detail, let us write abbreviate $\lambda w \lambda x . [|el(w)(x) = 1 \ \& \ \langle x, w \rangle \in F' \Rightarrow |gl(w)(x) = 1]$ as R_F , and show that $P = R_F$. Clearly, for all $\langle x, w \rangle \in D \times W$,

(i) if $|el(w)(x) = |gl(w)(x) = 1$, $R_F(w)(x) = 1$;

(ii) if $|el(w)(x) = 0$, $R_F(w)(x) = 0$;

(iii) if $|el(w)(x) = 1$ and $|gl(w)(x) = 0$, $R_F(w)(x) = 0$ if $P(w)(x) = 0$ and $R_F(w)(x) = 1$ otherwise.

(i) and (ii) guarantee R_F satisfies the general requires on P ((a) and (b) above in the definition of $R'(e, g)$), and line (iii) shows that R_F agrees with P on its characteristic behavior ((c) above in the line following the definition of $R'(e, g)$).