Zero N: number features and ⊥ [1]

Luisa Martí
luisa.marti@qmul.ac.uk
Queen Mary University of London
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Comments welcome!

1 Introduction

Why does *zero* combine with morphologically plural nouns in languages like English (*zero apples* vs. **zero apple*) or Spanish? Why does it combine with morphologically singular nouns in other languages, such as Turkish and Hungarian? Why is *zero N* impossible in yet other languages, such as Western Armenian? Is this related to how the other numerals in the language combine with nouns? In this squib I show that there is an independently-justified, compositional semantics answer to these questions that combines Martí's (2017a) account of the morphology and semantics of the numeral+noun construction, based on Harbour's (2014) number features and on Scontras (2014), and Bylinina and Nouwen's (2017) semantics for *zero*, which is accompanied by the postulation of an existential operator **E**. I show that once these assumptions are in place, the number marking on the noun that accompanies *zero* falls out without further stipulation. Given that Martí (2017a) shows that a Sauerland-style approach to plurality (Sauerland 2003, Spector 2007, and others) is not necessary in the account of the morphology and semantics of the numeral+noun construction (Martí 2017b, building on Farkas and de Swart 2010, shows it is not necessary in the account of plurality more generally), this squib also shows that a Sauerland-style approach to plurality is not necessary in the account of the properties of *zero N*, contra Bylinina and Nouwen (2017).

2 Number marking on nouns: data

One well-known and cross-linguistically common pattern, illustrated for English below, requires the use of a morphologically plural noun in the numeral[2]+noun construction with all numerals distinct from 1 (cf. Borer 2005, Krifka 1989):

(1) One apple/*apples
  Zero/two/fifty-five apples/*apple

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[1] Thanks to Klaus Abels, Amy Rose Deal and Nilüfer Şener for discussion and data, and specially to Greg Scontras for correcting important errors in an earlier version of this paper. All remaining ones are, of course, my own.

[2] I focus on cardinals in this discussion and put aside decimals, ordinals, and fractions. It does not seem difficult to extend the account proposed below to at least decimals, as suggested by Amy Rose Deal (p.c.), but I leave that for a future occasion.

[3] I focus on cardinals in this discussion and put aside decimals, ordinals, and fractions. It does not

A second attested pattern, illustrated for Turkish below, requires the use of a morphologically singular noun for all numerals in this construction (see Bale, Gagnon and Khanjian 2011, Martí 2017a, Scontras 2014):

(2) *Turkish*
Sıfır/bir/iki/üç/yirmi üç çocuk/*çocuk-lar
Zero/one/two/three/twenty-three boy.SG/boy-PL
'Zero/one/two/three/twenty-three boy(s)'

A third pattern is illustrated by Western Armenian. In Western Armenian, *zero N* is ungrammatical, (3)a, whether the accompanying noun is morphologically singular or plural (Bale and Khanjian 2014: 5, ft. 4), even though numerals greater than 1 usually allow both options, (3)c (Bale, Gagnon and Khanjian 2011, Donabédian 1993, Sigler 1997):

(3) *Western Armenian*
a. *Zero dəgha/dəgha-ner 'Zero boys'
b. Meg dəgha/*dəgha-ner
one boy.SG/boy-PL 'One boy'
c. Yergu dəgha/dəgha-ner
two boy.SG/boy-PL 'Two boys'

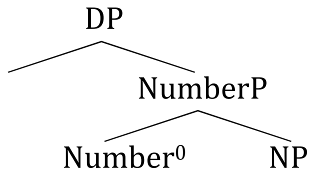
Any account of these facts should explain why *one* is special in English and Western Armenian, but not in Turkish, why *zero* is special in Western Armenian but not in English or Turkish, and why different languages make different number marking choices for numerals other than *one* (cf. (3)c with its counterparts in the other languages). Such an account also needs to get the semantics of the numeral+noun construction right in all of these cases. A compositional semantic account needs to additionally explain how the semantics of the parts that form that numeral+noun contruction contribute uniformly to the semantics of the construction. It is to these challenges that we now turn.

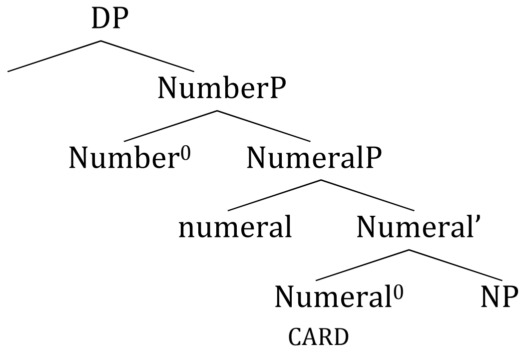
3 Martí's (2017a) account of the numeral+noun construction

The first ingredient of the explanation proposed below is Martí's (2017a) account of the pattern in (1)-(3) minus the *zero* facts. Once this account is combined with the semantics of *zero* in Bylinina and Nouwen (2017), as well as their existential operator **E**, introduced in section 4, the full pattern illustrated in (1)-(3) follows in a compositional fashion without further assumption.

Martí's (2017a) account of the numeral+noun construction is as follows. First, she assumes the following syntax; (4) is for noun phrases without numerals, (5) is for phrases with numerals ('…' indicates that other material is possible but not necessary)(cf. Borer 2005[3], Harbour 2014, Scontras 2014, and many others)[4]:

[3] Borer (2005: 114-118) proposes an explanation of these facts within the exoskeletal approach she defends there. Among other differences, in her account, plural morphology is not semantically plural. One can view my proposal here as an alternative to hers in which it is.
[4] I assume that these phrases are DPs, though nothing in the account here follows from this choice of label. Material irrelevant for our purposes here is possible between DP and NumberP.

(4)


(5)


NP in both (4) and (5) denotes a join semilattice (cf. Link 1983) in all cases. For just three individuals, a, b, and c, we have:

(6) [[NP]] = {a, b, c, ab, ac, bc, abc)

NumeralP is realized only in (5), with the numeral (*one*, *two*, etc.) generated as its specifier. Numeral^0 hosts Scontras' (2014) cardinality predicate (cf. Hackl 2001, and others), in (7), a function which takes a predicate P, furnished by NP, and a number n, furnished by the numeral, and returns a new predicate such that each of its members is in P and of numerosity n:

(7) [[CARD]] = $\lambda P \lambda n \lambda x.\ P(x)\ \&\ \#x = n$

For example:

(8) [[two CARD NP]] = $\lambda x.$ [[NP]]$(x)\ \&\ \#x = 2$

NumberP is the projection of number features, which, following Harbour (2014), are both semantically contentful and morpho-syntactically relevant. This projection is realized in both trees, given that it is necessary in the account of number marking found in noun phrases both with and without a numeral. The semantics for the number features we need, [±atomic] and [±minimal], from Harbour (2011, 2014), is as follows[5]:

(9) [[+atomic]] = $\lambda P \lambda x.\ P(x)\ \&\ \mathrm{atom}(x)$
[[−atomic]] = $\lambda P \lambda x.\ P(x)\ \&\ \neg \mathrm{atom}(x)$

(10) [[+minimal]] = $\lambda P \lambda x.\ P(x)\ \&\ \neg \exists y\ P(y)\ \&\ y<x$
[[−minimal]] = $\lambda P \lambda x.\ P(x)\ \&\ \exists y\ P(y)\ \&\ y<x$

[5] These denotations are simplified here in ways that don't affect matters in an important way. See Martí (2017a) for more on this.

The feature [±atomic] is sensitive to the atomic nature of the members of [[NP]], in (6), as follows:

(11) $[[+atomic]]([[NP]]) = \lambda x. [[NP]](x) \& atom(x)$ (={a, b, c})
$[[-atomic]]([[NP]]) = \lambda x. P(x) \& \neg atom(x)$ (={ab, ac, bc, abc})

[±Minimal] is sensitive to whether the members of the denotation of NP have ([−minimal]) or do not have ([+minimal]) proper parts:

(12) $[[+minimal]]([[NP]]) = \lambda x. [[NP]](x) \& \neg\exists y\ [[NP]](y) \& y<x$ (={a, b, c})
$[[-minimal]]([[NP]]) = \lambda x. [[NP]](x) \& \exists y\ [[NP]](y) \& y<x$ (={ab, ac, bc, abc})

Perhaps surprisingly, [±atomic] and [±minimal] give rise to the same result in the basic case. However, Harbour (2011) shows that [±atomic] and [±minimal] come apart in a number of interesting cases, including pronominal systems with an exclusive and inclusive first person distinction (where [[+atomic]](P)≠[[+minimal]](P)), number systems with a dual (which combine the two features, so that dual is [[+minimal]]([[−atomic]](P))), and number systems with a trial (where [±minimal] repeats, so that trial is [[+minimal]]([[−minimal]]([[−atomic]](P)))). Martí (2017a) argues that one further case where [±atomic] and [±minimal] come apart is precisely in their combination with numerals, as shown below.

Her account for English is as follows (cf. Scontras 2014), a language in which [+atomic] is realized as ∅ and [−atomic] is realized as *-s*:

(13)

a. [[[+atomic] [$_{NP}$ boy]]] = λx. [[boy]](x) and atom(x) → *boy*
b. [[[−atomic] [$_{NP}$ boy]]] = λx. [[boy]](x) and ¬atom(x) → *boys*
c. #[[[+atomic] two CARD [$_{NP}$ boy]]] → **two boy*
d. [[[−atomic] two CARD [$_{NP}$ boy]]] = λx. [[boy]](x) & card(x) = 2 → *two boys*
e. [[[+atomic] one CARD [$_{NP}$ boy]]] = λx. [[boy]](x) & card(x) = 1 → *one boy*
f. #[[[−atomic] one CARD [$_{NP}$ boy]]] → **one boys*

(13)a is the only source for the singular DP *boy*, and it gives rise, correctly, to a singular semantics for it. In (13)a, the NP *boy* and the resulting DP *boy* have different syntactic structure and different semantics, despite sounding the same. This is in part because [+atomic] in English is spelled out as ∅. (13)b gives rise to the plural form *boys* and assigns it an exclusive plural semantics, more on which below. (13)c is ill-formed, as there are no atoms in a set of, exclusively, plural individuals of numerosity 2 (or 'twosomes', for short). (13)c is also the only source of *two boy*, so *two boy* is ungrammatical. (13)d is the only source for *two boys* and gives rise, correctly, to a set of boy twosomes as its semantics. (13)e is the only well-formed source for *one boy*, and it also gives rise to the correct semantics. (13)f is ill-formed, since [[one CARD NP]] is a set of atoms, and [−atomic] cannot combine with it. It is the only source for *one boys*, which is thus correctly predicted to be ungrammatical. Notice that the denotation of NP is assumed to be as in (6) in all cases, contra Scontras (2014)—whether the noun surfaces in its singular or plural form is determined by the interaction of that denotation with the semantics of the Number^0 and Numeral^0 heads in (13).

Notice that the English use of morphologically singular and plural forms in this paradigm follows from an interaction between morphological and semantic assumptions. More precisely, that numerals greater than 1 combine with morphologically plural nouns in English follows from the fact that only in the case of such numerals does [[numeral CARD [$_{NP}$ boy]]] satisfy the requirements of [−atomic]. *One*, on the other hand, is the only numeral where [[numeral CARD [$_{NP}$ boy]]] satisfies [+atomic]—this is how its special status in English is derived.

[−Atomic] generates only an exclusive semantics for plural forms in English, as we saw in (13)b, where exclusive plurals are concerned with non-atoms only, and inclusive plurals are concerned with both atoms and non-atoms. As is well-known, English has both inclusive and exclusive plurals, so a legitimate question to ask is how inclusive plurals are to be accounted for in an analysis like (13). One popular analysis, proposed by Sauerland (2003)[6] and which Scontras (2014) uses in his analysis, takes it that singular features presuppose singularity and plural features are semantically vacuous. In this analysis, there isn't a feature like [−atomic] alongside [+atomic] that generates an exclusive reading for plural forms. Instead, plural forms are always semantically weak, with exclusive, stronger readings arising pragmatically. I call this and related analyses the Sauerland-style view of plurality in what follows. Martí (2017b) has shown that this view of plurality is incompatible with Harbour (2011, 2014). However, as suggested by Martí (2017a) and shown above, it is precisely Harbour's features that we are using in the account of the numeral+noun construction—one important reason for doing so is that it makes the account more principled (for details of this argument, see Martí 2017a). Thus, abandoning the Sauerland-style view of plurality, Martí (2017a) shows that it is possible to account for the number marking and semantics of the numeral+noun construction in English *and* for its exclusive and inclusive plurals by assuming that plural forms are ambiguous between an exclusive, [−atomic]-based semantics, and an inclusive semantics. In this approach, inclusive plurals in English arise from the possibility in this language not to generate NumberP in numeral-less noun phrases. (14) is the syntax of inclusive plurals then:

(14) DP
/ \
NP

English has both inclusive and exclusive plurals because plural forms in this language can spell out either (14) or (4) (with [−atomic], as in (13)b). The choice between the two is regulated, as in Farkas and de Swart (2010), by the Strongest Meaning Hypothesis. Readers not convinced that the ambiguity approach of Martí (2017a) and others is the right approach to exclusive and inclusive plurality can take the analysis presented here as a demonstration that the Sauerland-style view of plurality is not necessary in the account of the full pattern in (1)-(3). The account proposed here works with either approach to plurality.

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[6] Many others have made similar proposals. See Krifka (1989, 1995), Lasersohn (1998, 2011), Sauerland, Anderssen and Yatsushiro (2005), Spector (2007), Yatsushiro, Sauerland and Alexiadou (2017) and Zweig (2009); cf. Farkas and de Swart (2010) and Grimm (2012). Kiparsky and Tonhauser (2012) provide a useful overview of the main issues.

Martí's (2017a) analysis of the Turkish pattern (minus the *zero* facts) in (2) is as follows. Turkish is a [±minimal] system in this account: [+minimal] spells out as Ø and [−minimal] spells out as *–lAr*. We thus have (for *iki* 'two', *bir* 'one', and *çocuk* 'boy'):

(15)

a. [[[+minimal] [$_{NP}$ çocuk]]] = λx. [[çocuk]](x) & ¬∃y [[çocuk]](y) & y<x → *çocuk*

b. [[[−minimal] [$_{NP}$ çocuk]]] = λx. [[çocuk]](x) & ∃y [[çocuk]](y) & y<x → *çocuklar*

c. [[[+minimal] iki CARD [$_{NP}$ çocuk]]] = λx. [[iki CARD [$_{NP}$ çocuk]]](x) & ¬∃y [[iki CARD [$_{NP}$ çocuk]]](y) & y<x → *iki çocuk*

d. #[[[−minimal] iki CARD [$_{NP}$ çocuk]]] → * *iki çocuklar*

e. [[[+minimal] bir CARD [$_{NP}$ çocuk]]] = λx. [[bir CARD [$_{NP}$ çocuk]]](x) & ¬∃y [[bir CARD [$_{NP}$ çocuk]]](y) & y<x → *bir çocuk*

f. #[[[−minimal] bir CARD [$_{NP}$ çocuk]]] → * *bir çocuklar*

(15)a and (15)b result, respectively, in a singular semantics for the DP *çocuk* 'boy', and an exclusive plural semantics for the DP *çocuklar* 'boys', as desired. As Harbour (2011) notes, and as noted above, [±atomic] would have given the same result (see (13)a and (13)b). However, we obtain a different result in combination with numerals. For *iki çocuk* 'two boys' in (15)c, we obtain a set of boy twosomes (they have no proper parts in [[iki CARD [$_{NP}$ çocuk]]], which contains only boy twosomes). This is the only possible source for *iki çocuk*, so its correct morphology and semantics are derived. (15)e denotes a set of boy individuals composed of exactly one atom, these atomic boy individuals having no proper parts in [[bir CARD [$_{NP}$ çocuk]]] (which contains only boy atoms). This is the only possible source for *bir çocuk* 'one boy'. [−Minimal] never gives rise to a well-formed result when combined with a numeral, as shown in (15)d and (15)f), since [−minimal] selects from its input P those individuals that have proper parts in P, and there are no such parts in [[iki CARD [$_{NP}$ çocuk]]], [[bir CARD [$_{NP}$ çocuk]]], etc. Thus, that all numerals combine with morphologically singular nouns in Turkish follows from the fact that, for any numeral, [[numeral CARD [$_{NP}$ çocuk]]] satisfies the requirements of only [+minimal], not [−minimal].

For Western Armenian, Martí (2017a), following Scontras, assumes that either [±atomic], giving rise to the English pattern, or [±minimal], giving rise to the Turkish pattern, may be generated in Number^0. In either case, morphologically plural nouns for the numeral *one* are ruled out.

To summarize. The *zero*-less pattern in (1)-(3) follows in Martí's (2017a) system from the semantics of Harbour's (2011, 2014) [±atomic] and [±minimal] and their spell out as Ø vs. *–s*/*-lAr*/*-ner* and Martí's (2017b) Harbour-based analysis of exclusive and inclusive plurality. I now explain Bylinina and Nouwen's (2017) semantics for *zero* which, when added to this set of assumptions, will derive the full pattern in (1)-(3).

4 Bylinina and Nouwen's (2017) semantics for *zero*

Bylinina and Nouwen (2017) argue, first, that *zero* is not a more emphatic version of the negative quantifier *no*. *Zero* and *no* differ in distribution ((16)-(18)), polarity ((19)-(23)) and ability to license NPIs ((24)-(28)), and in split scope ((29)-(30)):

(16) John owns four cars. Bill owns zero (*ones)
John owns four cars. Bill owns thirteen (*ones)
John owns four cars. Bill owns *no/none

(17) There are zero/thirteen/??no litres of milk in the fridge
(18) John visited his grandmother zero/thirteen/??no times

(19) John doesn't love her, does/*doesn't he?
(20) John loves her, *does/doesn't he?
(21) No students love her, do/*don't they?
(22) Most students love her, *do/don't they?
(23) Zero people love her, *do/don't they? (De Clercq 2011)

(24) No student has visited me in years

(25) *Zero students have visited me in years
(26) No student said anything
(27) ?Zero students said anything (Gajweski 2011)
(28) *Zero students bought any car (Zeiljstra 2007)

(29) The company has fire no employees (Potts 2000)
#'It is <u>not</u> the case that the company is obligated to fire <u>an</u> employee'

(30) The company has fire zero employees
'It is <u>not</u> the case that the company is obligated to fire <u>an</u> employee'

Instead, they argue for a treatment of *zero* in which, just like other numerals, it denotes a number, 0, which is what we want from the perspective of the proposal in section 3. Bylinina and Nouwen propose that the denotation of count nouns is not a (join) semilattice, as is standardly assumed, but a full lattice, which includes the bottommost element, ⊥, as well. ⊥ is of cardinality 0 and has no proper parts. Their proposal is to reconsider our view of pluralization as full lattice formation. (6), repeated here, is replaced with (32) (in order to keep these denotations distinct, our earlier, Harbour semantics will be referred to using a subscript 'H' (cf. Link 1983), and the newer, Bylinina-Nouwen semantics will be referred to using a subscript 'BN'):

(31) [[NP]] = {a, b, c, ab, ac, bc, abc) (NP_H)
(32) [[NP]] = { ⊥, a, b, c, ab, ac, bc, abc) (NP_{BN})

The truth-conditions for a sentence like (33), instead of being those in (34), are now those in (35), where the new version of pluralization is assumed to apply to predicates other than count nouns (e.g., *in the text*) as well:

(33) There are typos in the text
(34) $\exists x\ [typo_H(x)\ \&\ in\_the\_text_H(x)]$
(35) $\exists x\ [typo_{BN}(x)\ \&\ in\_the\_text_{BN}(x)]$

One important issue that Bylinina and Nouwen address is that, while a semantics like that in (34) requires there to be at least one typo in the text, correctly, (35) is a tautology, since for any predicate P, $P(\perp) = 1$. The same holds for the numeral+noun construction:

(36) Zero students passed the test
(37) $\exists x[\#x = 0\ \&\ student_{BN}\ (x)\ \&\ pass\_the\_test_{BN}\ (x)]$

(37) is always true, independently of the number of students who passed the test, since one can always decide that $x = \perp$. In informal terms, the problem is that the truth-conditions for (37) are predicted to be those of *zero or more students passed the test*, which can never be falsified.

The solution proposed for (36) is to note that the semantics that this view provides for numerals is an *at least* semantics, and that exhaustification can generate the required stronger, *exactly* readings. The idea is as follows. Given the truth-conditions in (37), statements with other numerals ("one or more", "two or more") are stronger. Uttering (36) signals that those stronger statements are false. We thus have, for (36):

(38) $\neg\exists y\ [\ \#y > 0\ student_{BN}\ (x)\ \&\ pass\_the\_test_{BN}\ (x)]$

Taken together, (37) and (38) result in an *exactly* reading: there are zero or more students who passed the test, and there are no more than zero students who passed the test—so exactly zero did. Unlike other numerals, exhaustification is obligatory for *zero*, since no exhaustification leads to a defective, tautological interpretation. And, since the semantics of *zero* is not stronger in downward-entailing environments (the negation of a tautology is a contradiction), the *exactly* implicature still obtains in such contexts (cf. *Nobody read zero books*). In fact, an *at least* semantics is what the accounts in Martí (2017a) and Scontras (2014) generate for numerals—so it is no problem for these accounts to adopt Bylinina and Nouwen's solution.

The solution for the more general problem that arises in (33), where there is no numeral to trigger exhaustification, is to assume that the existential quantifier that operates on statements without numerals is not classical ∃ but **E**, as in (39). This takes into account the fact that the denotation of NP now includes $\perp$ and results in the contingent (40) for (33):

(39) $\mathbf{E}x[\varphi] \Leftrightarrow \exists x[\#x > 0\ \&\ \varphi]$

(40) **E**x [$typo_{BN}$ (x) & $in\_the\_text_{BN}$ (x)]

More precisely, Bylinina and Nouwen assume that both the **E**-operator and the **∃**-operator may apply in sentences such as (33), but that, following Landman (2011), a contingent statement is better than a trivial one, that is, that a pragmatic principle against triviality is generally at work in natural language. The postulation of the **E**-operator, which is necessary for sentences such as (33) once we assume ⊥, seems rather stipulative. In addition, the classical **∃**-operator still needs to be assumed in this system, as use of the **E**-operator in sentences such as (36) results in a contradiction. Bylinina and Nouwen argue that ⊥ is desirable also in the case of sentences with downward monotone degree quantifiers such as *fewer than n*, *at most n*, etc. A sentence like *fewer than ten students passed the exam* will fail to come out true in situations in which no students passed the test unless ⊥ is assumed to be part of the denotation of count nouns. Furthermore, the polarity behavior of *zero N*, as they show, can be explained once ⊥ is assumed. Despite the stipulative flavor of the **E**-operator, it seems necessary once we include ⊥ in the denotation of common nouns, which is in itself needed.

Another issue is where the two existential operators are used. While Bylinina and Nouwen (2017), following Hackl (2001), assume that a MANY predicate has the function of introducing **∃**-quantification and combining numerals and predicates, Scontras and Martí, as discussed in section 3, assume that **∃**-quantification is introduced elsewhere in the structure (cf. CARD in (7)). This difference does not have consequences for us. It is still the case in both views that the distribution of the **∃** and **E** operators is different (**∃** for numerals, **E** in other cases)—embedding the **∃**-operator as part of the semantics of MANY does not change this. Below, I assume (7) and, as far as existential quantification that comes from elsewhere in the structure is concerned, the **∃**-operator for numerals, and the **E**-operator for noun phrases without numerals. As we will see, as far as the denotation of features is concerned, in particular, [±minimal], the **E**-operator is necessary.

It will be important in the analysis in section 5 to bear in mind the following. In full lattices, atoms are defined as follows (see Davey and Priestley 2002: 113):

(41) Let L have a bottom element ⊥. An element x of L is an atom iff ⊥ < x and there exists no element y of L such that ⊥ < y < x

That is, in a full lattice, ⊥ is not an atom. That's because ⊥ does not have ⊥ as a proper part, since ⊥ = ⊥. If it is not an atom, then it is a non-atom. ⊥ has no proper parts, since the only part ⊥ has is ⊥ itself, and ⊥ = ⊥.

5 The morphology and semantics of *zero N*

Given these assumptions, the account for the full pattern in (1)-(3) is as follows. For English, to the derivations in (13), repeated as (43)a-(43)f, we add (43)h and (43)i. Recall that we are assuming (32), repeated here for convenience:

(42) [[NP]] = {⊥, a, b, c, ab, ac, bc, abc) (NP_{BN})

(43)

a. [[[+atomic] [NP boy]]] = λx. [[[NP boy]]](x) and atom(x) → *boy*
b. [[[−atomic] [NP boy]]] = λx. [[[NP boy]]](x) and ¬atom(x) → *boys*
c. #[[[+atomic] two CARD [NP boy]]] → **two boy*
d. [[[−atomic] two CARD [NP boy]]] = λx. [[[NP boy]]](x) & card(x) = 2 → *two boys*
e. [[[+atomic] one CARD [NP boy]]] = λx. [[[NP boy]]](x) & card(x) = 1 → *one boy*
f. #[[[−atomic] one CARD [NP boy]]] → **one boys*
g. [[zero CARD [NP boy]]] (= {⊥})
h. #[[[+atomic] zero CARD [NP boy]]] (= ∅) → **zero boy*
i. [[[−atomic] zero CARD [NP boy]]] (= {⊥}) → *zero boys*

Notice, first, that using (42)/(32) instead of (31)/(6) does not change our earlier results. To see this for a case with just three individuals a, b and c, together with ⊥, we have:

(44) [[+atomic]]({⊥, a, b, c, ab, ac, bc, abc}) = [[+atomic]]({a, b, c, ab, ac, bc, abc}) = {a, b, c}

That is, ⊥ is not an atom. We do obtain a different result with [−atomic], of course:

(45) [[−atomic]]({⊥, a, b, c, ab, ac, bc, abc}) = {⊥, ab, bc, ac, abc} ≠ [[−atomic]]({a, b, c, ab, ac, bc, abc}) = {ab, bc, ac, abc}

This is unproblematic, however. Exclusive plurals as in (43)b (and inclusive ones) now include ⊥, but the solution Bylinina and Nouwen invoke in (40) applies here. In (43)c/(43)d and (43)e/(43)f, since |⊥| = 0, ⊥ is neither in [[two CARD [NP boy]]] nor in [[one CARD [NP boy]]], so the results when the number features get added is as before.

For English *zero*, we have the following. Just as it was the case for sets of non-atoms, the only member of the set containing {⊥}, which arises from (43)g, does not satisfy the requirements of [+atomic] ((43)h): ⊥ is not an atom. If ⊥ is not an atom, then it is a non-atom, so (43)g satisfies the requirements of [−atomic] ((43)i). Thus, the reason why with both *zero* and any numeral greater than 1 English uses the plural morphological marker on the noun is the same: both non-atoms and ⊥ are non-atoms.

Turning now to Turkish, recall that the semantics of Harbour's feature [±minimal], repeated here, makes use of the ∃-operator:

(46) [[+minimal]] = λPλx. P(x) & ¬∃y P(y) & y<x
[[−minimal]] = λPλx. P(x) & ∃y P(y) & y<x

Since we are adopting Bylinina and Nouwen's system, the question arises as to whether this semantics needs revision. Given that, with the introduction of $\bot$, atoms now have proper parts ($\bot$ is a proper part of any atom), the semantics for [±minimal] that we need is as in (47), which uses the **E**-operator[7]:

(47) $[[+\text{minimal}]] = \lambda P \lambda x.\ P(x)\ \&\ \neg \mathbf{E}y\ P(y)\ \&\ y<x$
$[[-\text{minimal}]] = \lambda P \lambda x.\ P(x)\ \&\ \mathbf{E}y\ P(y)\ \&\ y<x$

For Turkish we now have the following (for *iki* 'two', *bir* 'one', *sıfır* 'zero' and *çocuk* 'boy'):

(48)

a. [[[+minimal] [$_{NP}$ çocuk]]] = λx. [[[$_{NP}$ çocuk]]](x) & ¬**E**y [[[$_{NP}$ çocuk]]](y) & y<x → *çocuk*

b. [[[−minimal] [$_{NP}$ çocuk]]] = λx. [[[$_{NP}$ çocuk]]](x) & **E**y [[[$_{NP}$ çocuk]]](y) & y<x → *çocuklar*

c. [[[+minimal] iki CARD [$_{NP}$ çocuk]]] = λx. [[iki CARD [$_{NP}$ çocuk]]](x) & ¬**E**y [[iki CARD [$_{NP}$ çocuk]]](y) & y<x → *iki çocuk*

d. #[[[−minimal] iki CARD [$_{NP}$ çocuk]]] → * *iki çocuklar*

e. [[[+minimal] bir CARD [$_{NP}$ çocuk]]] = λx. [[bir CARD [$_{NP}$ çocuk]]](x) & ¬**E**y [[bir CARD [$_{NP}$ çocuk]]](y) & y<x → *bir çocuk*

f. #[[[−minimal] bir CARD [$_{NP}$ çocuk]]] → * *bir çocuklar*

g. [[sıfır CARD [$_{NP}$ çocuk]]] (= {$\bot$})

h. [[[+minimal] sıfır CARD [$_{NP}$ çocuk]]] (= {$\bot$}) → *sıfır çocuk*

i. #[[[−minimal] sıfır CARD [$_{NP}$ çocuk]]] (= ∅) → * *sıfır çocuklar*

Let's begin with (48)a and (48)b. With the semantics in (47), we have, for three elements a, b and c, plus $\bot$:

(49) [[+minimal]]({$\bot$, a, b, c, ab, ac, bc, abc}) = {$\bot$, a, b, c}

(50) [[−minimal]]({$\bot$, a, b, c, ab, ac, bc, abc}) = {ab, ac, bc, abc}

Since both $\bot$ and atoms have no proper parts of numerosity greater than 0 ($\bot$ has no proper parts at all, and atoms have only $\bot$ as a proper part, but $\bot$ does not have numerosity greater than 0), they count as minimal and are included in (49). Plural individuals are not, since they do have proper parts of numerosity greater than 0. Since neither $\bot$ nor atoms have proper parts of numerosity greater than 0, they are excluded in (50). Plural individuals are, on the other

[7] Thanks to Greg Scontras for pointing this out. It remains to be seen what consequences this change in the semantics of [±minimal] has in Harbour's system. I demonstrate below that number systems that use just this feature on nouns can be accounted for as before. Since $\bot$ is excluded by [−minimal], any complex number value based on [−minimal] is derived without interference by $\bot$, as before. Complex number values where [+minimal] is not the first feature that operates on NP will also work as before. There might an issue with number values of pronouns based on [+minimal] or [−atomic] (cf. (45)), where $\bot$ will be present, since it might be asked whether the **E**-operator applies with pronouns, but I do not explore this issue here.

hand, included now because they have proper parts of numerosity greater than 0. The denotation of morphologically singular nouns in (48)a now includes ⊥; the use of the **E**-operator that Bylinina and Nouwen invoke for bare plurals in English is invoked here as well, and we correctly predict that DPs such as *çocuk* are semantically singular, as before (cf. (15)a). The denotation of morphologically plural DPs such as *çocuklar* is just as before (cf. (15)b).

In (48)c/(48)d and (48)e/(48)f, since |⊥| = 0, ⊥ is neither in [[iki CARD [$_{NP}$ çocuk]]] nor in [[bir CARD [$_{NP}$ çocuk]]], so the results when the number features get added are as before.

Turning now to the account of *zero N* in Turkish, the only member of the set containing {⊥} ((48)g) satisfies the requirements of [+minimal], since ⊥ has no proper minimal parts at all. Thus (48)h gives rise to the correct morphology and semantics for *sıfır çocuk* 'zero boys'. On the other hand, ⊥ is not non-minimal, so [−minimal] cannot successfully apply to {⊥}, and we obtain the result in (48)i, namely, **sıfır çocuklar*. Thus, the reason why with any numeral including *zero* Turkish uses the singular morphological marker on the noun stays the same: atoms, non-atoms and ⊥ are all proper-part-less once the numeral has combined with [CARD [$_{NP}$ çocuk]].

Finally, for Western Armenian, there are at least two approaches we can take. One possibility, suggested by Bylinina and Nouwen (2017: ft. 1), is that Western Armenian does not license *zero* in prenominal position. According to this hypothesis, the semantics of NP in Western Armenian is as in Turkish and English above; in particular, it includes ⊥, and the reason why nouns do not combine with *zero* is syntactic. Another possibility is to assume that, while the language does have the numeral *zero* (which, as before, denotes 0 and can be used to talk about mathematical calculations, for example), the semantics of NP never contains ⊥, and the only possible derivations for Western Armenian would be as in (13)/(15). This second possibility would have important consequences for Western Armenian semantics. For example, it predicts that the Western Armenian equivalent of *fewer than/at most ten students passed the exam* will be false when no students passed the exam. Finding out which analysis is better for Western Armenian and languages like it[8] is a matter for another time.[9]

6 Conclusion

In this squib I have proposed a semantic explanation for the pattern of number marking on nouns in the numeral+noun construction in three different languages. The reason why the N of *zero N* shows the same number marking (plural) as the noun that combines with numerals greater than 1 in English is that ⊥, the bottommost element in the full lattice of N, is not an atom (like the non-atoms of English plural forms, absent with singular forms). In turn, the

[8] Western Armenian is not the only language lacking a *zero* for the numeral+noun construction—e.g., Hungarian (Csirmaz and Szabolcsi 2012) or Slovenian also lack it (Lanko Marušič, p.c.).

[9] The account of the full pattern with *zero* in (1)-(3) works in Scontras' (2014) original account as well, where, recall, Scontras' account is based on a Sauerland-style view of plurality. Scontras' original account does not make use of Harbour's features, but it is compatible with Bylinina and Nouwen's treatment of *zero*.

reason why the N of *zero N* shows the same number marking (singular) as the noun that combines with all numerals in Turkish is that ⊥ does not have proper parts of numerosity greater than 0 (like the members of the denotation of Turkish singular forms, absent with plural forms). This explanation makes sense in a theory where English number is sensitive to atomicity, and Turkish number is sensitive to minimality, as argued in Martí (2017a) and Scontras (2014). The reason why Western Armenian *zero* doesn't combine with nouns might be that nouns in this language do not include ⊥ in their denotation (whereas, following Bylinina and Nouwen's (2017) proposal for English, both English and Turkish nouns do), or it might be due to a syntactic constraint.

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