Import-Export and 'And'[*]

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Import-Export says that a conditional of the form $\ulcorner p > (q > r) \urcorner$ is always equivalent to the corresponding conditional $\ulcorner (p \wedge q) > r \urcorner$. I argue that *Import-Export* does not sit well with a classical approach to conjunction: given some plausible and widely accepted principles about conditionals, *Import-Export* together with classical conjunction leads to absurd consequences. My main goal is to draw out these surprising connections. In concluding, I argue that the best response to these facts is to reject *Import-Export* and adopt instead a limited version of that principle which better fits natural language data, still accounts for the intuitions that motivate *Import-Export*, and sits more easily with a classical conjunction.

1. Introduction

Gibbard (1981) showed that *Modus Ponens* and *Import-Export*—two *prima facie* plausible principles about natural language conditionals—do not happily co-exist: theorists of the conditional must pledge allegiance to at most one of these, if they do not want to interpret the natural language conditional as the material conditional. McGee (1985) showed that there is good reason to take seriously the possibility that *Modus Ponens* (*MP*) is false and *Import-Export* (*IE*) true. Neglected in the subsequent literature has been the fact that adopting *IE* has striking consequences for a seemingly unrelated issue: the semantics of conjunction. In this paper I argue that *IE* does not sit well with the classical Boolean semantics for 'and': if we adopt *IE* together with some attractive and widely accepted principles about the conditional, then the classical rules of conjunction introduction and elimination cannot be valid.

My main goal is to point out the surprising relation between *IE* and 'and'. In concluding, however, I will suggest that the right conclusion to draw about the conditional is that it does not validate *IE* in full generality, but rather validates a slightly weaker principle, which I argue better fits the natural language data; accounts for all the intuitions that motivate *IE* in the first place; and sits more easily with a classical conjunction.

2. *Modus Ponens* and *Import-Export*

Let us work with a toy language $\mathcal{L}$ containing arbitrarily many atomic sentences and closed under the one-place operator '¬' and two-place connectives '∧', '>', and '⊃'. I use capital

italics to range over atoms in the language, lower-case italics to range over arbitrary sentences of the language and lower-case Greek letters to range over propositions (sets of possible worlds). '$\wedge$' and '$\neg$' are intended to correspond to 'and' and 'not', as usual. $\ulcorner p > q \urcorner$ corresponds to the natural language conditional $\ulcorner$ If p, then $q \urcorner$ (on either its indicative or subjunctive reading; everything I say here goes for both kinds of conditional, *modulo* the points in Footnote 24). And $\ulcorner p \supset q \urcorner$ corresponds to the material conditional, given its standard semantics: $\ulcorner p \supset q \urcorner$ is true just in case p is false or q true.[1] Call the fragment of our language without '$>$' the *conditional-free fragment*, or $\mathcal{L}_{CF}$ for brevity. We assume a stock of possible worlds $\mathcal{W}$ and a valuation function $\mathfrak{I}$ which takes atomic sentences to subsets of $\mathcal{W}$.[2] Let $[\![p]\!]^{f_1,f_2...f_n,w}$ be the truth-value of p at w relative to the sequence of parameters $\langle f_1, f_2 ... f_n \rangle$ (specified differently in the various theories we will explore), and let $[\![p]\!]^{f_1,f_2...f_n}$ be the proposition p expresses at $\langle f_1, f_2 ... f_n \rangle$ (the set of possible worlds where p is true relative to $\langle f_1, f_2 ... f_n \rangle$, i.e. $\{ w \in \mathcal{W} : [\![p]\!]^{f_1,f_2...f_n,w} = 1 \}$). We assume that for all atoms A, $[\![A]\!]^{f_1,f_2...f_n} = \mathfrak{I}(A)$. Finally, in addition to $\mathcal{W}$, we have an absurd world λ at which every sentence in the language is true.

Our two key principles about the conditional run as follows. ('c' ranges over contexts, which are sequences of parameters—again, specified differently in different semantic theories we will explore—provided by concrete speech situations for the evaluation of modal content.)

Modus Ponens (MP): $\forall c : \forall p, q \in \mathcal{L} : ([\![p]\!]^c \cap [\![p > q]\!]^c) \subseteq [\![q]\!]^c$

Import-Export (IE): $\forall c : \forall p, q, r \in \mathcal{L} : [\![p > (q > r)]\!]^c = [\![(p \wedge q) > r]\!]^c$

MP says, for instance, that if both 'It is raining' and 'If it is raining, the picnic is cancelled' are true, then 'The picnic is cancelled' is also true; and likewise that if both 'It rained yesterday' and 'If it had rained yesterday, the picnic would have been cancelled' are true, then 'The picnic was cancelled' is also true. And *IE* says, for instance, that 'If the picnic was cancelled, then if Mark was excited for the picnic, he was disappointed' always expresses the same thing (in a given context) as 'If the picnic was cancelled and Mark was excited for the picnic, he was disappointed'; likewise that 'If the picnic had been cancelled, then if Mark had been excited for the picnic, he would have been disappointed' always expresses the same thing in a given context as 'If the picnic had been cancelled and Mark had been excited for the picnic, he would have been disappointed'.

Both these principles are *prima facie* very natural. But Gibbard (1981) showed that these two principles entail that the natural language conditional is the material conditional, provided we also assume that for any c, if $[\![p]\!]^c \subseteq [\![q]\!]^c$, then $[\![p > q]\!]^c = \mathcal{W}$; i.e. when p entails q, $\ulcorner p > q \urcorner$ is a theorem. This latter assumption, which I call *Conditional Deduction*, seems essentially beyond reproach.[3] Briefly, Gibbard's proof goes as follows

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[1] I leave disjunction out of the story, for simplicity. We could define it as usual out of negation and conjunction; or, if we opt for a non-classical conjunction, we could combine that with a classical disjunction, giving up de Morgan equivalences.

[2] I will write e.g. '$A(w)=1$' or '$A(w)=0$' to indicate $w \in \mathfrak{I}(A)$ or $w \notin \mathfrak{I}(A)$.

[3] At least when p and q are conditional-free. It would be more in the spirit of the theories we are exploring here to state *Conditional Deduction* with q relativized to a context parameter updated with p—PM and PK do not actually validate *Conditional Deduction* as stated here when q contains a conditional, since, as I discuss below, $\ulcorner p > p \urcorner$ is not a theorem for them when p is in the conditional fragment—but this will not affect present interests, as Gibbard's result, and those we explore below, are decisive even if we focus on conditional-free p and q. One grounds for rejecting *Conditional Deduction* even in that case comes from theorists of conditionals with impossible antecedents who argue that for some inconsistent p and some q, $\ulcorner p > q \urcorner$ is false, but nonetheless p entails q; see Nolan 1997. I will not explore this possibility here.

(following presentation in Khoo 2013): for any context c and sentences p, q, under the assumption that $\ulcorner \neg p \wedge p \urcorner$ is nowhere true,[4] it follows that $[\![\neg p \wedge p]\!]^c \subseteq [\![q]\!]^c$, and thus by *Conditional Deduction* that $[\![(\neg p \wedge p) > q]\!]^c = \mathcal{W}$, and thus, by *IE*, that $[\![\neg p > (p > q)]\!]^c = \mathcal{W}$. By *MP*, it follows that $[\![\neg p]\!]^c \subseteq [\![p > q]\!]^c$. Next, under the assumption that $[\![q \wedge p]\!]^c \subseteq [\![q]\!]^{c,5}$ we know by *Conditional Deduction* that $[\![(q \wedge p) > q]\!]^c = \mathcal{W}$. By *IE*, $[\![q > (p > q)]\!]^c = \mathcal{W}$. By *MP*, $[\![q]\!]^c \subseteq [\![p > q]\!]^c$. Given the meaning of the material conditional '$\supset$', on which, again, $[\![p \supset q]\!]^c = [\![\neg p]\!]^c \cup [\![q]\!]^c$, it follows by set theory that $[\![p \supset q]\!]^c \subseteq [\![p > q]\!]^c$. *MP* guarantees that $[\![p > q]\!]^c \subseteq [\![p \supset q]\!]^c$. Thus $\forall c : \forall p, q \in \mathcal{L} : [\![p > q]\!]^c = [\![p \supset q]\!]^c$: that is, the natural language conditional '$>$' is the material conditional '$\supset$'.

The problem is that '$>$' is *not* the material conditional. This is almost universally accepted by those who study conditionals.[6] A quick argument: if '$>$' were the material conditional, then the negation of $\ulcorner p > q \urcorner$ would entail p. But it does not. For instance, (1) is true whether or not Patch turns out to be a rabbit:

(1)   It's not the case that, if Patch is a rabbit, she is a rodent. (Khoo & Mandelkern 2018)

That is, (1) does not entail that Patch is a rabbit—contrary to the predictions of a theory which said that the indicative conditional in (1) is the material conditional. It is equally clear, for parallel reasons, that the subjunctive conditional is not the material conditional, since (2) clearly does not entail that Patch is a rabbit.

(2) It's not the case that, if Patch had been a rabbit, she would have been a rodent.

Arguments like this can be easily multiplied, though I will stop here, since the point is well established: '$>$' is not '$\supset$'. Given the plausibility of *Conditional Deduction*, the consensus is that we must therefore validate at most one of *MP* and *IE*.[7]

Again, both principles are *prima facie* plausible. There is much to say in favor of, and against, both. I will not go into the details of this debate here. My main goal in this paper is not to argue for either of these principles, but rather to point out some downstream consequences of one possible choice between them, namely the choice to go with *IE*. McGee (1985), as we will review in more detail below, argued that there is reason to take seriously the possibility that *MP* is invalid and *IE* is valid; and Khoo & Mandelkern

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[4]   As we discuss below, $\ulcorner \neg p \wedge p \urcorner$ can be true in the PK or PM semantics given below, in particular when p is itself a conditional; even in those frameworks, though, $\ulcorner \neg p \wedge p \urcorner$ is never true when p is conditional-free, and so even in those frameworks, Gibbard's result will show that $\ulcorner p > q \urcorner$ is equivalent to $\ulcorner p \supset q \urcorner$ whenever p itself is conditional-free, provided *MP*, *IE*, and *Conditional Deduction* are all valid.

[5]   We will call into question right conjunction elimination below, but not left.

[6]   Some, like Lewis (1976), Jackson (1979), Grice (1989), Rieger (2006), have argued that contrasts in assertability between material conditionals and indicative conditionals can be explained pragmatically. But these accounts, even if successful, would not explain embedding data like (1). Moreover, to my knowledge no one has defended the thesis that the subjunctive conditional is the material conditional; but Gibbard's proof is *prima facie* just as worrisome for the subjunctive conditional as for the indicative (since, at first blush, both seem to validate *Conditional Deduction*, *MP*, and *IE*).

[7]   Kratzer (1986) claims to evade Gibbard's proof by challenging an implicit premise—that '$>$' is a two-place operator—but Khoo (2013) shows persuasively that this does not in fact allow Kratzer to avoid Gibbard's result, and that she does so because, and only because, she invalidates *MP*.

(2018) argue that the subsequent literature has not produced a convincing counterexample to *IE*, at least for indicative conditionals. So it at least seems open, given the present state of the literature, that *IE* is valid and *MP* invalid; and this suffices to motivate exploration of the consequences of validating *IE*.

3. Validating *Import-Export*

There are a number of theories which validate *IE*. I will spell out three representative theories—one from the literature, and two close variations on existing theories—to focus our investigation here. The first is due to McGee 1985. McGee builds on Stalnaker (1968), Stalnaker & Thomason (1970)'s theory of conditionals, which validates *MP*, not *IE*. McGee modifies the theory so that it validates *IE*, not *MP*. On McGee's theory, sentences of our language are evaluated relative to two modal parameters. The first is a Stalnakerian *selection function f* from propositions and worlds to worlds, which (i) takes any proposition and world to a world where that proposition is true (intuitively, the 'closest' world where that proposition is true); (ii) takes a proposition and world to the absurd world λ just in case the proposition in question is inconsistent; (iii) takes $\langle \varphi, w \rangle$ and $\langle \psi, w \rangle$ to the same world if $f(\varphi, w) \in \psi$ and $f(\psi, w) \in \varphi$; and (iv) takes $\langle \varphi, w \rangle$ to w just in case $w \in \varphi$. The second parameter is *a hypothesis set*: a set of sentences Γ, which will serve to keep track of conditional antecedents. Context will always supply an empty hypothesis set; the only role of the hypothesis set is to keep track of intra-sentential dynamics in conditionals.[8] Then we have the following semantic rules:

McGee Semantics:

- $[\![A]\!]^{f,\Gamma,w} = 1$ iff $f(\bigcap_{p \in \Gamma} [\![p]\!]^{f,\varnothing}, w) \in \mathfrak{I}(A)$

- $[\![\neg p]\!]^{f,\Gamma,w} = 1$ iff $[\![p]\!]^{f,\Gamma,w} = 0$

- $[\![p \wedge q]\!]^{f,\Gamma,w} = 1$ iff $[\![p]\!]^{f,\Gamma,w} = 1$ and $[\![q]\!]^{f,\Gamma,w} = 1$

- $[\![p > q]\!]^{f,\Gamma,w} = [\![q]\!]^{f,\Gamma \cup \{p\},w}$

Relative to an empty hypothesis set, atomic sentences, negation, and conjunction have their standard classical interpretations. The role of 'if'-clauses in conditionals is solely to add material to the hypothesis set: the conditional's consequent is then evaluated relative to an updated hypothesis set which includes its antecedent. The members of the hypothesis set are each interpreted relative to the matrix selection function and an empty hypothesis set; their intersection then serves as an argument for a Stalnakerian selection function. For simple conditionals—conditionals of the form $\ulcorner p > q \urcorner$ where $p, q \in \mathcal{L}_{CF}$ (the conditional-free fragment)—the resulting logic coincides with Stalnaker's: $\ulcorner p > q \urcorner$ is true just in case the closest p world is a q world. But for complex conditionals, the accounts diverge: in McGee's system, a conditional of the form $\ulcorner p > (q > r) \urcorner$ is true just in case $\ulcorner q > r \urcorner$ is true relative to a hypothesis set to which

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[8] All these conditions are left implicit in McGee's own presentation, but I assume they are intended. Quantification over contexts throughout is thus just quantification over selection functions, since the hypothesis set remains empty in all contexts. Likewise for PM and PK, *mutatis mutandis*.

p has been added, just in case r is true relative to a hypothesis set to which p and q have both been added. This renders $\ulcorner p > (q > r) \urcorner$ equivalent to $\ulcorner (p \wedge q) > r \urcorner$, sufficing to validate *IE*. By contrast, *MP* will not be valid: an easy way to see this is that $\ulcorner p > (\neg p > r) \urcorner$ will always be true, for any p and r; but there are worlds where p is true and $\ulcorner \neg p > r \urcorner$ false.

The second theory I explore here is a close variant on McGee's theory, which I'll call *Pseudo-McGee* (*PM*), and runs as follows. The modal parameters in this case are a selection function, as above, plus a *premise set* Φ: a set of propositions, which again we assume always starts out empty, and whose only role, again, is to keep track of the intra-sentential dynamics of information.

Pseudo-McGee Semantics:

- $[\![A]\!]^{f,\Phi,w} = 1$ iff $f(\bigcap \Phi, w) \in \mathfrak{I}(A)$

- $[\![\neg p]\!]^{f,\Phi,w} = 1$ iff $[\![p]\!]^{f,\Phi,w} = 0$

- $[\![p \wedge q]\!]^{f,\Phi,w} = 1$ iff $[\![p]\!]^{f,\Phi,w} = 1$ and $[\![q]\!]^{f,\Phi \cup \{[\![p]\!]^{f,\Phi}\},w} = 1$

- $[\![p > q]\!]^{f,\Phi,w} = [\![q]\!]^{f,\Phi \cup \{[\![p]\!]^{f,\Phi}\},w}$

This semantics is exactly like McGee's, except that what gets added to our modal parameters as we process a conditional is a proposition, not a sentence. This means that what gets added to premise sets are the denotations of sentences interpreted relative to their local premise sets, rather than uninterpreted sentences which will ultimately be interpreted relative to an empty premise set. This makes a crucial difference when it comes to importing and exporting a sentence which is itself a conditional, since what will get added to the premise set will be the conditional *as interpreted relative to a non-empty premise set*. Thus, in order to validate *IE* in that case, we must adopt the non-classical conjunction here, which ensures that a right conjunct is evaluated relative to a premise set updated with the content of the left conjunct. *MP* will be invalid; the same example which showed this for McGee will show this for PM.

The final theory I will put on the table here is due to Kratzer (1981, 1986), augmented with a conjunction along the lines suggested in Khoo & Mandelkern 2018; call this theory *Pseudo-Kratzer* (*PK*). This theory again has two modal parameters. The first is a *modal base function g* that takes each world to a set of propositions. The role of the modal base, like hypothesis and premise sets, is to keep track of information in conditionals; we thus again assume that any context provides a modal base which takes every world to the empty set. The second element is a function $\preceq$ from worlds to well-founded partial pre-orders on worlds such that for any world w, w is strictly minimal in $\preceq(w)$.[9] Finally, this approach augments our language with a covert modal 'ᴍ'.[10] Then:

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[9]   My presentation simplifies Kratzer's in moving directly to a pre-order rather than going by way of an ordering source, and in assuming that $\preceq(w)$ is well-founded for all w (the limit assumption). Both of these simplifications are harmless for present purposes. I also assume (the model-theoretic version of) strong centering, which is necessary for proving some of the facts below.

[10]   'ᴍ' counts as part of the conditional fragment of our language, so $\mathcal{L}_{CF}$ does not contain 'ᴍ'.

Pseudo-Kratzer Semantics:

- $[\![A]\!]^{g,\preceq,w} = 1$ iff $w \in \mathfrak{I}(A)$

- $[\![\neg p]\!]^{g,\preceq,w} = 1$ iff $[\![p]\!]^{g,\preceq,w} = 0$

- $[\![p \wedge q]\!]^{g,\preceq,w} = 1$ iff $[\![p]\!]^{g,\preceq,w} = 1$ and $[\![q]\!]^{g^{p^{g,\preceq}},\preceq,w} = 1$,
  where $g^{p^{g,\preceq}}$ is the smallest function such that $\forall w' : g^{p^{g,\preceq}}(w') = g(w') \cup \{[\![p]\!]^{g,\preceq}\}$

- $[\![p > q]\!]^{g,\preceq,w} = [\![q]\!]^{g^{p^{g,\preceq}},\preceq,w}$

- $[\![\mathrm{M}(p)]\!]^{g,\preceq,w} = 1$ iff $\forall w' \in \bigcap g(w) : w'$ is minimal in $\preceq_g(w) \rightarrow [\![p]\!]^{g,\preceq,w'} = 1$,
  where $\preceq_g(w)$ is the limitation of $\preceq(w)$ to $\bigcap g(w)$[11]

Atoms and negation are treated classically. The only role of a conditional antecedent is to add its information to the modal base (much as in McGee's theory). Crucially, Kratzer assumes that conditionals without overt modals (which is to say all conditionals in our language, which does not contain overt modals) always contain the covert modal 'M' which takes scope over everything which follows the most deeply embedded '>' in the conditional; thus e.g. where r does not contain 'M' or '>', instead of $\ulcorner p > r \urcorner$ we will have $\ulcorner p > \mathrm{M}r \urcorner$; likewise, instead of $\ulcorner p > (q > r) \urcorner$ we will have $\ulcorner p > (q > \mathrm{M}r) \urcorner$. $\ulcorner \mathrm{M}p \urcorner$ is true just in case p is true at all the minimal worlds in the intersection of the value of the modal base. Putting this all together, simple conditionals of the form $\ulcorner p > \mathrm{M}q \urcorner$ will be true just in case all the minimal p worlds consistent with the value of the modal base are q worlds.[12] When it comes to complex conditionals, the clause for conditionals, together with the assumption about covert modals, ensures that we keep track of consecutive antecedents, just as for McGee and PM. So $\ulcorner p > (q > \mathrm{M}r) \urcorner$ will be true just in case $\ulcorner q > \mathrm{M}r \urcorner$ is true relative to a modal base which includes the information that p is true, which holds just in case r is true at all the minimal worlds where p and q are true. When q is conditional-free, this suffices to guarantee that $\ulcorner p > (q > \mathrm{M}r) \urcorner$ is equivalent to $\ulcorner (p \wedge q) > \mathrm{M}r \urcorner$. And our clause for conjunction ensures that right conjuncts are evaluated relative to a modal base which includes the information in the left conjunct. This, in turn, suffices to validate *IE* in full generality, even when q is a conditional (more below). *MP*, by contrast, will not be valid; the same example given above shows this.

    All three of these theories, then, validate *IE*. They differ in a variety of ways, but what I want to focus on here is their treatment of 'and'. McGee's theory has a classical 'and'. But PM and PK give 'and' a decidedly non-classical treatment. On both those theories, 'and' is fully classical for the conditional-free fragment; but not when we have a conjunction whose right conjunct contains a conditional. Then the conditional is evaluated, in PM, relative to a premise set updated with the left conjunct; and, in PK, relative to a modal base which is updated with the left conjunct. To illustrate this divergence in their treatment of 'and', consider the conjunction $\ulcorner p \wedge (\neg p > q) \urcorner$ for any conditional-free p. In McGee's theory, this conjunction is true relative to $\langle f, \Gamma, w \rangle$ just in case both p and $\ulcorner \neg p > q \urcorner$ are true relative to $\langle f, \Gamma, w \rangle$, just as we would expect from a classical point of

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[11]   '$\rightarrow$' is the meta-language material conditional.

[12]   The resulting semantics for simple conditionals is a slight generalization of the one given in Lewis 1973 for subjunctive conditionals.

view. Consider next PK. We assume that in a conjunction like this, q has the form $\ulcorner \text{M} r \urcorner$. The conjunction $\ulcorner p \wedge (\neg p > \text{M} r) \urcorner$ is then true relative to $\langle g, \preceq, w \rangle$ just in case p is true relative to $\langle g, \preceq, w \rangle$ and $\ulcorner \neg p > \text{M} r \urcorner$ is true relative to *a different* point of evaluation, namely $\langle g^{p^{g, \preceq}}, \preceq, w \rangle$. And now notice something important: $\ulcorner \neg p > \text{M} r \urcorner$ is *always* true relative to *any* point of evaluation which contains $g^{p^{g, \preceq}}$ as its modal base parameter, *no matter what g is*, and no matter what $\preceq$ and w are. This is because $[\![\neg p > \text{M} r]\!]^{g^{p^{g, \preceq}}, \preceq, w} = 1$ just in case all $\preceq(w)$-minimal worlds in a certain set make r true —but what set? We get that set by taking the intersection of $g(w)$, then intersecting that with the intension of p, and then intersecting that with the intension of $\ulcorner \neg p \urcorner$. Whatever we started with, we'll end with the empty set. And so the quantification here ends up vacuous, and $\ulcorner \neg p > \text{M} r \urcorner$ is guaranteed to be true. That means that in PK, $\ulcorner p \wedge (\neg p > \text{M} r) \urcorner$ is true at an index just in case p is. And that, of course, means that 'and' is highly non-classical in this system. In particular, right conjunction elimination will be invalid: the truth of p at an index will suffice for the truth of $\ulcorner p \wedge (\neg p > \text{M} r) \urcorner$, but not for the truth of $\ulcorner \neg p > \text{M} r \urcorner$.[13] Conjunction introduction will also be invalid, for similar reasons: we can have p and $\ulcorner q > \text{M} r \urcorner$ both true relative to some index, while $\ulcorner p \wedge (q > \text{M} r) \urcorner$ is false relative to that same index, since in the conjunction, the right conjunct is evaluated relative to a shifted modal base (intuitively: for the conjunction to be true we would need not only p and $\ulcorner q > \text{M} r \urcorner$ to be true, but also for $\ulcorner (p \wedge q) > \text{M} r \urcorner$ to be true; and the truth of the latter does not follow from the truth of the former two in a variably strict framework like PK).[14] Things are parallel for PM: $\ulcorner p \wedge (\neg p > q) \urcorner$ will be true relative to any index just in case p is, for the right conjunct will be evaluated relative to a premise set updated with p; which means that q will be evaluated relative to the closest world which makes both p and $\ulcorner \neg p \urcorner$ true; which means it will be evaluated relative to the absurd world, which makes everything true. So right conjunction elimination will be invalid. And conjunction introduction will likewise be invalid, for parallel reasons.

That a non-classical 'and' is required in order for theories like Kratzer's to validate *IE* was first pointed out to me by Justin Khoo (p.c.), and first observed in the literature, to my knowledge, in Khoo & Mandelkern 2018. But why do PK and PM need a non-classical 'and' to validate *IE*? The intuition is the following. In both these theories, *IE* is validated because we keep track of the information in the antecedents of conditionals in our modal parameters as we process a conditional; when we have nested conditionals, we evaluate each successive antecedent relative to the information contained in the antecedents we have already processed. But that means that if we want to validate *IE* in full generality, conjunction must keep track of this information in exactly the same way. In other words, we evaluate q in $\ulcorner p > (q > r) \urcorner$ relative to modal parameters that include the information that p is true; if *IE* is to be valid for the case in which q is itself a conditional, we must likewise evaluate q in $\ulcorner (p \wedge q) > r \urcorner$ relative to modal parameters that are updated with p.

For an illustration, consider a conditional of the form $\ulcorner p > ((\neg p > q) > r) \urcorner$, with p and q conditional-free.[15] For both PM and PK, the second embedded conditional here,

[13]   Left conjunction elimination remains valid.

[14]   For a countermodel, consider a model with three possible worlds, x, y, z. Assume p, q, r are atoms. Let $g(z) = \varnothing$; let $\preceq(z)$ be the total order $\langle z, x, y \rangle$; and let $p(z) = 1$, $q(z) = 0$, $q(x) = r(x) = 1$, $p(x) = 0$, $q(y) = p(y) = 1$, $r(y) = 0$. Then p and $\ulcorner q > \text{M} r \urcorner$ are both true at $\langle g, \preceq, z \rangle$, but their conjunction is not.

[15]   In PK, we of course assume that q and r have the form $\ulcorner \text{M} s \urcorner$ and $\ulcorner \text{M} t \urcorner$ for some conditional-free and modal-free s and t. Likewise, *mutatis mutandis*, throughout what follows.

$\ulcorner \neg p > q \urcorner$, is interpreted relative to modal parameters that include the information that p is true, and thus is trivially true. It follows that, in PM and PK, $\ulcorner p > ((\neg p > q) > r) \urcorner$ is equivalent to $\ulcorner p > r \urcorner$. Now *IE* tells us that $\ulcorner p > ((\neg p > q) > r) \urcorner$ is semantically equivalent to $\ulcorner (p \wedge (\neg p > q)) > r \urcorner$. But, if we were to adopt all the semantic entries in PK or PM together with a *classical* 'and', then the latter would not be equivalent to $\ulcorner p > r \urcorner$, since the right conjunct of the antecedent of $\ulcorner (p \wedge (\neg p > q)) > r \urcorner$ would not be interpreted relative to modal parameters that are updated with $[\![p]\!]^c$, and so the right conjunct of the antecedent would not be at all trivial. This just follows from the fact that, if 'and' is classical, then $\ulcorner p \wedge (\neg p > q) \urcorner$ is not equivalent to p. And so $\ulcorner (p \wedge (\neg p > q)) > r \urcorner$ would not, after all, be equivalent to $\ulcorner p > ((\neg p > q) > r) \urcorner$: PK or PM amended so that it has a classical conjunction will invalidate *IE* when what is imported/exported is itself a conditional. By contrast, with the non-classical semantics for conjunction given above, both theories will, again, validate *IE* in full generality.

This makes it clear why PK and PM need a non-classical conjunction to validate *IE*. It also raises the converse question: how does McGee validate *IE* with a classical conjunction? McGee, like PK and PM, validates *IE* by stipulating that when we process a conditional with multiple antecedents, we keep track of the information in all the antecedents as we go, evaluating the consequent in light of all that information. But crucially, in McGee's semantics, we *don't* evaluate that information *as we go*, taking into account earlier information. Rather, we just keep track of all the sentences which are antecedents of the conditional, and then evaluate each on its own, relative to the empty hypothesis set. More concretely, consider $\ulcorner p > (q > r) \urcorner$. Rather than evaluating q relative to a hypothesis set which includes the information that p is true, and then evaluating r relative to a hypothesis set that includes the information that p is true and that q is true, we evaluate both p and q relative to the empty hypothesis set, and then evaluate r relative to the information derived that way. This means that we *don't* take into account p when evaluating q in $\ulcorner p > (q > r) \urcorner$; rather, q is evaluated as it would be when unembedded. This means that, in $\ulcorner p > ((q > r) > s) \urcorner$, $\ulcorner q > r \urcorner$ is evaluated the same way as when unembedded—it is evaluated relative to an empty hypothesis set—and thus it is evaluated in the same way as when it appears in $\ulcorner (p \wedge (q > r)) > s \urcorner$, where '$\wedge$' has classical semantics. This means that McGee can validate *IE* in full generality with a classical conjunction. In particular, in $\ulcorner p > ((\neg p > q) > r) \urcorner$, the embedded conditional $\ulcorner \neg p > q \urcorner$ will not be trivial—because it won't be evaluated relative to a hypothesis set updated with p —and so the whole conditional will not be equivalent to $\ulcorner p > r \urcorner$, but will be equivalent to $\ulcorner (p \wedge (\neg p > q)) > r \urcorner$, where '$\wedge$' has classical semantics.

PM and PK thus need a non-classical conjunction to validate *IE*; McGee manages to validate *IE* with a classical conjunction. Let me note, moreover, that PM and PK are the rule, not the exception: as far as I know, McGee's theory is the *only* extant theory of conditionals which validates *IE* with a classical conjunction. Every other theory I know of requires a non-classical 'and' along the lines of the conjunction in PM and PK in order to validate *IE*.

4. *Nothing Added*

Good for McGee and bad for PM and PK, we might think. Classical conjunction has many things going for it. If we want to validate *IE*, then (it is natural to think) we should do so without introducing excessive weirdness elsewhere in our semantics.

But this response is overhasty. Adopting *IE* together with a classical conjunction forces us to invalidate other plausible principles about the conditional. This counts against any approach which aims to validate *IE* while keeping conjunction classical. (In the end I will argue that the best response is not to accept PM/PK's non-classical conjunctions, but rather to reject *IE* in favor of a more limited principle; but more on that to come.)

I will discuss two principles in particular which we cannot reasonably validate if we adopt *IE* together with a classical conjunction. I call the first principle *Nothing Added*:

Nothing Added: $\forall p, q, r \in \mathcal{L} : (\forall c : [\![p > q]\!]^c = \mathcal{W}) \rightarrow (\forall c : [\![p > (q > r)]\!]^c = [\![p > r]\!]^c)$

Nothing Added says that, when $\ulcorner p > q \urcorner$ is a theorem, then $\ulcorner p > (q > r) \urcorner$ will always express the same thing as $\ulcorner p > r \urcorner$. The principle is very natural. Intuitively, if $\ulcorner p > q \urcorner$ is a theorem, then q tells us nothing beyond what p already tells us; but then $\ulcorner p > (q > r) \urcorner$ should have just the same status as $\ulcorner p > r \urcorner$.

But it turns out that, given some anodyne background assumptions, if we validate both *Nothing Added* and *IE* together with classical conjunction, we arrive at absurd conclusions. As an illustration to begin, note that McGee does not validate *Nothing Added*. For McGee, $\ulcorner p > (\neg p > q) \urcorner$ is a theorem, since q will be true relative to any hypothesis set which includes both p and $\ulcorner \neg p \urcorner$. *Nothing Added* would then say that, for any r, $\ulcorner p > ((\neg p > q) > r) \urcorner$ is equivalent to $\ulcorner p > r \urcorner$. But these are inequivalent in McGee's semantics. For instance, let p, q, and r be arbitrary different atoms. Suppose we evaluate $\ulcorner p > ((\neg p > q) > r) \urcorner$ and $\ulcorner p > r \urcorner$ relative to a selection function f, an empty hypothesis set, and a world w where both p and r are true. $\ulcorner p > r \urcorner$ will be true at this index no matter what (thanks to the fourth constraint on selection functions). But now suppose that $f([\![p]\!]^{f,\varnothing} \cap [\![\neg p > q]\!]^{f,\varnothing}, w) = w' \neq w$, and suppose that $r(w') = 0$ (these assumptions are perfectly consistent). Then $\ulcorner p > ((\neg p > q) > r) \urcorner$ will be false at $\langle f, \varnothing, w \rangle$. Since $\ulcorner p > r \urcorner$ is true at $\langle f, \varnothing, w \rangle$, it follows that $\ulcorner p > ((\neg p > q) > r) \urcorner$ is not equivalent to $\ulcorner p > r \urcorner$.

So *Nothing Added* is not valid in McGee's semantics—whereas, by contrast, *Nothing Added* is valid in PM and PK.[16] This divergence is not an accident: given two innocuous background assumptions, validating *IE* and *Nothing Added* while keeping conjunction classical leads us to absurd consequences. Those assumptions are, first, *Conditional Deduction*, the principle adverted to in Gibbard's proof which says that if p entails q, then $\ulcorner p > q \urcorner$ is a theorem; and, second, a principle I'll call *Equivalence*:

Equivalence: $\forall p, q \in \mathcal{L} : (\forall c : \forall r \in \mathcal{L} : [\![p > r]\!]^c = [\![q > r]\!]^c) \rightarrow \forall c : [\![p]\!]^c = [\![q]\!]^c$

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16 The proof in PM is as follows. Suppose $\forall f : [\![p > q]\!]^{f,\varnothing} = \mathcal{W}$. Then $\forall f : [\![p]\!]^{f,\varnothing} \subseteq [\![q]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}}$. Otherwise we would have: $\exists f : \exists w : [\![p]\!]^{f,\varnothing,w} = 1 \wedge [\![q]\!]^{f,\{[\![p]\!]^{f,\varnothing}\},w} = 0$. But then $[\![p > q]\!]^{f,\varnothing,w} = 0$, contrary to assumption. Now $\forall f : [\![p > (q > r)]\!]^{f,\varnothing} = [\![q > r]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}} = [\![r]\!]^{f,\{[\![p]\!]^{f,\varnothing},[\![q]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}}\}} = [\![r]\!]^{f,\{[\![p]\!]^{f,\varnothing} \cap [\![q]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}}\}} = [\![r]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}} = [\![p > r]\!]^{f,\varnothing}$. The proof in PK is similar. A parallel proof will not go through for McGee's semantics, because crucially, in McGee's semantics (if we were to equivalently reformulate it in terms of sets of propositions instead of sentences), $[\![q > r]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}}$ is not $[\![r]\!]^{f,\{[\![p]\!]^{f,\varnothing},[\![q]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}}\}}$; rather, $[\![q > r]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}} = [\![r]\!]^{f,\{[\![p]\!]^{f,\varnothing},[\![q]\!]^{f,\varnothing}\}}$.

Equivalence says that, given two sentences p and q, as long as $\ulcorner p > r \urcorner$ and $\ulcorner q > r \urcorner$ are semantically equivalent *no matter what r is*, then p and q must be semantically equivalent as well. This is a principle that is validated by every semantics I know for the conditional, and has not to my knowledge been questioned in the literature.[17]

Now suppose we take on board *IE*, *Nothing Added*, *Conditional Deduction*, and *Equivalence*. We also make a weak version of the classical *Quodlibet* assumption that, as long as p is conditional-free, $\ulcorner p \wedge \neg p \urcorner$ is nowhere true, and thus entails everything. Then for any $q \in \mathcal{L}$ and $p \in \mathcal{L}_{CF}$:[18]

i. $\forall c : [\![(p \wedge \neg p) > q]\!]^c = \mathcal{W}$ *Conditional Deduction, Quodlibet*

ii. $\forall c : [\![p > (\neg p > q)]\!]^c = \mathcal{W}$ *IE*, (i)

iii. $\forall c : \forall r \in \mathcal{L} : [\![p > ((\neg p > q) > r)]\!]^c = [\![p > r]\!]^c$ *Nothing Added*, (ii)

iv. $\forall c : \forall r \in \mathcal{L} : [\![(p \wedge (\neg p > q)) > r]\!]^c$

     $= [\![p > ((\neg p > q) > r)]\!]^c = [\![p > r]\!]^c$ *IE*, (iii)

v. $\forall c : [\![p]\!]^c = [\![p \wedge (\neg p > q)]\!]^c$ *Equivalence*, (iv)

Now suppose that conjunction is classical, in particular that it validates right conjunction elimination.[19] Since from (v) we can conclude that whenever p is true, so is $\ulcorner p \wedge (\neg p > q) \urcorner$, right conjunction elimination then lets us conclude that $\ulcorner \neg p > q \urcorner$ is also true. In other words, we arrive at the conclusion that p entails $\ulcorner \neg p > q \urcorner$; i.e. that $\forall c : \forall q \in \mathcal{L} : \forall p \in \mathcal{L}_{CF} : [\![p]\!]^c \subseteq [\![\neg p > q]\!]^c$. But this conclusion is clearly false: it is, in fact, one of the worst consequences of the material conditional analysis of the conditional. For this conclusion entails that the *falsity* of $\ulcorner \neg p > q \urcorner$ entails the falsity of p; more succinctly (given classical negation, which is not in dispute here), the falsity of $\ulcorner p > q \urcorner$ entails the truth of p. Call this principle *Ex Falso*:

     Ex Falso: $\forall c : \forall q \in \mathcal{L} : \forall p \in \mathcal{L}_{CF} : [\![\neg(p > q)]\!]^c \subseteq [\![p]\!]^c$

But *Ex Falso* is clearly unacceptable: as we saw above, for instance, the truth of 'It's not the case that if Patch is a rabbit, she is a rodent' does *not* entail that Patch is a rabbit. Likewise for subjunctives.

In sum: *Nothing Added*, *Conditional Deduction*, *Equivalence*, and *Quodlibet* entail *Ex Falso*, if 'and' is classical and *IE* is true. *Ex Falso* is clearly false, and *Conditional Deduction* and *Equivalence* look hard to challenge. So if 'and' is classical, *IE* is in direct tension with a very appealing principle, namely *Nothing Added*.

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[17]   Though its converse—that conditionals with semantically equivalent antecedents are equivalent—has been challenged; see Santorio 2018 and citations therein.

[18]   We limit our attention to p in the conditional-free fragment, because $\ulcorner p \wedge \neg p \urcorner$ will not always be a contradiction when p itself contains a conditional, according to PM and PK, as I discuss further below. Likewise in discussion of *Absurdum* below.

[19]   (v) on its own is already problematic, at least for subjunctives. I return to this point in §6.

5. *Absurdum*

We find similar tension between *IE*, classical 'and', and a principle which I'll call *Absurdum*:[20]

Absurdum: $\forall p, q \in \mathcal{L} : (\forall c : [\![p > q]\!]^c = \varnothing) \rightarrow (\forall c : [\![p > (q > \bot)]\!]^c = \mathcal{W})$

Absurdum says that if $\ulcorner p > q \urcorner$ is inconsistent, then $\ulcorner p > (q > \bot) \urcorner$ is a theorem. *Absurdum* is quite attractive. If $\ulcorner p > q \urcorner$ is inconsistent, then p must somehow guarantee that q is false—otherwise there would be some model where $\ulcorner p > q \urcorner$ is true. So p and q must be inconsistent. Under the assumption that p and the subsequent assumption that q, then, anything at all will follow; and so $\ulcorner p > (q > \bot) \urcorner$ should be a theorem.

PM and PK both validate *Absurdum*.[21] But, again given some anodyne background assumptions, there is no plausible way to validate *Absurdum* while validating *IE* and keeping conjunction classical. For the sake of illustration, note first that McGee invalidates *Absurdum*. For McGee, $\ulcorner p > (\neg p > q) \urcorner$ is a theorem; and so, assuming p is consistent, $\ulcorner p > \neg(\neg p > q) \urcorner$ is inconsistent. Then *Absurdum* says that $\ulcorner p > (\neg(\neg p > q) > \bot) \urcorner$ should be a theorem. But in McGee's framework, it's not. By *IE*, this will be equivalent to $\ulcorner (p \wedge \neg(\neg p > q)) > \bot \urcorner$. Since conjunction is classical for McGee, the antecedent of this conditional will be consistent, and so the whole conditional will be inconsistent. Thus McGee fails to validate *Absurdum*.

More generally, suppose we take on board the following background principles:

Theorem to Contradiction 1: $\forall p, q \in \mathcal{L} : (\forall c : [\![p]\!]^c \neq \varnothing \wedge [\![p > q]\!]^c = \mathcal{W}) \rightarrow (\forall c : [\![p > \neg q]\!]^c = \varnothing)$

Theorem to Contradiction 2: $\forall p, q \in \mathcal{L} : (\forall c : [\![p > \bot]\!]^c = \mathcal{W}) \rightarrow (\forall c : [\![p]\!]^c = \varnothing)$

Theorem to Contradiction 1 (*TC1*) says that, if p is consistent and $\ulcorner p > q \urcorner$ is a theorem, then $\ulcorner p > \neg q \urcorner$ is inconsistent. If p is consistent, then $\ulcorner p > q \urcorner$ will be a theorem only if, intuitively, the truth of p somehow guarantees the truth of q, no matter what the world is like; but then $\ulcorner p > \neg q \urcorner$ will always be false. *Theorem to Contradiction 2* (*TC2*) says that if $\ulcorner p > \bot \urcorner$ is a theorem, then p is inconsistent. This, again, seems very plausible: if $\ulcorner p > \bot \urcorner$ is a theorem, then this means that the truth of p somehow guarantees the truth of $\bot$; but since $\bot$ is never true, this can only hold if p is never true. Indeed, *TC2* follows from *TC1* given the very plausible assumption that $\ulcorner p > \top \urcorner$ is consistent for any p (a principle, again, which as far as I know everyone accepts). Suppose then that $\ulcorner p > \bot \urcorner$ is a theorem. If p were consistent, then, by *TC1*, it would follow that $\ulcorner p > \top \urcorner$ is inconsistent, contrary to our assumption. So it must be that p is inconsistent. *TC1* in turn is a close relative of *Conditional Deduction*. *TC1* and *TC2* are validated by every semantics for the conditional I know, and seem very plausible to me.

Now, taking these assumptions on board, consider any consistent $p \in \mathcal{L}_{CF}$ and any $q \in \mathcal{L}$:

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[20] $\bot$ is an atom true only at the absurd world; $\top$ is an atom everywhere true.

[21]   For PM, the proof is as follows. Suppose $\forall f : [\![p > q]\!]^{f,\varnothing} = \varnothing$. Then $\forall f : [\![p]\!]^{f,\varnothing} \cap [\![q]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}} = \varnothing$; else $\exists w : \exists f : [\![p]\!]^{f,\varnothing,w} = [\![q]\!]^{f,\{[\![p]\!]^{f,\varnothing}\},w} = 1$, but then $[\![p > q]\!]^{f,\varnothing,w} = 1$, contrary to assumption. Then $\forall f : [\![p > (q > \bot)]\!]^{f,\varnothing} = [\![q > \bot]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}} = [\![\bot]\!]^{f,\{[\![p]\!]^{f,\varnothing},[\![q]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}}\}} = [\![\bot]\!]^{f,\{[\![p]\!]^{f,\varnothing} \cap [\![q]\!]^{f,\{[\![p]\!]^{f,\varnothing}\}}\}} = [\![\bot]\!]^{f,\{\varnothing\}}$ $= \mathcal{W}$. The proof for PK is similar.

i. $\forall c : [\![(p \wedge \neg p) > q]\!]^c = \mathcal{W}$ *Conditional Deduction, Quodlibet*

ii. $\forall c : [\![p > (\neg p > q)]\!]^c = \mathcal{W}$ *IE*, (i)

iii. $\forall c : [\![p > \neg(\neg p > q)]\!]^c = \varnothing$ *TC1*, (ii)

iv. $\forall c : [\![p > (\neg(\neg p > q) > \bot)]\!]^c = \mathcal{W}$ *Absurdum*, (iii)

v. $\forall c : [\![(p \wedge \neg(\neg p > q)) > \bot]\!]^c = \mathcal{W}$ *IE*, (iv)

vi. $\forall c : [\![p \wedge \neg(\neg p > q)]\!]^c = \varnothing$ *TC2*, (v)

Now, if 'and' is classical, then from (vi) we can conclude that, whenever p is consistent and in $\mathcal{L}_{CF}$, we have $\forall c : \forall q \in \mathcal{L} : [\![p]\!]^c \subseteq [\![\neg p > q]\!]^c$ (otherwise we would have that $[\![p]\!]^c$ is consistent with $[\![\neg(\neg p > q)]\!]^c$ and thus that $[\![p \wedge \neg(\neg p > q)]\!]^c$ is consistent, contrary to (vi)). The same of course holds whenever p is inconsistent; and so we arrive again at *Ex Falso*. Once more, then, classical conjunction puts *IE* in tension with a very intuitive principle—this time *Absurdum*—given background assumptions (*TC1* and *TC2*, together with *Conditional Deduction* and *Quodlibet*) which seem difficult to challenge.

6. *Restricted Import-Export*

Import-Export thus does not sit easily with classical conjunction. Given some innocuous background assumptions, if we adopt classical conjunction, then we cannot validate *IE* together with *Nothing Added* or with *Absurdum*, at risk of validating the obviously false principle *Ex Falso*.

My main goal here has been to draw out the surprising connections between *IE* and conjunction. There are a variety of ways we could respond to these results. Accepting *Ex Falso* seems like a non-starter to me. So does rejecting one of the background assumptions (*Conditional Deduction, Quodlibet, Equivalence, TC1, TC2*), though proponents of non-classical logics may be inclined to go that way. It seems to me, then, that we must reject one of the following three: (i) classical conjunction, (ii) *IE*, and (iii) *Nothing Added*; and likewise we must reject one of the following three: (i) classical conjunction, (ii) *IE*, and (iii) *Absurdum*.

So what are we to do? In concluding, I will very briefly argue that the right response to these results is to reject *IE*, but on a limited basis. Let me start with some negative remarks. Following McGee's route—validating *IE* with a classical conjunction, and therefore invalidating *Nothing Added* and *Absurdum*—is *prima facie* unattractive insofar as those latter principles are very natural (though one could certainly deny them).

Following PM/PK's route—adopting a non-classical conjunction, together with *IE*, *Nothing Added*, and *Absurdum*—likewise has serious drawbacks. It should be acknowledged that the non-classical conjunctions under discussion behave classically except when a right conjunct is a conditional; and they bear a close resemblance to non-classical conjunctions which have been motivated and defended in the dynamic semantics literature on the basis of facts about anaphora, presupposition, and—most closely related to

present considerations—epistemic modality.[22] But there are serious drawbacks to following PM/PK's route. I will briefly highlight two. The first is that the non-classical 'and' advocated by PK and PM invalidates certain logical principles which seem very intuitive in natural language, even when 'and' conjoins conditionals. The most prominent of these is the principle of non-contradiction, which says that $\ulcorner p \wedge \neg p \urcorner$ is a contradiction, for any p. This principle is not valid in PK or PM: we will get countermodels when p itself is a conditional. In particular, sentences of the form $\ulcorner ((p > q) > r) \wedge \neg((p > q) > r) \urcorner$ are consistent in these frameworks (the key point is that the second, negated conditional is interpreted relative to modal parameters updated with the first, and thus can be false while the first is true, rendering the whole sentence true).[23] This seems like a bad result, one not borne out by natural language; (3) for instance seems contradictory:

(3) If the vase will break if dropped, we won't drop it; and it's not the case that if the vase will break if dropped, we won't drop it.

Second—as an anonymous referee for this journal and David Boylan have both pointed out to me—while adopting a non-classical conjunction lets us avoid *Ex Falso*, adopting *IE* together with *Nothing Added* still forces us to an unsavory conclusion, *whatever conjunction we adopt*. Namely, as our first result showed, given our background assumptions, these together entail that p and $\ulcorner p \wedge (\neg p > q) \urcorner$ are semantically equivalent for conditional-free p. This conclusion on its own is difficult to assess for indicative conditionals, which are generally felicitous only if their antecedents are compatible with the common ground (Stalnaker 1975); but is clearly unacceptable for subjunctive conditionals. For instance, suppose that it's sunny. And suppose that if it had rained the picnic would have been cancelled. Then (4) is true and (5) false:

(4) It's sunny, and if it had rained the picnic would have been cancelled.

(5) It's sunny, and if it had rained the picnic would not have been cancelled.

This is contrary to our predictions if p and $\ulcorner p \wedge (\neg p > q) \urcorner$ are semantically equivalent, in which case we predict both of these to be true. Thus *IE* and *Nothing Added* together are already in tension, whatever conjunction we adopt, suggesting that we cannot reasonably validate both of them.

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[22] See e.g. Heim 1982, 1983, Groenendijk et al. 1996. In Mandelkern 2018a, I defend a non-classical conjunction on the basis of considerations about epistemic modality. But that conjunction differs substantially from the one under discussion here, both in motivation and in its logical profile; in particular, unlike the one under consideration here, that conjunction Strawson-validates all classical logical laws.

[23] For an example in PK, consider a model with worlds $\{w, a, b, c, d, e, f, g, h, i\}$, with $\preceq$ returning total orders at each world, in particular $\preceq(w) = \langle w, a, b, h \rangle, \preceq(a) = \langle a, f \rangle, \preceq(f) = \langle f, g \rangle, \preceq(b) = \langle b, c, d \rangle, \preceq(c) = \langle c, e \rangle, \preceq(h) = \langle h, i \rangle$ and $p(a) = p(c) = p(d) = p(e) = p(f) = p(g) = p(i) = p(h) = 1, p(w) = p(b) = 0, r(w) = r(d) = r(g) = r(i) = 1, \quad r(a) = r(b) = r(e) = 0, \quad q(a) = q(d) = q(e) = q(g) = q(i) = 1, \quad q(c) = q(f) = q(h) = 0.$ $\ulcorner ((p > q) > r) \wedge \neg((p > q) > r) \urcorner$ will be true at w (relative to $\preceq$ and the modal base g which takes each world to $\varnothing$). This can easily be translated into a corresponding model in PM. See Mandelkern 2018b for a parallel result concerning dynamic 'and'. A related issue for PM and PK is that $\ulcorner p > p \urcorner$ will not be a theorem; in particular, it can be false when p is a conditional (the model just presented is a countermodel to $\ulcorner ((p > q) > r) > ((p > q) > r) \urcorner$). This issue does not have anything to do with conjunction, and seems independently worrisome.

Both of these options thus have clear drawbacks. There is an alternative, however. To motivate it, a bit of review is in order. The year is 1980: the Republican American presidential candidate Reagan is ahead in the polls, followed by the Democrat Carter. Well behind both is Anderson, a second Republican. McGee (1985) noted that the following conditional sounds clearly true in this situation:

(6) If a Republican wins the election, then if Reagan doesn't win, Anderson will.

But the following conditional sounds clearly false (or at least very unlikely):

(7) If Reagan doesn't win the election, Anderson will.

Since we nonetheless think it quite likely that a Republican will win the election, it looks like it can't be that 'A Republican will win the election', together with (6), entails that (7) is true. This is McGee's famous counterexample to *MP*. At the same time, McGee noted that (6) sounds equivalent to (8):

(8) If a Republican wins the election and Reagan doesn't win, Anderson will.

That felt equivalence counts as evidence in favor of *IE*.

The conclusion that *IE* is valid, however, is not unavoidable. Other researchers have indeed explored *IE* and have found no convincing (to my mind) counterexamples to the principle, at least for indicative conditionals.[24] But, as far as I know, none of those explorations has looked in particular at instances of *IE* where what is being imported or exported itself contains a conditional, e.g. at pairs of the form $\ulcorner p > ((q > r) > s) \urcorner$ and $\ulcorner (p \wedge (q > r)) > s) \urcorner$. *IE*, of course, predicts pairs of this form to be equivalent; and readers may have noticed that both of the proofs above make crucial use of precisely this instance of *IE* (in step (iv) of the first, and step (v) of the second). This means that those proofs would be blocked if we adopted a weaker version of *IE*, which says that the predicted equivalences of *IE* hold *except* when what is being imported or exported itself contains conditional language:

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[24] See Khoo & Mandelkern 2018 for a survey. When it comes to subjunctive conditionals, however, things may be otherwise, as Etlin 2008 discusses (thanks to an anonymous referee for this journal for bringing this to my attention). Etlin provides the following pair (slightly modified):

(9) If this match had lit at noon today, then if it had been soaked in water last night it would have lit at noon today.

(10) If this match had lit at noon today and it had been soaked in water last night, then it would have lit at noon today.

(9) has a salient reading on which it is not true, while (10) has only a true reading; these together thus constitute a *prima facie* counterexample to *IE* (and to *RIE*, the weaker principle I'll take up in a moment) for subjunctives —but not, significantly, for indicatives, since corresponding indicative pairs pattern together. One conclusion we could draw from cases like this is that *RIE* is false for subjunctives, but true for indicatives. This would open up two substantial explanatory questions: First, why do indicatives and subjunctives pattern differently here? Second, why does *RIE* seem to hold in so many cases even for subjunctives? A different strategy would be to argue that, appearances notwithstanding, (9) and (10) do not constitute a genuine counterexample to *RIE*. This would avoid opening up those two explanatory questions. I will not try to adjudicate between these options here. If we do ultimately reject *IE* and *RIE* for subjunctives, then the rest of the paper should be read as applicable only to indicative conditionals.

Restricted Import-Export (RIE):
$$\forall c : \forall p, r \in \mathcal{L} : \forall q \in \mathcal{L}_{CF} : [\![p > (q > r)]\!]^c = [\![(p \wedge q) > r]\!]^c$$

RIE says that $\ulcorner p > (q > r) \urcorner$ and $\ulcorner (p \wedge q) > r \urcorner$ will always be equivalent as long as q is conditional-free; when q contains a conditional, these may come apart.

The first thing to note about *RIE* is that it lets us escape the results presented so far: that is, there is no difficulty validating *RIE* with a classical conjunction, together with *Nothing Added* and *Absurdum*. Indeed, if we take either PM or PK and replace their non-classical conjunctions with classical conjunctions, we arrive at semantic theories which validate *RIE* but not *IE*; which validate *Nothing Added* and *Absurdum*, together with all the background principles assumed above; but which do not validate *Ex Falso* or predict an equivalence between p and $\ulcorner p \wedge (\neg p > q) \urcorner$.

RIE thus lets us avoid the tensions pointed out above. *RIE* at first blush looks hopelessly *ad hoc*. But this appearance is greatly diminished when we look at *RIE* from a semantic rather than a syntactic point of view: as we have just seen, from the perspective of certain theories of the conditional (like those given by PM and PK), there is a sense in which it is simpler to validate *RIE* than it is to validate *IE*.[25] And, more importantly, I will argue that intuitions about natural language actually match the predictions of *RIE*: conditionals $\ulcorner p > (q > r) \urcorner$ and $\ulcorner (p \wedge q) > r \urcorner$ are always felt to be equivalent, *unless q* itself contains a conditional, in which case intuitions about their truth-values can diverge. For the first part of this claim, I refer readers to the existing literature. My contribution is to the second part: when q contains a conditional, $\ulcorner p > (q > r) \urcorner$ and $\ulcorner (p \wedge q) > r \urcorner$ do not invariably strike us as equivalent.

A variation on McGee's case provides a nice illustration of this. Suppose that we know that Reagan is well ahead of both Carter and Anderson in the polls, but we don't know the relative standing of Anderson and Carter. Now consider the following conditional:

(11) If a Republican will win the election, and Anderson will win if Reagan doesn't win, then both Republicans are currently in a stronger position to win than Carter.

(11) strikes me as likely true in the scenario as described. If a Republican will win the election, presumably it will be Reagan, since we know he is in the strongest position to win. But if it's *also* the case that Anderson will win if Reagan doesn't, then that must be because Anderson is in a stronger position than Carter at present. And so we know that both Anderson and Reagan are in a stronger position to win than Carter: Reagan because we already know that he's ahead of Carter, and Anderson because the conditional fact that Anderson will win if Reagan doesn't, together with our background knowledge that Reagan is ahead, suggests that Anderson must be doing better than Carter. (11), then, strikes me as likely true.

But now consider (12):

(12) If a Republican will win the election, then if Anderson will win if Reagan doesn't, then both Republicans are currently in a stronger position to win than Carter.

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[25]   Another indirect piece of evidence in favor of *RIE* is the fact that a similarly restricted version of *MP* looks very appealing: namely, that $\{p, \ulcorner p > q \urcorner\}$ entails q if $q \in \mathcal{L}_{CF}$. In fact, the three theories introduced at the outset all validate exactly this restriction of *MP*, despite invalidating *MP*.

(12), *unlike* (11), does not seem likely to be true. After all, if a Republican wins the next election, then of course it already follows that Anderson will win if Reagan doesn't. And so that embedded conditional doesn't tell us anything more, beyond the assumption that a Republican will win the election. In other words, (12) strikes me as being equivalent to 'If a Republican will win the election, then both Republicans are currently in a stronger position to win than Carter'. But that does not strike me as having any better than even odds of being true: a Republican winning the election doesn't show that both Republicans are in a stronger position to win than Carter. In other words, the set-up described by 'If a Republican wins the election, then if Anderson will win if Reagan doesn't, then...' seems clearly consistent with the following situation: Reagan is well ahead; then Carter; and then Anderson. And in this situation, it's not the case that both Republicans are ahead. In short, my credence in (11) is very high; my credence in (12) is at best middling. If my credences in these can rationally come apart, then they do not express the same proposition.

I conclude that (11) and (12) are not semantically equivalent. But now note that (11) has the form $\ulcorner(p \land (q > r)) > s\urcorner$, and (12) has the form $\ulcorner p > ((q > r) > s)\urcorner$—with p = 'a Republican will win the election', q = 'Reagan doesn't win', r = 'Anderson will win', and s = 'both Republicans are currently in a stronger position to win than Carter'. That means that, if *IE* is valid, then (11) and (12) are semantically equivalent. And indeed, McGee, PM, and PK all predict this equivalence, by validating *IE*—though in different ways. McGee, by validating *IE* with a classical conjunction, predicts that both (11) and (12) have the truth conditions which, intuitively, only (11) has—i.e. that both conditionals are likely true in the situation as described. PM and PK validate *IE* by generalizing in the opposite direction: they predict that (11) and (12) both have the truth conditions which, intuitively, only (12) has—i.e. that neither conditional is likely true in the situation as described.

But, as we have seen, contrary to these predictions, (11) and (12) seem to mean different things. And so *IE* is not valid; for if *IE* were, then (11) and (12) would be semantically equivalent. In particular, this divergence shows that *IE* fails for pairs of the form $\ulcorner p > ((q > r) > s)\urcorner$ and $\ulcorner(p \land (q > r)) > s)\urcorner$, just as *RIE* predicts. Conditionals, it seems, cannot themselves be imported and exported *salva veritate*.

Pairs like (11) and (12) can be modified so as to target the importation direction of *IE* rather than the exportation direction. Let's revert to McGee's set-up: we've just received polling data that shows that Reagan is ahead, then Carter, and then far behind is Anderson. But now we're not entirely sure whether the data are correct. Consider the following conditionals:

(13) If a Republican will win the election, then if Anderson will win if Reagan doesn't, then the polling data we've just received are correct.

(14) If a Republican will win the election, and Anderson will win if Reagan doesn't, then the polling data we've just received are correct.

Here, it seems at least possible that (13) is true: the antecedents seem compatible with the polling data being correct, since the data are consistent with a Republican winning the election and Anderson winning if Reagan doesn't *under the supposition that a Republican will win*. By contrast, (14) seems clearly false: if Anderson will win if

Reagan doesn't, then the data we've just received, which show Anderson well behind Carter, must be wrong.

Let me give, finally, another scenario which furnishes parallel counterexamples to *IE*. Suppose that we hope to find out about John's preference ordering over apple, blueberry, and pecan pie. We know that John prefers apple over both blueberry and pecan. We don't know whether he prefers blueberry or pecan. Then we overhear a different person saying each of the following:

(15) If John will choose fruit, and he will choose blueberry if not apple, then he prefers both apple and blueberry to pecan.

(16) If John will choose fruit, then if he will choose blueberry if not apple, then he prefers both apple and blueberry to pecan.

My intuition is that, in this situation, (15) seems likely to be true, while (16) does not, for parallel reasons to those given above.

Thus I am inclined to think that the right way to resolve the tensions I have identified in this paper is the following. All the intuitions that have been adduced in the literature in favor of *IE* are in fact consistent with *IE* being invalid and just the weaker principle *RIE* being valid. Moreover, contrasts like that between (11) and (12), (13) and (14), and (15) and (16) show that *IE* fails precisely where *RIE* predicts it to: when we import and export a sentence which itself contains conditional language. So we should accept *RIE* instead of *IE*. Doing so lets us adopt a classical semantics for conjunction, while still validating *Nothing Added* and *Absurdum*, without arriving at the absurd conclusion that $\ulcorner \neg(p > q) \urcorner$ entails p or that p and $\ulcorner p \wedge (\neg p > q) \urcorner$ are equivalent. There are, again, many ways we could spell out an approach like this; two options are obtained, again, by replacing the non-classical conjunctions in PM and PK with classical conjunctions.

7. Conclusion

IE does not sit easily with classical 'and'. I have illustrated this by showing that, if we have classical 'and' and *IE*, there are two very natural principles which we cannot validate, at risk of absurdity. I have argued that the best resolution to this puzzle is to reject *IE* in favor of a slightly weaker principle, *RIE*, which better fits data from natural language and which allows us to hold onto both of those principles together with classical conjunction.

Others may prefer to find a different way out of this tangle; my main goal here has been to identify a mess which so far has been passed over in dignified silence.

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