Reverse proportionality without context dependent standards

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Abstract

In their so-called reverse proportional reading (Herburger 1997), the truth conditions of statements of the form *many/few* ϕ ψ appear to make reference to the ratio of the individuals that are in the extensions of both ϕ and ψ to the individuals that are in the extension of ψ. The analysis of such readings is controversial. One prominent approach (Büring 1996, de Hoop and Solà 1996, Romero 2015, 2016, Solt 2009) assumes that they are a symptom of *many* and *few* making reference to a context dependent standard of comparison. Elaborating on remarks in Partee (1989), we observe that this initially attractive approach systematically undergenerates, failing to capture pervasive reverse proportionality in environments that remove context dependency of the standard. We propose that reverse proportionality in such cases instead reflects the underspecification of the measure function underlying the meanings of *many* and *few* (Bale and Barner 2009, Wellwood 2014).

1 Introduction

Since Partee (1989), much work has assumed that *many* and *few* are lexically ambiguous between a *cardinal* and a *proportional* sense. Under the cardinal meaning, the truth of *many/few* ϕ ψ requires that the cardinality of $[\![\phi]\!] \cap [\![\psi]\!]$, the intersection of the extensions of ϕ and ψ, be above/below a contextually determined standard cardinality; under the proportional meaning, it requires that the ratio of individuals in $[\![\phi]\!] \cap [\![\psi]\!]$ to individuals in $[\![\phi]\!]$ is above/below a contextually determined standard proportion. This lexical ambiguity is posited to capture the range of interpretations that is illustrated for *few* by Partee's examples in (1).

(1) a. There were few faculty children at the 1980 picnic.
b. Few egg-laying mammals suckle their young

Partee presents (1a) as illustrating the cardinal sense of *few*. The sentence can be judged true even if *all* of the faculty children were at the 1980 picnic, on the grounds that at the time there were only few faculty children to begin with. This suggests that the sentence portrays the cardinality of the intersection of $[\![$*faculty children*$]\!]$ and $[\![$*at the party*$]\!]$ as falling below a contextually determined standard. In contrast, Partee reports that truth conditions of (1b) do not impose requirements on the cardinality of the intersection of $[\![$ *egg-laying mammals* $]\!]$ and $[\![$ *suckle their young* $]\!]$, and that the sentence is instead read as being about the ratio of individuals in that intersection to individuals in $[\![$ *egg-laying mammals* $]\!]$, portraying that ratio as falling below a contextually determined standard. Hence Partee takes (1b) to illustrate the proportional sense of *few*.

In the proportional reading identified by Partee (1989), the proportion that *many/few* $\phi\ \psi$ refers to, $|\,[\![\phi]\!] \cap [\![\psi]\!]\,| / |\,[\![\phi]\!]\,|$, has a denominator determined by the nominal argument of *many/few*. Refining the standard terminology, we refer to this reading as the *forward* proportional reading. This is to distinguish it from the *reverse* proportional reading that is the focus of our investigation. That *many* and *few* have a third lexical meaning, the said reverse proportional sense, was first proposed in Westerståhl (1985b). This proposal is motivated by observations about cases like example (2), from Herburger (1997).

(2) Few cooks applied.

Herburger reports that this sentence can be read as a statement about the ratio of the set of applicants that are cooks, the intersection of $[\![$ *cooks* $]\!]$ and $[\![$ *applied* $]\!]$, to the set of applicants, $[\![$ *applied* $]\!]$, stating that this ratio is below a contextually determined standard. In this reading, the proportion that *many/few* $\phi\ \psi$ refers to is $|\,[\![\phi]\!] \cap [\![\psi]\!]\,| / |\,[\![\psi]\!]\,|$, where the denominator is now determined by the *scope* of the quantifier that *many/few* forms, rather than by the noun phrase that serves as its restrictor.

The existence of reverse proportional readings of sentences with *many* and *few* appears to be beyond dispute. What is debated, however, is the analysis of such readings. Driven in part by considerations of theoretical parsimony, most authors reject Westerståhl's (1985b) assumption that reverse proportional readings are due to a reverse proportional lexical meaning of *many* and *few*. In one prominent school of thought, which we will refer to as the *standard-based* approach to reverse proportionality (Büring 1996, de Hoop and Solà 1996, Romero 2015, 2016, Solt 2009), reverse proportional readings are instead a symptom of *many* and *few* making reference to a context dependent standard of comparison, and are a natural consequence of this context dependency, under appropriate conditions, even in the absence of reverse proportional lexical entries for *many* and *few*.

However, the main objective of this paper is to demonstrate, elaborating on remarks in Partee (1989), that the standard-based approach systematically undergenerates, as it fails to capture pervasive reverse proportionality in environments that remove context dependency of the standard of comparison (section 3). Moreover, we aim to motivate an alternative, novel, approach to reverse proportionality in such cases, proposing that it reflects the underspecification of the measure function underlying the meanings of *many* and *few* (Bale and Barner 2009, Wellwood 2014; section 4). To set the stage for these arguments, we begin by spelling out in more detail the two analyses of reverse proportionality hinted at above, the lexical ambiguity analysis and the standard-based analysis (section 2).[1]

[1]In this paper, we do not address the interactions between syntax, semantics and focus structure with regards to the interpretation of *many* and *few*. As far as we can see, the conclusions we reach in this paper stand regardless of how these issues are resolved. Given that we discuss readings previously unexplored in the literature, future work will have to explore how the new range of semantic interpretations interact with these factors. For a discussion of these interactions, see Büring 1996; de Hoop and Solà 1996; Cohen 2001; Herburger 1997; Partee 1989; Romero 2015, 2016, among others.

2 Reverse proportionality from context dependent standards

The literature develops the standard-based approach into different detailed analyses that diverge on important particulars (Büring 1996, de Hoop and Solà 1996, Romero 2015, Romero 2016, Solt 2009). However, since our argument will apply to the standard-based approach as a whole, there is no need here for a comprehensive review of these different proposals. We will instead introduce this general approach by outlining one particular possible rendition. This rendition is discussed (although ultimately not endorsed) in Westerståhl 1985b, and it also follows closely the line of reasoning developed in Solt 2009.

As a baseline, we first define the family of lexical entries that captures the two types of readings associated with *many* and *few* that Partee (1989) argued for. Treating *many* and *few* as forming generalized quantifiers in the sense of Barwise and Cooper (1981), the cardinal and forward proportional sense of *many* and *few* are given in (3) and (4), where n and p are contextually given standards of cardinality and proportion, respectively.

(3) a. $[\![\textit{many}_1]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}| > \mathrm{n}$
b. $[\![\textit{few}_1]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}| < \mathrm{n}$

(4) a. $[\![\textit{many}_2]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{X}| > \mathrm{p}$
b. $[\![\textit{few}_2]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{X}| < \mathrm{p}$

Following Westerståhl (1985b), reverse proportional readings could be captured in a straightforward way by positing the pair lexical entries in (5), obtained from those in (4) by replacing the first set argument with the second in the denominator of the fraction that the truth conditions refer to.

(5) a. $[\![\textit{many}_3]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{Y}| > \mathrm{p}$
b. $[\![\textit{few}_3]\!](\mathrm{X})(\mathrm{Y}) \Leftrightarrow |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{Y}| < \mathrm{p}$

However, as also noted by Westerståhl (1985b), given that the standard proportion p in these meanings is context dependent, it can be argued that conventionally encoded reference to reverse proportions is dispensable. This is because the right sides of the equivalencies in (5) can be restated as in (6).

(6) a. $|\mathrm{X} \cap \mathrm{Y}|/|\mathrm{Y}| > \mathrm{p} \;\Leftrightarrow\; |\mathrm{X} \cap \mathrm{Y}| > \mathrm{n}$, where $\mathrm{n} := \mathrm{p} \times |\mathrm{Y}|$
b. $|\mathrm{X} \cap \mathrm{Y}|/|\mathrm{Y}| < \mathrm{p} \;\Leftrightarrow\; |\mathrm{X} \cap \mathrm{Y}| < \mathrm{n}$, where $\mathrm{n} := \mathrm{p} \times |\mathrm{Y}|$

Indeed, as Westerståhl (1985b) observes, the forward proportional readings, too, could be accounted for by manipulating the contextual standard, as shown in (7).

(7) a. $|\mathrm{X} \cap \mathrm{Y}|/|\mathrm{X}| > \mathrm{p} \;\Leftrightarrow\; |\mathrm{X} \cap \mathrm{Y}| > \mathrm{n}$, where $\mathrm{n} := \mathrm{p} \times |\mathrm{X}|$
b. $|\mathrm{X} \cap \mathrm{Y}|/|\mathrm{X}| < \mathrm{p} \;\Leftrightarrow\; |\mathrm{X} \cap \mathrm{Y}| < \mathrm{n}$, where $\mathrm{n} := \mathrm{p} \times |\mathrm{X}|$

Thus, *many* and *few* could have a univocal, cardinal meaning with polysemy rooted in an independently motivated contextually determined standard.

This theoretically parsimonious option places the burden of proof on those wishing to argue for the existence of forward and reverse proportional lexical senses like those defined in (4) and (5).[2] Here we focus on the reverse proportional reading, though, which is also the one that is more

[2]Westerståhl (1985b) warns against such an appeal to parsimony, noting that this would require enriching our

commonly collapsed into a cardinal interpretation.[3]

Accepting this burden of proof, we will now proceed to establish that reverse proportionality is not in fact dependent on the presence of a contextually determined standard of comparison. While we do not know of any reasons, empirical or conceptual, for assuming that reverse proportionality can *never* be due to a particular setting of the standard, we will see that standard setting is at least not the only source of the relevant readings.[4]

3 Reverse proportionality without context dependent standards

Bresnan (1973) proposed that *many* and *few* function in the same way as gradable predicates. This proposal suggests itself for *few*, which combines with degree morphology in the characteristic way, in particular forming comparative and superlative forms *few+er* and *few+est*. Bresnan extends this type of analysis to *many* by analyzing *more* and *most* as *many+er* and *many+est*. Hackl (2000) further motivated this proposal by providing compelling semantic arguments that support decomposition.[5] In this section, we explore some of the consequences of this point of view, first reviewing a somewhat standard proposal for how to treat gradable adjectives before turning our attention back to cardinal and proportional interpretations of *many* and *few*, in particular interpretations that do not involve a comparison to some kind of standard.

In one prominent analysis of gradable adjectives (see Cresswell 1976, von Stechow 1984, among others), gradable predicates—the elements which most commonly combine with comparative and superlative morphemes—are analyzed using measurements and degrees. For example, gradable adjectives like *tall* can be interpreted as comparing a measurement of height to a degree of some sort, e.g., $[\![\textit{tall}]\!] = \lambda d.\, \lambda x.\ \mu_{\text{ht}}(x) \geqslant d$, where μ_{ht} maps individuals to the degree of their height. The use of such predicates in constructions like *John is six feet tall* is rather straightforward ($[\![\textit{John is six feet tall}]\!] = \mu_{\text{ht}}([\![\textit{John}]\!]) \geqslant [\![\textit{six feet}]\!]$). However, the analysis of sentences without an overt degree argument requires a phonologically null operator, often called *POS* (see von Stechow

semantics so that multiple contextual standards could be set within the same sentence. For example, Westerståhl (1985b) cites Barbara Partee's example *Many boys date many girls*, where it is apparent that the contextual standard of what counts as *many* in the first DP is much higher than what counts as *many* in the second. However, as Westerstahl notes in his work with respect to context sets (Westerståhl, 1985a), it seems to be a general property of language that contextually sensitive variables can receive distinct values for different DPs within the same sentence.

[3]There is some motivation in the literature to resist, in particular, having a reverse proportional lexical entry. For example, unlike the forward proportional lexical entry, a reverse proportional entry would not be *conservative* in the sense of van Benthem 1984. See the discussion in Westerståhl 1985b.

[4] Westerståhl (1985b) had initially detected reverse proportionality in the now famous example *Many Scandinavians have won the Nobel prize in literature*. However, subsequent authors argued that this sentence does not actually allow for the reverse proportional truth conditions of the sort derived by the lexical entry in (5a) (Cohen 2001, Romero 2015, 2016). Romero 2015, 2016 argues that the actual interpretation of Westerståhl's example crucially requires reference to the setting of the context dependent standard, which is to be calculated with reference to focus values in the sense of Rooth (1985). We are inclined to agree with Romero's assessment, which is compatible with the conclusions we draw in this paper. Again, it seems very plausible to us that the setting of a contextual standard can yield reverse proportionality or similar effects. What we deny is that standard setting is the only source of reverse proportionality.

[5]Some of the more compelling evidence that Hackl (2000) presents are instances of *split scope*. There are certain sentences that have a reading that is only compatible with truth conditions where the comparative morpheme scopes above an intensional operator while cardinal measurement function scopes below. For example, the sentence *A professor is required to write fewer than two books in order to get tenure* can be true in a context where a professor is only required to write at least one book to get tenure, although the professor is allowed to write more than one.

1984 and Kennedy 1999, among others), which takes an abstracted degree predicate as an argument. The *POS* operator compares the maximal value of the degree predicate to a contextually set standard. For example, let's suppose that that the sentence in (8) has an LF structure like the one in (8a), where *POS* has a meaning similar to the one represented in (8b), where STND is a contextually set standard. With this kind of structure, (8) would have the truth conditions in (8c).

(8) John is tall.
   a. [*POS* λd John is d tall]
   b. $[\![POS]\!] = \lambda D.\ \textsc{max}(D) > \textsc{stnd}$
   c. $\textsc{max}(\{d : \mu_{\text{ht}}([\![John]\!]) \geqslant d\}) > \textsc{stnd} \quad \Leftrightarrow \quad \mu_{\text{ht}}([\![John]\!]) > \textsc{stnd}$

Criticially, the contextually set standard is not an integral part of the semantics of the degree expression itself.[6] Not only is it absent when explicit measurement phrases are used (as with *six feet* in the example above), but it is also absent in comparative constructions. Although the details are not important for our purposes, for concreteness we will sketch a standard view on which the comparative morpheme *-er* denotes a function like (9), taking two degree properties as arguments, one obtained by abstraction in the *than*-clause and the other from the main clause after covert movement of the degree phrase formed by *-er* and the *than*-clause.[7]

(9) $[\![\textit{-er}]\!] = \lambda D_2.\ \lambda D_1.\ \textsc{max}(D_1) > \textsc{max}(D_2)$

The argument D_2 and D_1 will be furnished by the *than*-clause and the main clause, respectively. To illustrate, a sentence like (10) would have an LF structure similar to the one in (10a), resulting in truth conditions like those represented in (10b).

(10) Mary is taller than Bill is.
   a. [$_{\text{DEGP}}$ -er λd than Bill is d tall] λd[$_{\text{S}}$ Mary is d tall]
   b. $\textsc{max}(\{d : \mu_{\text{ht}}([\![Mary]\!]) \geqslant d\}) > \textsc{max}(\{d : \mu_{\text{ht}}([\![Bill]\!]) \geqslant d\})$

Such truth conditions compare two degrees that are explicitly determined by two clausal arguments, hence they do not make reference to a contextually set standard of comparison.

On this approach, the analysis of *many* and *few* as gradable expressions requires a revision of the lexical entries for *many* and *few* that separates the introduction of a contextually determined standard from the degree expression. Specifically, instead of the lexical entries for *many* in (3a) and (4a), we could now have those in (11).

(11) a. $[\![many_1]\!] = \lambda\text{d}.\ \lambda\text{X}.\ \lambda\text{Y}.\ |\text{X} \cap \text{Y}| \geqslant \text{d}$

[6]For the sake of simplicity, we ignore the issue of vagueness in terms of setting a value for the standard. For an adequate discussion of vagueness with respect to a standard, see the discussions in Kennedy 2007, Klein 1980, Kamp 1975 and references therein.

[7]As argued by Heim (2000), there are two main facts that support a movement analysis of degree phrases headed by the comparative morpheme. One is that such movement can account for scope ambiguities with intensional operators. For example, there is a reading of *Mary read 5 pages and John is required to to read exactly 2 more pages than that*, which means that the number of pages that John is minimally required to read is exactly two pages more than what Mary read. The other main argument stems from Antecedent Contained Deletion with comparatives (see also Bresnan 1973, among others). For example, ACD is acceptable in sentences like *John was climbing taller buildings than Mary was*. However, it is unacceptable (or at least strained) in sentences like *John was climbing buildings that Mary was*. Movement of the Degree Phrase [-er than Mary was] out of the VP would create the right environment for VP ellipsis.

b. $[\![\mathit{many}_2]\!] = \lambda \mathrm{d}.\ \lambda \mathrm{X}.\ \lambda \mathrm{Y}.\ |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{X}| \geqslant \mathrm{d}$

Hackl (2000) called these types of meanings *parameterized determiners*. Note that for the sake of simplicity, we will limit our discussion here to *many*, but similar interpretations can be given for *few*.[8] Just like adjectives, these lexical entries compare a measurement (of cardinality or proportion) to a degree. In order to introduce some kind of contextually determined standard, the *POS* operator would need to be introduced. For example, the sentence in (12) has a cardinal interpretation as represented in (12a) and a forward proportional interpretation as represented in (12b).

(12) Many students cheated.

a. for a contextually determined cardinality STND,
$[\![\mathit{POS}\ \lambda d.\ d\ \mathit{many}_1\ \mathit{students\ cheated}]\!] \Leftrightarrow$
$\textsc{max}(\{d : |[\![\mathit{students}]\!] \cap [\![\mathit{cheated}]\!]| \geqslant d\}) > \textsc{stnd} \Leftrightarrow$
$|[\![\mathit{students}]\!] \cap [\![\mathit{cheated}]\!]| > \textsc{stnd}$

b. for a contextually determined proportion STND,
$[\![\mathit{POS}\ \lambda d.\ d\ \mathit{many}_2\ \mathit{students\ cheated}]\!] \Leftrightarrow$
$\textsc{max}(\{d : |[\![\mathit{students}]\!] \cap [\![\mathit{cheated}]\!]|/|[\![\mathit{students}]\!]| \geqslant d\}) > \textsc{stnd} \Leftrightarrow$
$|[\![\mathit{students}]\!] \cap [\![\mathit{cheated}]\!]|/|[\![\mathit{students}]\!]| > \textsc{stnd}$

As with regular gradable predicates like *tall*, it is predicted that reference to a contextually determined standard should be absent in comparative constructions. Thus, comparative constructions provide an natural testing ground for whether reverse proportional readings (and proportional readings in general for that matter) are always derived by manipulating a contextual standard.

With this in mind, consider the example in (13), where the positive form of *few* in Herburger's (1997) classic example of a reverse proportional meaning (see (2) above) is replaced by the comparative form of *many*, accompanied by a *than*-phrase, with contrasting phrases *our program* and *yours*.

(13) More cooks applied to our program than to yours.

We submit, that (13) can be read as comparing two ratios, viz. the ratio of applicants to our program that are cooks relative to the total number of applicants to our program and the ratio of applicants to your program that are cooks relative to the total number of applicants to your program, stating that the former ratio is greater. Such a comparison can explain why (13) can be judged as true on the basis of no information about the sets of cooks and applicants to the two programs other than that cooks represent a greater proportion of the applicants to our program, say 20%, compared to the proportion of the applicants to yours, say 10%. (Thus, given what is known, the cardinal and forward proportional interpretation might not be true.) If we let X be the set of cooks, and Y1 and Y2 be the sets of applicants to our program and to your program, respectively, we can state the truth conditions of (13) as in (14).

(14) $|\mathrm{X} \cap \mathrm{Y1}|/|\mathrm{Y1}| > |\mathrm{X} \cap \mathrm{Y2}|/|\mathrm{Y2}|$

[8] The difference between *many* and *few* is akin to the difference between gradable antonyms like *short* and *tall*. Kennedy (1999), based off of a degree ontology introduced by von Stechow (1984), suggests that the difference between antonymous degrees is how they extend: positive degrees extend from zero to a measurement whereas negative degrees extend from a measurement to infinity. Such a solution can be adopted here for *few*. The details would closely follow Kennedy's analysis of the difference between *tall* and *short*.

Thus, (13) allows for a reading that is reverse proportional in the same sense as the relevant reading of (2) described by Herburger (1997), that is, a reading where both the main clause and the comparative phrase make reference to the ratio of the members of the set given by the intersection of the noun phrase and the scope to the members of set given by the scope alone.

We postpone until the next section the question how exactly these truth conditions arise. What is clear enough, however, is that in the absence of a contextually determined standard, reverse proportionality in (13) shows that reverse proportional readings are not after all dependent on the presence of a contextually determined standard, and therefore are not in general a symptom of the malleability of such a standard, contra the proposals in a whole branch of work on reverse proportionality (Büring 1996, de Hoop and Solà 1996, Romero 2015, 2016, Solt 2009). In drawing this conclusion, we are in fact stepping in the footprints of Partee (1989), who presented data much like (13). In concluding remarks, Partee presents comparative data that include the example in (15), providing the comments quoted below.

(15) There are more illiterate people in small rural towns than in large cities.

> "Such sentences are potentially valuable sources of data, since comparatives generally remove the ambiguity of vague predicates, and clear truth-conditional differences can then show up between cardinal and proportional readings. However, I think that judgments about the range of possible readings for [such] sentences [...] show a surprising range of possibilities, including a non-CN-based proportional reading for [(15)]." (Partee 1989, p. 400)

We take it that Partee employs *non-CN-based proportional reading* to refer to the reverse proportional reading discussed above. Indeed, it seems clear that (15) can be judged true on the basis of no other information than the assumption that small towns have a larger proportion of illiterate inhabitants than large cities, in analogy to what we have described for (13).

As is clear from the first sentence in the passage quoted above, Partee also hinted at the very same conclusion regarding reverse proportionality that we have drawn on the basis of (13). Given Hackl's (2000) semantic arguments for the analysis of *many* as a gradable predicate and for decomposition of *more*, bolstering Bresnan's (1973) earlier syntactic arguments, this conclusion in fact looks even more unavoidable now than it did at the time of Partee's writing.

Comparatives expectedly are not the only type of degree construction that this conclusion can be based on. Reference to contextually determined standards is also known to be removed in, for example, degree questions, equatives, or cases with demonstrative *that* used as a measure phrase. In such constructions, too, reverse proportional readings can be detected, as the examples in (16) serve to illustrate.

(16) a. Julia found out how many cooks applied.
b. That many cooks had never applied before.
c. Twice as many cooks applied last year.

We take it, if this year, 10% of the applicants were cooks, (16a) could be judged as true in virtue of Julia having found out that that was the case, without implying that Julia found out about any other cardinalities or proportions, including the absolute number of applicant cooks; similarly, (16b) can be true in virtue of the mere fact that in previous years the proportion of cooks among the applicants always remained below 10%, independently of any other cardinalities or proportions; and (16c) can be understood as conveying that last year 20% of the applicants were cooks, again without supporting inferences about other cardinalities or proportions.

It has moreover been proposed that bare numerals, such as *five* in *Five cooks applied*, fill the degree argument position of a silent version of *many* (e.g., Hackl 2000, Nouwen 2010). On this analysis, data like those in (13), (15), and (16) might lead us to expect that proportions too can serve the role of such bare numerals. This expectation is correct, as illustrated by the (attested) German example in (17).

(17) In den EU-Mitgliedstaaten leben 93,3 Prozent eigene Staatsbürger.
in the EU-member.states live 93.3 percent own citizens
'Citizens of the EU member states comprise 93.3 percent of their inhabitants.'

In this example, the numeral *93,3 Prozent* identifies the ratio of EU citizens that inhabit the EU to all EU inhabitants. So the truth conditions of (17) refer to a proportion whose denominator, the cardinality of the set of EU inhabitants, is given by the main clause content, hence they are reversely proportional in the relevant sense.[9]

To reiterate, we conclude from such data that there exists a source of reverse proportionality (and perhaps proportionality in general) other than contextually determined standards of comparison. We have no reason for doubting that reverse proportionality can in principle be due to the setting of the standard. But we have argued, following Partee (1989), that such standard setting is insufficient to capture all instances of reverse proportionality.

4 Non-standard based sources of reverse proportionality

The question that remains is how to properly analyze reverse proportionality in cases like (13). Below we briefly map out the range of answers emerging from the literature. Adding to our main argument above, we then present novel data suggesting that these answers, too, are insufficiently general. Extending arguments presented in Bale and Barner (2009) and Wellwood (2014), those data lead us to propose that reverse proportionality can reflect the underspecification of the measure function underlying the meanings of *many* and *few*.

4.1 Lexical and syntactic argument switching

Analyzing *many* and *few* as gradable expressions, and applying the analysis of comparatives outlined above, we are led to assign to (13) a logical form like (18).

(18) [-er λd. than [[d many] cooks] [applied to your program]] λd.[$_S$ [[d many] cooks] [applied to our program]]

Consider now the lexical entry for *many* in (19), which adapts Westerståhl's (1985b) reverse proportional entry proposed in (5a) to the assumed degree based semantics, in parallel to the entries

[9] The periphrastic English translation given here reflects the fact that structures parallel to (17) do not seem acceptable in English. We are not sure about the reasons for this cross-linguistic contrast, which we leave as a topic for future research.

for cardinal and forward proportional entries in (11). Applied to the structure in (18), this entry delivers the intended truth conditions in (19), truth conditions equivalent to those formulated in (14).

(19) $[\![\, many_3 \,]\!] = \lambda \mathrm{d}.\, \lambda \mathrm{X}.\, \lambda \mathrm{Y}.\ |\mathrm{X} \cap \mathrm{Y}|/|\mathrm{Y}| \geqslant \mathrm{d}$

(20) $\textsc{max}(\{d : |[\![\, \textit{cooks} \,]\!] \cap [\![\, \textit{applied to our program} \,]\!]|/|[\![\, \textit{applied to our program} \,]\!]| \geqslant d\}) >$
$\textsc{max}(\{d : |[\![\, \textit{cooks} \,]\!] \cap [\![\, \textit{applied to your program} \,]\!]|/|[\![\, \textit{applied to your program} \,]\!]| \geqslant d\})$

So, while reverse proportional readings in comparatives are beyond the scope of the standard-based approach, their existence is correctly predicted on an a lexical analysis, where *many* and *few* are gradable expressions with reverse proportional lexical entries.

That said, the literature also offers a second non-standard based route to reverse proportional readings that is compatible with the existence of such readings in comparatives and other standard-fixing constructions. This approach, pursued in Herburger (1997) and Greer (2014), rejects the proliferation of lexical entries required in the lexical analysis, and locates the added complexity in the syntax-semantics interface. Note that the forward proportional entry for *many* in (11b) above can be mapped to the reverse proportional entry in (19) by switching the order of two degree property arguments. Rejecting reverse proportional entries, Herburger (1997) and Greer (2014) argue that rather than by lexical meaning, this switch is accomplished by syntax or focus marking at the syntactic level. We will refrain here from reviewing these accounts—let's call them the *syntactic mapping* analyses—in more detail. The point we wish to make is merely that for the present purposes, syntactic mapping analyses of reverse proportionality are like lexical ambiguity analyses in that they do not rely on the presence of a contextually determined standard of comparison. Therefore, such analyses, too, are not challenged by reverse proportionality in comparatives and other standard-fixing constructions.

However, supplementing our primary argument about the standard based-approach, we will now argue in addition that, just like the standard-based approach, syntactic mapping analyses are insufficient to capture the full range of reverse proportional interpretations. The next subsection is dedicated to making this point.

4.2 Contextual proportionality

The example sentences in (21) permit interpretations that are similar to the reverse proportional readings that we have been discussing.

(21) a. There are more boats on Lake Ontario than on Lake Superior.
b. There are more knots in the blue rope than in the red one.
c. Your manuscript has more typos than my manuscript.

Sentence (21a) can be read as comparing the number of boats on Lake Ontario and Lake Superior in proportion to their surface areas. With the surface area of Lake Superior being about four times that of Lake Ontario, (21a) can be true in a scenario where there are, for example, exactly 1000 boats on each lake. Similarly, (21b) can be true in a scenario where, for example, each of the two ropes has exactly 20 knots in it, but where the red rope is, say, three times longer than the blue one; and (21c) can be true in a scenario where there are, for example, 100 typos in each of the two manuscripts, but where the word count of my manuscript is, say, ten times the word count of yours.

In these readings, then, it is not cardinalities that are being compared. Instead, the sentences in (21) appear to allow for truth conditions of the form (22) below. In (21a), X is the set of boats, X∩Y1 and X∩Y2 are the sets of boats on Lake Ontario and on Lake Superior, and m1 and m2 are the surface areas of the two lakes; in (21b), X is the set knots, X∩Y1 and X∩Y2 are the sets of knots in the blue rope and in the red rope, and m1 and m2 are the lengths of the two ropes; and in (21c), X is the set of typos, X∩Y1 and X∩Y2 are the sets of typos in your manuscript and my manuscript, and m1 and m2 are the word counts of the two manuscripts.

(22) $|\mathrm{X} \cap \mathrm{Y1}|/\mathrm{m1} > |\mathrm{X} \cap \mathrm{Y2}|/\mathrm{m2}$

Comparison of the truth conditions in (22) with those in (14) above reveals that the relevant readings of the sentences in (21) differ minimally from canonical reverse proportional readings. In both types of cases, the numerators of the fractions on the two sides of the inequality are given by parallel syntactic constituents. The only differences concern the denominators of the two fractions. In canonical reverse proportional readings of comparative sentences like (13), the denominators are the cardinalities of the sets determined by the denotation of the scope of *many* in the main clause and the *than*-phrase. In contrast, in the cases in (21), the denominators are certain measurements associated with the denotations of contrasting expressions within the scope of *many*.

The crucial observation is that these measurements are not referred to in the conventional meaning of the syntactic environment in which *many* appears. That is, we take it that there are no constituents in (21a) that refer to a lake's surface area, just like there are no constituents in (21b) and (21c) that refer to a rope's length or a manuscript's word count. We conclude, therefore, that the proportions referred to in the meanings of proportional interpretations are not always fixed by semantic content. We will therefore refer to these readings as *contextually proportional*.[10]

The discovery of contextual proportionality leads us to the lexical entry for *many* in (23). This entry again follows Hackl (2000) in positing that the denotation of *many* takes a degree argument. The interpretation refers to a fraction whose numerator is formed by the cardinality of the intersection of the two set arguments X and Y. The denominator of this fraction is given by the free meta-language variable m, a measurement whose content is underspecified in the sense of not being fixed by conventional meaning. A similar meaning can be given for *few* but we will forego the details here.[11]

(23) Where m is a contextually determined denominator,
$[\![\textit{many}]\!] = \lambda \mathrm{d}.\ \lambda \mathrm{X}.\ \lambda \mathrm{Y}.\ |\mathrm{X} \cap \mathrm{Y}|/\mathrm{m} \geqslant \mathrm{d}$

We can capture the relevant readings of (21) by allowing for m to be set to any value that is salient in the context of an utterance. We take it that in (21), the mention of the lakes, ropes, and manuscripts raises the salience of the relevant surface areas, lengths, and page counts respectively, and hence that m can take on the values specified above for m1 and m2, capturing the readings in question.

[10] Expectedly, contextual proportionality is not limited to comparatives. For example, in parallel to example (16c) above, *There are twice as many boats on Lake Ontario as there are on Lake Superior* can be read as conveying that the proportion of number of boats on Lake Ontario to the surface area of Lake Ontario is two times the proportion of number of boats on Lake Superior to the surface area of Lake Superior.

[11] As noted earlier, an interpretation for *few* can be given that is basically the same as the entry for *many*, modulo the semantics of gradable antonymy. Such a semantics could involve reversing the ordering of the degrees either by reversing the comparative relation (e.g., $[\![\textit{few}]\!] = \lambda \mathrm{d}.\ \lambda \mathrm{X}.\ \lambda \mathrm{Y}.\ |\mathrm{X} \cap \mathrm{Y}|/\mathrm{m} < \mathrm{d}$) or by interpreting degrees as intervals (as in Kennedy 1999).

The lexical entry in (23) delivers ordinary reverse proportional readings of examples like (2) or (13) as a special case, viz. the case where m is set to the cardinality of the set determined by the scope of *many*, that is where in (23), m is set to $|Y|$. In fact, it is apparent that the entry is general enough to accommodate all of the readings described above. The forward proportional that Partee (1989) detected in examples like (1b) obtains when m is set to $|X|$ and the cardinal reading attested in (1a), when m is set to 1.

These observations suggest that the seemingly obvious analysis of contextual proportionality put forth here is general enough to cover the full range of readings that *many* is perceived to participate in. We propose, therefore, that the existence of contextual proportionality places the burden of proof on those who wish to argue, following Westerståhl (1985b), Herburger (1997), and Greer (2014), that canonical reverse proportional readings are a matter of conventional meaning fixed by either lexical meaning of *many* or *few* alone (Westerståhl 1985b) or by the interaction of lexical meaning with the mapping of syntactic material to the argument positions of *many* or *few* (Herburger 1997, Greer 2014). In fact, more generally, we take contextual proportionality to present a new challenge to those wishing to argue, following Partee (1989), that *many* or *few* are lexically ambiguous.

While we seem to be first to discuss contextual proportionality, the relevant interpretations of the cases in (21) are reminiscent of certain familiar data points, discussed in Cresswell (1976) and Bale and Barner (2009), regarding the interpretation of *much* plus mass nouns. Contextualizing our findings reported in this subsection, we will conclude in the next and final subsection by identifying this connection and its possible consequences.

4.3 Measurements and proportionality with mass nouns

There is an interesting parallel between the context sensitivity of *many*, as described above, and the behaviour of mass nouns in comparative constructions. We will briefly summarize the facts with respect to mass nouns before proposing a general interpretation of *many/much* that integrates the count and mass interpretations into one *parameterized determiner*. It should be noted that our point here is rather modest, namely that it is possible to account for the patterns in comparatives by having a single lexical entry for *much/many* with a context sensitive measurement function. This possibility simplifies our lexical entries even further and, all else being equal, should be preferred to a theory that has multiple lexical entries to account for the different readings of comparative sentences.

As thoroughly discussed in Cresswell 1976 and Bale and Barner 2009, comparatives that modify mass nouns involve truth conditions that specify fundamentally different types of measurements. For example, to judge the comparison in (24a), one normally needs to know the volume of water in the two buckets. In contrast, to adequately judge the comparisons in (24b) and (24c), one needs to know the length of the two strings and the number of items of furniture in the two rooms, respectively.

(24) a. John's bucket has more water than Mary's. (comparison of volume)
b. John has more string in his desk than Mary. (comparison of length)
c. John's bedroom has more furniture than Mary's. (comparison of number)

If we assume that *more* in these sentences decomposes into *much+er* (on analogy to the analysis of *many+er* as discussed in Bresnan 1973), we would need to hypothesize a meaning for *much* that has a context sensitive measure function.

(25) $[\![\textit{much}]\!] = \lambda$d. λX. λY. $\mu(\text{X} \cap \text{Y}) \geqslant d$,
where μ can yield a measure of length, weight, volume, number etc.[12]

Note that the variability in the measurement function is not completely determined by the nominal complement. As noted by Cresswell (1976) and Bale and Barner (2009), one and the same nominal complement can induce truth conditions that rely on different types of measures. For example, in contexts where weight contrasts with volume, the sentence in (24a) can be judged as both true and false, depending on which type of measure is contextually emphasized. Similarly, consider the sentences in (26).

(26) a. This ring has more gold in it than that necklace.
b. This bottle of wine has more alcohol in it than that bottle.

If we assume that the ring is small whereas the necklace is rather large and we further assume that the ring is slightly closer to being "pure gold", the sentence in (26a) can be judged as both true and false. It can be true if the relevant measure is taken to be the proportion of gold in the ring versus the proportion of gold in the necklace, but it can be false if the relevant measure is taken to be the weight/volume of gold in the ring versus the weight/volume of gold in the necklace.

A similar observation can be made about (26b). If we assume that the first bottle only has a litre of wine but has a higher alcohol percentage, whereas the second has two litres of wine but a slightly lower alcohol percentage, then the sentence in (26b) can be both true and false. It can be true if the relevant measure is taken to be the proportion of alcohol in the wine, but it can be false if the relevant measure is taken to be the overall weight/volume of alcohol in the wine.

Hence, the measure function can take on different values with respect to the same nominal complement much like the variety of readings of *many* demonstrated in the previous subsection. This naturally leads to the question of whether *much* and *many* are allomorphs of a single lexical entry, as independently argued for by Chierchia (1998) and Wellwood (2014) for morphosyntactic reasons. This could be represented as in (27).

(27) $[\![\textit{much/many}]\!] = \lambda$d. λX. λY. $\mu(\text{X} \cap \text{Y}) \geqslant d$,

where μ has a contextually set value (e.g., one of μ_{WT}, μ_{VOL}, μ_{LENGTH}, $\mu_{\#}$, $\mu_{\frac{\text{VOL}}{\text{VOL-OF-X}}}$, $\mu_{\frac{\text{VOL}}{\text{VOL-OF-Y}}}$, $\mu_{\frac{\#}{\#\text{-OF-X}}}$, $\mu_{\frac{\#}{\#\text{-OF-Y}}}$, $\mu_{\frac{\#}{\text{LENGTH-OF-ROPE}}}$, $\mu_{\frac{\#}{\text{AREA-OF-LAKE}}}$, etc.).

If something like (27) is on the right track, then the main task that we, as researchers, face is to explain why certain types of measurements are unavailable in certain contexts. For example, why are measurements of temperature never available? Why is volume available when measuring water but not when measuring boats? Why is length available when measuring rope but not when measuring people?

Some of these questions have already been answered in Schwarzschild 2002, where it was noted that such measure functions must be monotonic with respect to the subgroup/subaggregate relation inherent in the nominal complement (which, for example, rules out measurements of tem-

[12] Although μ in (25) applies to a set (the intersection of X and Y), it ultimately can be understood as a measurement of a plurality, namely the measurement of the supremum of the intersection. See Bale and Barner 2009 for details.

perature). Wellwood (2014) attempts to develop a stronger constraint than monotonicity, one that maintains that the relevant measure function is invariant under all automorphisms on the denotation of the nominal complement. Such a constraint would explain why count nouns cannot be measured in terms of weight or volume, but yet permit measurements of number and proportions. For now, we will simply note that this is an active and interesting area of research. We think that the proportional data discussed above will play a critical role in determining whether a univocal meaning for *much/many* is plausible and, if so, what type of constraints are needed to limit the number of contextually available measure functions.

5 Conclusion

We have argued that, while the standard-based approach to reverse proportionality with *many* and *few* is motivated by considerations of theoretical parsimony, the finding that reverse proportionality is attested in standard-fixing constructions such as comparatives shows this approach to be insufficient. Based on the discovery of contextual proportionality, we have moreover argued that proportionality in general is due to the fact that sentences with *many* and *few* do not semantically fix the measure that determines what value is being compared to the standard of comparison. Taking into account a broader range of data, then, considerations of theoretical parsimony suggest that the underspecification of this measure is the key to the meaning of *many* and *few*, and raise the question whether anything more needs to be said about *many* and *few* to capture the readings that have been posited in the literature.

We of course do not pretend to have offered a conclusive answer to this question. One prominent issue that remains to be investigated consists in grammatical constraints on cardinal and proportional readings that have been described in the literature. For example, Partee (1989) reports that cardinal readings are excluded when *many* and *few* appear in partitives or as subjects of individual-level predicates in the sense of Carlson (1977). Also, Büring (1996), Cohen (2001), Herburger (1997), and Romero (2015, 2016) all discuss the interaction of certain readings with focus structure. On the approach we have proposed, any such constraints would have to be interpreted constraints on the setting of the underspecified measure. We will leave an assessment of the prospects of such a reinterpretation to future work.

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