Zero N: number features and ⊥

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May 2020

1 Introduction

In some languages, the numeral *zero* combines with morphologically plural nouns, as exemplified in (1) for English (cf. Borer 2005, Krifka 1989). In others, such as Turkish ((2)), it combines with morphologically singular nouns (Turkish requires the use of a morphologically singular noun for all numerals, despite having morphologically plural nouns; Bale, Gagnon and Khanjian 2011, Martí 2020a, Scontras 2014, among others):

(1) *English*
One {apple | *apples}
Zero/two/fifty-five {apples | *apple}

(2) *Turkish*
Sıfır/bir/iki/üç/yirmi üç {çocuk | *çocuk-lar}
Zero/one/two/three/twenty-three boy.SG boy-PL
'Zero/one/two/three/twenty-three boy(s)'

In this squib I demonstrate that there is an explanation of these patterns that combines Martí's (2020a) account of the morphology and semantics of the numeral[1]+noun construction with Bylinina and Nouwen's (2018) semantics for *zero* and which does not need to appeal to any further principles (e.g., agreement, further discussion below in section 5).

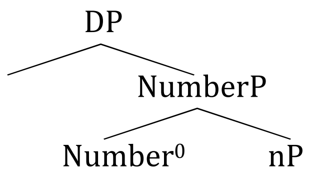
Section 2 introduces the crucial ingredients of Martí's proposal. Section 3 introduces the semantics for *zero* argued for in Bylinina and Nouwen (2018). Section 4 puts that together with the technology in section 2 to derive the *zero* facts. Section 5 discusses issues related to plurality, agreement, and the typology of grammatical number that the account in section 4 raises. Section 6 is the conclusion.

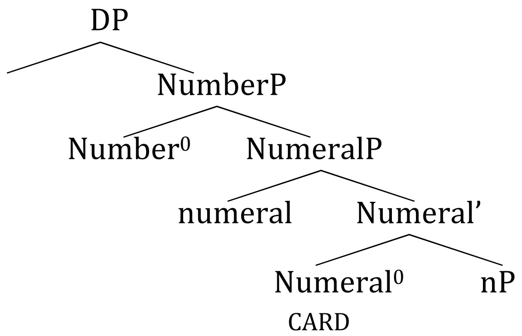
2 Martí's (2020a) account of the numeral+noun construction

Martí's (2020a) account of the pattern in (1)-(2) minus the *zero* facts is as follows. Martí assumes the syntax in (3) for noun phrases without numerals, and that in

[1] I focus on cardinals in this discussion and put aside decimals and ordinals. It does not seem difficult to extend the account proposed below to at least decimals, as suggested by Amy Rose Deal (p.c.), but I leave that task for a future occasion.

(4) for phrases with numerals (cf. Borer 2005[2], Harbour 2014, Scontras 2014, and many others)[3]:

(3)


(4)


Following Harbour (2014), nP in both (3) and (4) denotes a join semilattice (cf. Link 1983) in all cases. For just three individuals, a, b, and c, we have:

(5) ⟦nP⟧ = {a, b, c, ab, ac, bc, abc)

NumeralP is realized only in (4), with the numeral (*one*, *two*, etc.) generated as its specifier. Numeral^0 hosts Scontras' (2014) cardinality predicate (cf. Hackl 2001, and others), in (6), a function which takes a predicate P, furnished by nP, and a number, furnished by the numeral, and returns a new predicate such that each of its members is in P and is of that numerosity ('#x' stands for 'the numerosity of x'):

(6) ⟦CARD⟧ = $\lambda P \lambda n \lambda x.\ P(x)\ \&\ \#x = n$

(7) ⟦two CARD nP⟧ = $\lambda x.$ ⟦nP⟧$(x)\ \&\ \#x = 2$

NumberP hosts number features, which, following Harbour (2014), are both semantically contentful and morpho-syntactically relevant. These number features are thus taken to be responsible for the number semantics of nouns (and other nominal entities, such as pronouns or demonstratives in the languages where these show number morphology) as well as for their morphological shape. NumberP is realized in both trees, given that it is necessary in the account of number marking found in noun phrases both with and without a numeral. Martí follows Harbour's (2014) theory of NumberP-projecting features, where only

[2] Borer (2005: 114-118) proposes an explanation of these facts within the exoskeletal approach she defends there. Among other differences, in her account plural morphology is not semantically plural. One can view my proposal here as an alternative to hers in which it is.

[3] I assume that these phrases are DPs, though nothing in the account here follows from this choice of label. Material irrelevant for our purposes is possible between DP and NumberP. nP is a nominal sub-constituent that is taken to contain a root and a nominalizer, n^0, which coverts that root into a noun and gives rise to the denotation in (5).

three features are possible: [±atomic], [±minimal], and [±additive] (for the first two, see also Harbour 2011). With these three features, and a number of additional constraints, Harbour generates all and only the attested grammatical number systems found in the languages of the world, that is, the full cross-linguistic typology of number (see section 5 for more discussion of this point). Only two of those features will be necessary for us, [±atomic] and [±minimal], whose semantics is as follows[4]:

(8) $[\![+\text{atomic}]\!] = \lambda P.\lambda x.\ P(x)\ \&\ \text{atom}(x)$
$[\![-\text{atomic}]\!] = \lambda P.\lambda x.\ P(x)\ \&\ \neg\text{atom}(x)$

(9) $[\![+\text{minimal}]\!] = \lambda P.\lambda x.\ P(x)\ \&\ \neg\exists y\ P(y)\ \&\ y\sqsubset x$
$[\![-\text{minimal}]\!] = \lambda P.\lambda x.\ P(x)\ \&\ \exists y\ P(y)\ \&\ y\sqsubset x$

The feature [±atomic] is sensitive to the atomic nature of the members of $[\![\text{nP}]\!]$:

(10) $[\![\text{NumberP}]\!] = [\![+\text{atomic}]\!]([\![\text{nP}]\!]) = \lambda x.\ [\![\text{nP}]\!](x)\ \&\ \text{atom}(x) = \{a, b, c\}$
(11) $[\![\text{NumberP}]\!] = [\![-\text{atomic}]\!]([\![\text{nP}]\!]) = \lambda x.\ [\![\text{nP}]\!](x)\ \&\ \neg\text{atom}(x) = \{ab, bc, ac, abc\}$

[±Minimal] is sensitive to whether the members of the denotation of its argument have ([−minimal]) or do not have ([+minimal]) proper parts in it:

(12) $[\![+\text{minimal}]\!]([\![\text{nP}]\!]) = \lambda x.\ [\![\text{nP}]\!](x)\ \&\ \neg\exists y\ [\![\text{nP}]\!]\ (y)\ \&\ y\sqsubset x = \{a, b, c\}$
(13) $[\![-\text{minimal}]\!]([\![\text{nP}]\!]) = \lambda x.\ [\![\text{nP}]\!](x)\ \&\ \exists y\ [\![\text{nP}]\!]\ (y)\ \&\ y\sqsubset x = \{ab, bc, ac, abc\}$

In the simplest case, the argument of [±minimal] is nP, hence (12) and (13), though, as we will see shortly, this is not always the case. The argument of [±atomic] is always nP. [±Atomic] and [±minimal] give rise to the same result whenever their argument is nP. However, Harbour (2011) shows that [±atomic] and [±minimal] come apart in a number of interesting cases, including pronominal systems with an exclusive and inclusive first person distinction (where $[\![+\text{atomic}]\!](P)\neq[\![+\text{minimal}]\!](P)$), number systems with a dual (which combine the two features, so that dual number arises from the feature combination $[\![+\text{minimal}]\!]([\![-\text{atomic}]\!](P))$), and number systems with a trial (where [±minimal] repeats, so that trial number arises from the feature combination $[\![+\text{minimal}]\!]([\![-\text{minimal}]\!]([\![-\text{atomic}]\!](P)))$). Martí (2020a) argues that one further case where [±atomic] and [±minimal] come apart is precisely in their combination with numerals, as shown below. For a more detailed explanation of these arguments, see Martí (2020a: 9-14).

Martí's account for English is as follows (cf. Scontras 2014), a language in which [+atomic] is realized as Ø and [−atomic] is realized as *-s*:[5]

[4] These denotations are simplified here in ways that don't affect matters in any important way. See Martí for more on this. A full account for the numeral+noun construction across languages will need to take the remaining feature, [±additive], into account. See section 5 for more on this.
[5] Anomalous combinations are marked with the symbol '✗' here and throughout.

(14)

a. ⟦[+atomic] [$_{\text{nP}}$ boy]⟧ = λx. ⟦[$_{\text{nP}}$ boy]⟧(x) and atom(x) → *boy*

b. ⟦[−atomic] [$_{\text{nP}}$ boy]⟧ = λx. ⟦[$_{\text{nP}}$ boy]⟧(x) and ¬atom(x) → *boys*

c. ✗ ⟦[+atomic] two CARD [$_{\text{nP}}$ boy]⟧ → ~~*two boy*~~

d. ⟦[−atomic] two CARD [$_{\text{nP}}$ boy]⟧ = λx. ⟦[$_{\text{nP}}$ boy]⟧(x) & card(x) = 2 → *two boys*

e. ⟦[+atomic] one CARD [$_{\text{nP}}$ boy]⟧ = λx. ⟦[$_{\text{nP}}$ boy]⟧(x) & card(x) = 1 → *one boy*

f. ✗ ⟦[−atomic] one CARD [$_{\text{nP}}$ boy]⟧ → ~~*one boys*~~

(14)a is the only source for the singular DP *boy* and gives rise, correctly, to a singular semantics for it. In (14)a, the NP *boy* and the resulting DP *boy* have different syntactic structure and different semantics, despite sounding the same. This is in part because [+atomic] in English is spelled out as ∅. (14)b gives rise to the plural form *boys* and assigns it an exclusive plural semantics, more on which in section 5. (14)c is empty, as there are no atoms in a set of, exclusively, plural individuals of numerosity 2 (or 'twosomes', for short), so it is assumed to lead to ungrammaticality[6]. Thus, *two boy* is ungrammatical in English. (14)d is the only source for *two boys* and gives rise, correctly, to a set of boy twosomes as its semantics. (14)e is the only well-formed source for *one boy*, and it also gives rise to the correct semantics. (14)f is ill-formed, since ⟦one CARD nP⟧ is a set of atoms, and [−atomic] cannot combine with it. It is the only source for *one boys*, which is thus correctly predicted to be ungrammatical. Notice that the denotation of nP is assumed to be as in (5) in all cases—whether the noun surfaces in its singular or plural form is determined by the interaction of that denotation with the semantics of the Number^0 and Numeral^0 heads in (14). The English use of morphologically singular and plural forms in this paradigm thus follows from an interaction between morphological and semantic assumptions. More precisely, that numerals greater than 1 combine with morphologically plural nouns in English follows from the fact that only in the case of such numerals does ⟦numeral CARD [$_{\text{nP}}$ boy]⟧ satisfy the requirements of [−atomic]. *One*, on the other hand, is the only numeral where ⟦numeral CARD [$_{\text{nP}}$ boy]⟧ satisfies [+atomic]—this is how its special status in languages like English is derived.

Martí's analysis of the Turkish pattern (minus the *zero* facts) in (2) is as follows. Turkish is a [±minimal] system in this account: [+minimal] spells out as ∅ and [−minimal], as *–lAr*. We have (for *iki* 'two', *bir* 'one', and *çocuk* 'boy'):

[6] Following Gajewski's (2002) argument about the relationship between L-analyticity and ungrammaticality, Martí (2020a: 10, ft. 14) takes outputs such as (14)c to be ungrammatical.

(15)

a. $[\![[+\text{minimal}]\ [_{nP}\ \text{çocuk}]]\!] = \lambda x.\ [\![[_{nP}\ \text{çocuk}]]\!](x)\ \&$
$\neg\exists y\ [\![[_{nP}\ \text{çocuk}]]\!](y)\ \&\ y<x$ → *çocuk*

b. $[\![[-\text{minimal}]\ [_{nP}\ \text{çocuk}]]\!] = \lambda x.\ [\![[_{nP}\ \text{çocuk}]]\!](x)\ \&$
$\exists y\ [\![[_{nP}\ \text{çocuk}]]\!](y)\ \&\ y<x$ → *çocuklar*

c. $[\![[+\text{minimal}]\ \text{iki}\ \text{CARD}\ [_{nP}\ \text{çocuk}]]\!] = \lambda x.\ [\![\text{iki}\ \text{CARD}\ [_{nP}\ \text{çocuk}]]\!](x)\ \&$
$\neg\exists y\ [\![\text{iki}\ \text{CARD}\ [_{nP}\ \text{çocuk}]]\!](y)\ \&\ y<x$ → *iki çocuk*

d. ✗ $[\![[-\text{minimal}]\ \text{iki}\ \text{CARD}\ [_{nP}\ \text{çocuk}]]\!]$ → ~~*iki çocuklar*~~

e. $[\![[+\text{minimal}]\ \text{bir}\ \text{CARD}\ [_{nP}\ \text{çocuk}]]\!] = \lambda x.\ [\![\text{bir}\ \text{CARD}\ [_{nP}\ \text{çocuk}]]\!](x)\ \&$
$\neg\exists y\ [\![\text{bir}\ \text{CARD}\ [_{nP}\ \text{çocuk}]]\!](y)\ \&\ y<x$ → *bir çocuk*

f. ✗ $[\![[-\text{minimal}]\ \text{bir}\ \text{CARD}\ [_{nP}\ \text{çocuk}]]\!]$ → ~~*bir çocuklar*~~

(15)a and (15)b result, respectively, in a singular semantics for the DP *çocuk* 'boy', and an exclusive plural semantics for the DP *çocuklar* 'boys', as desired.[7] As Harbour (2011) notes, and as noted above, [±atomic] would have given the same result (see (14)a and (14)b). However, we obtain a different result in combination with numerals. For *iki çocuk* 'two boys' in (15)c, we obtain a set of boy twosomes (they have no proper parts in $[\![\text{iki CARD}\ [_{nP}\ \text{çocuk}]]\!]$, which contains only boy twosomes). This is the only possible source for *iki çocuk*, so its correct morphology and semantics are derived. (15)e denotes a set of boy individuals composed of exactly one atom, these atomic boy individuals having no proper parts in $[\![\text{bir CARD}\ [_{nP}\ \text{çocuk}]]\!]$ (which contains only boy atoms). This is the only possible source for *bir çocuk* 'one boy'. [−Minimal] never gives rise to a well-formed result when combined with a numeral, as shown in (15)d and (15)f), since [−minimal] selects from its input P those individuals that have proper parts in P, and there are no such parts in $[\![\text{iki CARD}\ [_{nP}\ \text{çocuk}]]\!]$, $[\![\text{bir CARD}\ [_{nP}\ \text{çocuk}]]\!]$, etc. Thus, that all numerals combine with morphologically singular nouns in Turkish follows from the fact that, for any numeral, $[\![\text{numeral CARD}\ [_{nP}\ \text{çocuk}]]\!]$ satisfies the requirements of only [+minimal], not [−minimal].

3 Bylinina and Nouwen's semantics for *zero*

Bylinina and Nouwen argue that *zero* is not a more emphatic version of the negative quantifier *no*. *Zero* and *no* differ in distribution ((16)-(17)), polarity ((18)-(19)) and ability to license NPIs ((20)-(21)), among other things (De Clercq 2011, Gajweski 2011, Zeiljstra 2007):

(16) John owns four cars. Bill owns zero/thirteen (*ones)
(17) John owns four cars. Bill owns *no/none
(18) No students love her, do/*don't they?
(19) Zero people love her, *do/don't they?
(20) No student has visited me in years
(21) *Zero students have visited me in years

[7] An issue that is not fully settled in the literature is whether Turkish has inclusive plurals or not (see Renans *et al.* 2017 for one view, and Görgülü 2012 for another). If it does, then all the account in (15) needs is the additional possibility in (42).

They argue for a treatment of *zero* in which, just like other numerals, it denotes a number, 0. Bylinina and Nouwen propose that the denotation of count nouns is not a (join) semilattice, as standardly assumed, but a full lattice, which includes the bottommost element, $\perp$. $\perp$ is of numerosity 0 and has no proper parts. Their proposal is to reconsider our view of pluralization as full lattice formation. (5), repeated here as (22), is replaced with (23) (in order to keep these denotations distinct, our earlier, Harbour semantics will be referred to with a subscript 'H' (cf. Link 1983); the Bylinina and Nouwen-inspired semantics will be referred to with a subscript 'BN'):[8]

(22) $[\![nP_H]\!]$ = {a, b, c, ab, ac, bc, abc)

(23) $[\![nP_{BN}]\!]$ = {$\perp$, a, b, c, ab, ac, bc, abc)

The truth-conditions for a sentence like (24), instead of being those in (25), are now those in (26), where the new version of pluralization is assumed to apply to predicates other than count nouns (e.g., *in the text*) as well:[9]

(24) There are typos in the text
(25) $\exists$x [$typo_H$(x) & $in\_the\_text_H$(x)]
(26) $\exists$x [$typo_{BN}$(x) & $in\_the\_text_{BN}$(x)]

One important issue that Bylinina and Nouwen address is that, while a semantics like that in (25) requires there to be at least one typo in the text, correctly, (26) is a tautology, since for any predicate P, P($\perp$) = 1. The same holds for the numeral+noun construction:

(27) Zero students passed the test
(28) $\exists$x[#x = 0 & $student_{BN}$ (x) & $pass\_the\_test_{BN}$ (x)]

(28) is always true, independently of the number of students who passed the test, since one can always decide that x = $\perp$. In informal terms, the problem is that the truth-conditions for (28) are predicted to be those of *zero or more students passed the test*, which can never be falsified.

The solution proposed for (27) is to note that the semantics that this view provides for numerals is an *at least* semantics, and that exhaustification can generate the required stronger, *exactly* readings. Given the truth-conditions in

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[8] Bylinina and Nouwen do not decompose nouns into the more sophisticated structures I have assumed here. (23) corresponds to their assumptions about the meaning of NPs.

[9] Just as in other accounts, predicates other than nouns, such as *in the text* or *pass the test*, need to be given the appropriate semantics if they are to compose appropriately with arguments whose denotation contains both atomic and non-atomic individuals. In Link (1983) and many others, the *-operator is in charge of this job. Bylinina and Nouwen replace that with an operator, which we can call the ^{x}-operator, that takes $\perp$ into account. Harbour and Martí are different, since for them the basic denotation of nP already includes atomic and non-atomic individuals. The *-operator (or the ^{x}-operator, if we take into account Bylinina and Nouwen's arguments) is still needed in these accounts for the treatment of predicates that are not nouns (hence, $in\_the\_text_H$ and $in\_the\_text_{BN}$ in (25)/(26)). Depending on what one assumes to be the internal semantics of nPs, one might still postulate an *-operator (or ^{x}-operator) for nouns.

(28), statements with other numerals are stronger. Uttering (27) signals that those stronger statements are false. We thus have, for (27):

(29) $\neg\exists y$ [#y > 0 student_{BN} (x) & $\text{pass\_the\_test}_{BN}$ (x)]

Taken together, (28) and (29) result in an *exactly* reading: there are zero or more students who passed the test, and there are no more than zero students who passed the test—so exactly zero did. Unlike other numerals, exhaustification is obligatory for *zero*, since no exhaustification leads to a defective, tautological interpretation. And, since the semantics of *zero* is not stronger in downward-entailing environments (the negation of a tautology is a contradiction), the *exactly* implicature still obtains in such contexts (cf. *Nobody read zero books*).

The solution for the more general problem that arises in (24), where there is no numeral to trigger exhaustification, is to assume that the existential quantifier that operates on statements without numerals is not classical ∃ but **E**, as in (30). This takes into account the fact that the denotation of NP now includes ⊥ and results in the contingent (31) for (24):

(30) **E**x[φ] ⇔ ∃x[#x > 0 & φ]
(31) **E**x [typo_{BN} (x) & in_the_text_{BN} (x)]

More precisely, Bylinina and Nouwen assume that both the **E**-operator and the ∃-operator may apply in sentences such as (24), but that, following Landman (2011), a contingent statement is better than a trivial one, that is, that a pragmatic principle against triviality is generally at work in natural language.

The postulation of the **E**-operator, which is necessary for sentences such as (24) once we assume ⊥, seems rather stipulative. In addition, the classical ∃-operator still needs to be assumed in this system, as use of the **E**-operator in sentences such as (27) results in a contradiction. Bylinina and Nouwen argue that ⊥ is desirable also in the case of sentences with downward monotone degree quantifiers such as *fewer than n*, *at most n*, etc. A sentence like *fewer than ten students passed the exam* will fail to come out true in situations in which no students passed the test unless ⊥ is assumed to be part of the denotation of count nouns. Furthermore, the polarity behavior of *zero N*, as they show, can be explained once ⊥ is assumed. Despite the stipulative flavor of the **E**-operator, it seems necessary once we include ⊥ in the denotation of common nouns.

Another issue is where the two existential operators are used. While Bylinina and Nouwen, following Hackl (2001), assume that a MANY predicate has the function of introducing ∃-quantification and combining numerals and predicates, Scontras and Martí, as discussed in section 3, assume that ∃-quantification is introduced elsewhere in the structure (cf. CARD in (6)). This difference does not have consequences for us. It is still the case in both views that the distribution of the ∃ and **E** operators is different (∃ for numerals, **E** in other cases)—embedding the ∃-operator as part of the semantics of MANY does not change this. Below, I assume (6) and, as far as existential quantification that comes from elsewhere in the structure is concerned, the ∃-operator for numerals, and the **E**-operator for noun phrases without numerals. As we will see, as far as the denotation of features is concerned, the **E**-operator is necessary.

Importantly, in full lattices, $\perp$ is not considered an atom (for something to count as an atom, it has to have $\perp$ as its only proper part; since, $\perp = \perp$, $\perp$ cannot be a proper part of $\perp$) (cf. Davey and Priestley 2002: 113). If $\perp$ is not an atom, then it is a non-atom.

4 The morphology and semantics of *zero N*

Given these assumptions, the account for the full pattern in (1)-(2) is as follows. For English, to the derivations in (14), repeated as (33)a-(33)f, we add (33)h and (33)i. Recall that we are assuming (23), repeated here for convenience:

(32) $[\![nP_{BN}]\!] = \{\perp, a, b, c, ab, ac, bc, abc)$

(33)

a. $[\![$[+atomic] [$_{nP}$ boy]$]\!]$ = λx. $[\![$[$_{nP}$ boy]$]\!]$(x) and atom(x) → *boy*
b. $[\![$[−atomic] [$_{nP}$ boy]$]\!]$ = λx. $[\![$[$_{nP}$ boy] $]\!]$(x) and ¬atom(x) → *boys*
c. ✗ $[\![$[+atomic] two CARD [$_{nP}$ boy]$]\!]$ → ~~*two boy*~~
d. $[\![$[−atomic] two CARD [$_{nP}$ boy]$]\!]$ = λx. $[\![$[$_{nP}$ boy]$]\!]$(x) & card(x) = 2 → *two boys*
e. $[\![$[+atomic] one CARD [$_{nP}$ boy]$]\!]$ = λx. $[\![$[$_{nP}$ boy]$]\!]$(x) & card(x) = 1 → *one boy*
f. ✗ $[\![$[−atomic] one CARD [$_{nP}$ boy]$]\!]$ → ~~*one boys*~~
g. $[\![$zero CARD [$_{nP}$ boy]$]\!]$ (= $\{\perp\}$)
h. ✗ $[\![$[+atomic] zero CARD [$_{nP}$ boy]$]\!]$ (= $\emptyset$) → ~~*zero boy*~~
i. $[\![$[−atomic] zero CARD [$_{nP}$ boy]$]\!]$ (= $\{\perp\}$) → *zero boys*

Using (32)/(23) instead of (22)/(5) does not change our earlier results. To see this for a case with just three individuals a, b and c, together with $\perp$, we have:

(34) $[\![+\text{atomic}]\!](\{\perp, a, b, c, ab, ac, bc, abc\}) = [\![+\text{atomic}]\!](\{a, b, c, ab, ac, bc, abc\}) = \{a, b, c\}$

That is, $\perp$ is not an atom. We obtain a different result with [−atomic]:

(35) $[\![-\text{atomic}]\!](\{\perp, a, b, c, ab, ac, bc, abc\}) = \{\perp, ab, bc, ac, abc\} \neq$
$[\![-\text{atomic}]\!](\{a, b, c, ab, ac, bc, abc\}) = \{ab, bc, ac, abc\}$

This is unproblematic, however. Exclusive plurals as in (33)b (and inclusive ones) now include $\perp$, but the solution Bylinina and Nouwen invoke in (31) applies here. In (33)c/(33)d and (33)e/(33)f, since $|\perp| = 0$, $\perp$ is neither in $[\![$two CARD [$_{NP}$ boy]$]\!]$

nor in ⟦one CARD [$_{NP}$ boy]⟧, so the results when the number features get added is as before.[10]

For English *zero*, we have the following. Just as it was the case for sets of non-atoms, the only member of the set containing {⊥}, which arises from (33)g, does not satisfy the requirements of [+atomic] ((33)h): ⊥ is not an atom. If ⊥ is not an atom, then it is a non-atom, so (33)g satisfies the requirements of [−atomic] ((33)i). Thus, the reason why with both *zero* and any numeral greater than 1 English uses the plural morphological marker on the noun is the same: both non-atoms and ⊥ are non-atoms.

Turning now to Turkish, recall that the semantics of Harbour's feature [±minimal], repeated here, makes use of the ∃-operator:

(36) ⟦+minimal⟧ = $\lambda P \lambda x.\ P(x)\ \&\ \neg\exists y\ P(y)\ \&\ y<x$
⟦−minimal⟧ = $\lambda P \lambda x.\ P(x)\ \&\ \exists y\ P(y)\ \&\ y<x$

Since we are adopting Bylinina and Nouwen's system, the question arises as to whether this semantics needs revision. Given that, with the introduction of ⊥, atoms now have proper parts (⊥ is a proper part of any atom), the semantics for [±minimal] that we need is as in (37), which uses the **E**-operator[11]:

(37) ⟦+minimal⟧ = $\lambda P \lambda x.\ P(x)\ \&\ \neg\mathbf{E}y\ P(y)\ \&\ y<x$
⟦−minimal⟧ = $\lambda P \lambda x.\ P(x)\ \&\ \mathbf{E}y\ P(y)\ \&\ y<x$

For Turkish we now have (for *iki* 'two', *bir* 'one', *sıfır* 'zero' and *çocuk* 'boy'):

[10] As a reviewer correctly points out, this makes ⟦[−atomic] [$_{nP}$]⟧ pick out a discontinuous area of the lattice (since ⟦−atomic⟧({⊥, a, b, c, ab, ac, bc, abc}) = {⊥, ab, bc, ac, abc}) and, thus, non-convex (similarly to Landman 2011, Link 1983). This very interesting observation takes us back to Harbour's (2014: 210-212) discussion of convex meanings. Harbour's Convexity condition (his (32)), that all basic meanings be convex, might seem at odds with the proposal in the text, but a way out of this problem is to view the condition as applying only to the feature that Harbour is concerned with here, [±additive]. The issue, however, deserves more careful consideration, something that I leave for future research.

[11] Thanks to Greg Scontras for discussion of this point. The question arises as to what consequences this change in the semantics of [±minimal] has in Harbour's system. I demonstrate below that number systems that use just this feature on nouns can be accounted for as before. Since ⊥ is excluded by [−minimal], any complex number value based on [−minimal] is derived without interference by ⊥, as before. Complex number values where [+minimal] is not the first feature that operates on NP will also work as before. There might be an issue with number values of pronouns based on [+minimal] or [−atomic] (cf. (35)), where ⊥ will be present, since it might be asked whether the **E**-operator applies with pronouns, but I do not explore this issue here.

(38)

a. ⟦[+minimal] [$_{nP}$ çocuk] ⟧ = λx. ⟦[$_{nP}$ çocuk]⟧(x) &
¬**E**y ⟦[$_{nP}$ çocuk]⟧(y) & y<x → *çocuk*

b. ⟦[−minimal] [$_{nP}$ çocuk]⟧ = λx. ⟦[$_{nP}$ çocuk]⟧(x) &
Ey ⟦[$_{nP}$ çocuk]⟧(y) & y<x → *çocuklar*

c. ⟦[+minimal] iki CARD [$_{nP}$ çocuk]⟧ = λx. ⟦iki CARD [$_{nP}$ çocuk]⟧(x) &
¬**E**y ⟦iki CARD [$_{nP}$ çocuk]⟧(y) & y<x → *iki çocuk*

d. ✗ ⟦[−minimal] iki CARD [$_{nP}$ çocuk]⟧ → ~~*iki çocuklar*~~

e. ⟦[+minimal] bir CARD [$_{nP}$ çocuk]⟧ = λx. ⟦bir CARD [$_{nP}$ çocuk]⟧(x) &
¬**E**y ⟦bir CARD [$_{nP}$ çocuk]⟧(y) & y<x → *bir çocuk*

f. ✗ ⟦[−minimal] bir CARD [$_{nP}$ çocuk]⟧ → ~~*bir çocuklar*~~

g. ⟦sıfır CARD [$_{nP}$ çocuk]⟧ (= {⊥})

h. ⟦[+minimal] sıfır CARD [$_{nP}$ çocuk]⟧ (= {⊥}) → *sıfır çocuk*

i. ✗ ⟦[−minimal] sıfır CARD [$_{nP}$ çocuk]⟧ (= ∅) → ~~*sıfır çocuklar*~~

Let's begin with (38)a and (38)b. With the semantics in (37), we have, for three elements a, b and c, plus ⊥:

(39) ⟦+minimal⟧({⊥, a, b, c, ab, ac, bc, abc}) = {⊥, a, b, c}

(40) ⟦−minimal⟧({⊥, a, b, c, ab, ac, bc, abc}) = {ab, ac, bc, abc}

Since both ⊥ and atoms have no proper parts of numerosity greater than 0 (⊥ has no proper parts at all, and atoms have only ⊥ as a proper part, but ⊥ does not have numerosity greater than 0), they count as minimal and are included in (39). Plural individuals are not, since they do have proper parts of numerosity greater than 0. Since neither ⊥ nor atoms have proper parts of numerosity greater than 0, they are excluded in (40). Plural individuals are, on the other hand, included now because they have proper parts of numerosity greater than 0. The denotation of morphologically singular nouns in (38)a now includes ⊥; the use of the **E**-operator that Bylinina and Nouwen invoke for bare plurals in English is invoked here as well, and we correctly predict that DPs such as *çocuk* are semantically singular, as before (cf. (15)a). The denotation of morphologically plural DPs such as *çocuklar* is just as before (cf. (15)b).

In (38)c/(38)d and (38)e/(38)f, since |⊥| = 0, ⊥ is neither in ⟦iki CARD [$_{nP}$ çocuk]⟧ nor in ⟦bir CARD [$_{nP}$ çocuk]⟧, so the results when the number features get added are as before.

Turning now to the account of *zero N* in Turkish, the only member of the set containing {⊥} ((38)g) satisfies the requirements of [+minimal], since ⊥ has no proper minimal parts at all. Thus (38)h gives rise to the correct morphology and semantics for *sıfır çocuk* 'zero boys'. On the other hand, ⊥ is not non-minimal, so [−minimal] cannot successfully apply to {⊥}, and we obtain the result in (38)i, namely, **sıfır çocuklar*. Thus, the reason why with any numeral including *zero*

Turkish uses the singular morphological marker on the noun stays the same: atoms, non-atoms and ⊥ are all proper-part-less once the numeral has combined with [CARD [$_{nP}$ çocuk]].[12, 13, 14]

5 Issues: plurality, agreement and the typology of grammatical number

Facts such as those in (1) and (2) in section 1 are often thought of as morpho-syntactic facts, usually in terms of agreement (or concord) between the noun and the numeral. For example, English may be taken to show that nouns agree in the plural with the numeral in this construction, and that the numeral *one* is special in that it does not support such agreement. Turkish can be taken to show that in some languages agreement is lacking, or that it is singular (by default) (cf. Alexiadou 2019, Ionin and Matushansky 2006, 2018). On the other hand, the account proposed above, following Martí (2020a), does not appeal to agreement or concord. There are two advantages that Martí's proposal has above one that invokes an independent mechanism to explain the number morphology of the noun in the numeral+noun construction. First, the set of tools needed to explain the number morphology of the noun (singular or plural in the case of English or Turkish) is the same set of tools that accounts for its semantics—that's because Harbour's number features, which are at the core of the proposal, have both semantic (they are semantically contentful) and morphological (they are realized morphologically) implications. An explanation that relies on an independent mechanism, such as agreement or concord, to explain the morphological make up of the noun still needs to be complemented by an account of the semantics of the construction. In the proposal above, one and the same set of tools is responsible for both aspects of the construction, which, everything else being equal, is

[12] The account of the full pattern with *zero* in (1)-(2) works in Scontras' (2014) original account as well, where, recall, Scontras' account is based on a Sauerland-style view of plurality. Scontras' account does not use Harbour's features but is compatible with Bylinina and Nouwen's analysis.

[13] In Western Armenian (Bale and Khanjian 2014: 5, ft. 4), Hungarian (Csirmaz and Szabolcsi 2012) or Slovenian (Lanko Marušič, p.c.), *zero* never combines with nouns:

(i) *Western Armenian*
*Zero {dəgha | dəgha-ner}, meg {dəgha | *dəgha-ner}, yergu {dəgha | dəgha-ner}
zero boy.SG boy-PL one boy.SG boy-PL two boy.SG boy-PL
'Zero boys, one boy, two boys'

Scontras (2014) and Martí account for the *zero*-less pattern in Western Armenian as well; Martí argues that languages like Western Armenian have access to both the English, [±atomic] number system and the Turkish, [±minimal] number system. Bylinina and Nouwen (ft. 1) suggest that this language does not license *zero* syntactically in the numeral+noun construction. Another possibility is to assume that, while the language does have the numeral *zero* (which, as before, denotes 0 and can be used to talk about mathematical calculations), the semantics of its noun phrases never contains⊥. This would predict that the Western Armenian equivalent of *fewer than/at most ten students passed the exam* will be false when no students passed the exam, a prediction that remains to be explored.

[14] *Zero* seems able to combine with mass nouns in English, though perhaps not in all languages: *zero tolerance*, *zero sugar*, etc. The question arises as to what the account of these facts is, given in particular that mass nouns in languages like English don't combine with numerals directly (e.g., **three water*). I do not have a proposal to make in this regard, but I do note the special emphatic character of these expressions (e.g., *I have zero tolerance for that behavior*, which may be paraphrased as *I have absolutely no tolerance for that behavior*).

preferable. The same argument applies here: there is no need to appeal to agreement, concord or some other morphosyntactic process to explain the *zero*+noun facts above, as one and the same set of tools accounts for both the semantics and the morphology we observe in this construction.

Second, Harbour's number features have ample independent justification, since Harbour's system derives the complex set of generalizations that characterize the typology of grammatical number across languages. This typology, known since Greenberg (1966) and discussed in great detail in Corbett (2000), is concerned not just with the number values that are attested cross-linguistically (singular and plural but also dual, trial, paucal, minimal, augmented, greater plural, global plural, and others), but with the kinds of numbers systems that are attested and unattested in the languages of the world. For example, one cross-linguistically robust generalization is that there is no language that has the dual number value without also have the plural value. Another one is that there is no language that has the trial number value without also having the dual. Thus (see Harbour 2014: 186):

(41) Trial requires dual
Dual requires singular
Singular requires plural
Plural requires singular or minimal
Unit augmented requires augmented
Minimal requires augmented or plural
Augmented requires minimal
Greater paucal requires (lesser) paucal
Paucal requires plural
Greater (and global) plural requires plural or augmented

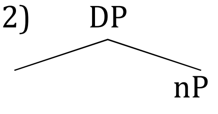
Harbour's proposal accounts for the generalizations in (41), that is, for the meaning, expression and combination of grammatical number values, with a remarkably small set of tools, which includes the features and assumptions discussed in section 2, in addition to the feature [±additive] and the assumption that one and the same number feature may repeat (neither of which is discussed there because the number values they help to account for, such as paucal or trial, are not expressed in English or Turkish). Thus, the account proposed above allows us to see the numeral+noun construction, including the *zero* facts, as part of a much larger explanation, that of the semantics and morphology of grammatical number more generally. The advantage of this is not just that the tools used to account for that construction are justified independently, it's that we now have a series of expectations about possible and impossible morphological marking on the noun in the construction as a factor of the grammatical number of the language in question. For singular-plural languages, the facts may be as in (1) or (2), with no other combinations allowed. Specific and testable predictions will be made for languages with more number values. I know of no other account of the numeral+noun construction that has the power to make cross-linguistic predictions in this fashion.

Another important issue is concerned with the semantics of plurality assumed in this account. [−Atomic] generates only an exclusive semantics for plural forms, as we saw in (14)b, where exclusive plurals are concerned with non-atoms only, and inclusive plurals, with both atoms and non-atoms. As is well-known, however, English has, descriptively, both inclusive and exclusive plurals, so a legitimate question to ask is how inclusive plurals are to be accounted for in an analysis like (14).

One popular analysis, proposed by Sauerland (2003)[15] and which Scontras (2014) uses in his analysis, takes it that singular features presuppose singularity and plural features are semantically vacuous, which is at odds with (14). In this analysis, there isn't a feature like [−atomic] alongside [+atomic] that generates an exclusive reading for plural forms. Instead, plural forms are always semantically weak, with exclusive, stronger readings arising pragmatically. I call this and related analyses the Sauerland-style view of plurality in what follows.

Martí (2020b) shows, however, that this view of plurality is incompatible with Harbour (2011, 2014). Her argument is as follows: if a language with inclusive plurals is analyzed as not making use of [−atomic], the prediction Harbour makes is that that language should have no number values that are built on [−atomic], such as dual or paucal. This prediction is wrong, since languages with inclusive plurals and duals (or paucals) do exist. Therefore, if one is to keep both Harbour's account of number, which is the only one currently capable of accounting for the crosslinguistic typology of number, and an account of inclusive plurality, one must choose the ambiguity account of plurality. The other option is to give up Harbour altogether, but then we lose the account of the typology of number in (41). Readers not convinced by this argument would presumably still agree that it is worthwhile pointing out this tension between the semantics of plurality and the cross-linguistic typology of number as it comes to be realized in current theorizing.[16]

Thus, abandoning the non-ambiguity, Sauerland-style view of plurality, Martí shows that it is possible to account for the number marking and semantics of the numeral+noun construction in English *and* for its exclusive and inclusive plurals by assuming that plural forms are ambiguous between an exclusive, [−atomic]-based semantics, and an inclusive semantics. In this approach, inclusive plurals in English arise from the possibility not to generate NumberP in numeral-less noun phrases. That is, English has both inclusive and exclusive plurals because its numeral-less plural forms spell out either (42) or (3) (with [−atomic], as in (14)b):

(42)


[15] Many others have made similar proposals. See Krifka (1989, 1995), Ivlieva (2013), Lasersohn (1998, 2011), Mayr (2015), Sauerland, Anderssen and Yatsushiro (2005), Spector (2007), Yatsushiro, Sauerland and Alexiadou (2017) and Zweig (2009); cf. Farkas and de Swart (2010) and Grimm (2012). Kiparsky and Tonhauser (2012) provide a useful overview of the main issues.

[16] The interpretation of plurals in *exactly*-phrases (e.g., *exactly one student brought wine bottles to the party*) is a well-known problem for ambiguity accounts (see Farkas and de Swart 2010: 34, footnote 25, Spector 2007) that the current account inherits. Non-ambiguity accounts deal with such problems more easily. However, non-ambiguity accounts are not compatible with Harbour (2014), as argued above, so giving it up to account for the *exactly* facts involves giving up the account of the cross-linguistic typology of number that Harbour manages to achieve. Either account has its drawbacks, and I take it that at the very least, it is a good thing to have accounts of the numeral+noun construction that are compatible with either approach to plurality.

The choice between the two is regulated e.g., by the Strongest Meaning Hypothesis (Farkas and de Swart 2010). Martí (2020a) further assumes that NumberP has to be generated if NumeralP is, as otherwise the account runs into trouble with the numeral *one*. Again, readers not convinced that the approach of Martí (2020b) and others is the right approach to plurality can take the analysis presented here as a demonstration that the Sauerland-style view of plurality is not necessary in the account of the full pattern in (1)-(2), and thus, that an account of the numeral+noun construction need not rely on a particular view of plurality, which seems like a worthy point to make.

6 Conclusion

In this squib I have added *zero* to Martí's (2020a) account of the numeral+noun construction. Two of the main types of languages that Martí considers are exemplified by English and Turkish, and it is those two types of patterns with *zero* that the analysis proposed here has been shown to account for. Importantly, the proposal makes use of the same technology (number features, a certain structural relationship between them and numerals) that Martí uses to account for the numeral+noun construction more generally, assumptions that are combined with Bylinina and Nouwen's analysis of the semantics of *zero*. Together with Martí, this squib demonstrates that the theory of grammatical number in Harbour (2014), from which the number features used here are taken, can be extended quite straightforwardly to cover a new empirical domain in a range of languages. That a small number of assumptions can account for such a large array of data, i.e., the numeral+noun construction *plus* the cross-linguistic typology of grammatical number that the features were originally designed to capture, should be seen as one of its major advantages.

In a nutshell, the noun of English *zero*+noun shows the same number marking (plural) as the noun that combines with numerals greater than 1 because $\bot$ is not an atom (like the non-atoms of English plural forms, absent with singular forms). The noun shows the same number marking (singular) as the noun that combines with all numerals in Turkish because $\bot$ does not have proper parts of numerosity greater than 0 (like the members of the denotation of Turkish singular forms, absent with plural forms). Crucial in this explanation is the idea that, while English number is sensitive to atomicity, Turkish number is sensitive to minimality, an idea inherited from Scontras' account. Once these ideas are properly implemented, the number marking on the noun that accompanies *zero* (and other numerals) follows without further stipulation.

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