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The Liar paradox and ungrammaticality

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Consider two statements.

(1) This statement is false.

(2) This sentence is ungrammatical.

Sentence (1) is known as an example of the Liar Paradox. It is impossible to assign a truth value to it, because assuming it to be either true or false leads to logical contradiction. However, a statement must be true or false. This leads to a logical paradox. There exist many other self-referent sentences similar to (1) that are paradoxical (Barber Paradox, Pinocchio Paradox, Russell's Paradox etc.). Now let us turn to (2). It is self-referential like (1) so one might expect it to be paradoxical. To check whether or not it is a paradox, we can simply assume it to be true and see if it leads to logical contradiction. If (2) is true then it must be the case that the sentence is ungrammatical. This is precisely what is expressed in the sentence so our conclusion does not contradict our hypothesis. Therefore, (2) is true. However, any native English speaker would disagree and say that (2) is perfectly grammatical[1] and is therefore false. It means that (2) must be paradoxical. Can we find a solution to avoid the paradox?

To understand that, let us outline several solutions to the problem (1) that have been suggested in literature:

a) The statement does not convey any information and is thus nonsensical. By virtue of being nonsensical it cannot bear a truth value at all.

b) The statement is not self-referent and refers to another statement in the discourse that may be false.

c) Prior (1976) suggested that every statement implicitly asserts its truth. Thus sentence (1) is equivalent to "It is true, that this statement is false". It can also be paraphrased as "This statement is true and this statement is false" which is a proposition of the form "$P \wedge \neg P$". This proposition is a simple contradiction and is

[1] By "grammatical" we mean "acceptable by a native speaker as a sentence of their native language" or "consistent with speaker's/listener's knowledge of language".

false by laws of logic. Therefore there is no paradox at all. Mills (1998) suggests a somewhat different but similar solution.

Other solutions exist, such as that of Tarski or Goedel, but are of little relevance to our discussion, as well as technical details and few complications that each of them meets.

Let us try to apply these solutions to (2).

(a) is clearly not a solution because unlike (1) it certainly does convey some information and thus it is meaningful. (b) is reasonable but it does not mean that (2) cannot be self-referent. The self-referential case of (2) is exactly what we are interested in. Moreover it is simply impossible to appeal to the discourse because there does not exist such a sentence that would validate the grammaticality status of (2). Solution (c) allows (2) to assert its own truth. Then we can say (2) is equivalent to

(3) It is true, that this sentence is ungrammatical.

However it does not allow us to use the same "trick" as in the Liar. It is important to note that what is asserted is the truth of the proposition that (2) expresses, not the truth of the sentence itself. Thus the only way to paraphrase it would be (4) or (5).

(4) That this sentence is ungrammatical is true and this sentence is ungrammatical.

(5) This sentence is ungrammatical and the previous clause is true.

Neither (3) nor (4) or (5) are of the form "$P \wedge \neg P$".

We can see that (2) is clearly different from (1), because it does not (explicitly) refer to its own truth value. Suppose we modify Prior's solution and say that every sentence also asserts its grammaticality. However, it would mean that any sequence of words is inherently grammatical. Imagine a Russian student who does not speak Italian and sees a sentence written in Italian that is ungrammatical (but he is unaware of it). Our modification would allow him to wrongly conclude that the sequence is grammatical. This is very different from the assumption that every statement asserts its truth, because a truth value can depend on contingent facts when a statement is not self-referent (e.g. "There are two cats under the table"). In other words, Prior's (or Mill's) original solution does not in fact "force" us to decide the truth value in some cases. Moreover, even if we rewrite (2) as "This sentence is grammatical and this sentence is ungrammatical", "this" refers only to a clause in which it occurs, so we are still unable to represent (2) in a solvable form. Thus such a modification is not a satistying solution.

Instead, the "$P \wedge \neg P$" form of (2) can be derived in one more way.

Suppose that it is not a sentence itself that asserts its grammaticality but a speaker-hearer that *tacitly* assigns grammaticality value to every sentence of his native language[2]. While the assignment is unconscious, our speaker-hearer always has access to the value. Note that his judgment depends solely on his knowledge of language and not the discourse. Let us now derive the "P $\wedge \neg$P" form of (2). We already saw that (2) seems to be inherently true since it does not logically contradict itself in self-reference. This constitues the left side of the formula. We also know that there is a non-contingent judgement made by the speaker-hearer. Native English speakers would uniformly judge (2) as grammatical. This means that under interpretation (2) is false and constitutes the right side of the formula.

Sentence (2) is then equivalent to (6):

(6) It is true that this sentence is grammatical and it is false that this sentence is grammatical.

(6) is of the form "P $\wedge \neg$P" which is a logical contradiction and is therefore false. This solution avoids the paradox. It does so without appealing to its meaningfulness or discourse and relies solely on speaker-hearer's knowledge of language. If one assumes that no binary value can be assigned to a sentence by a speaker-hearer's judgment than it seems impossible to solve this paradox.

References

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[2] (2) in fact preserves its seeming paradoxicality even if traslated into some other language. The weaker form of (2) would say something like "This sentence is not a sentence of English".