# Decomposing Quantifier Phrases and Split Scope 

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June 2018


#### Abstract

In this paper, we propose to analyze split scope in terms of quantification over covert number/amount expressions, so that a quantifier phrase like no employees can have a syntactic and semantic analysis similar to the pseudo-partitive no number of employees. More specifically, we propose to syntactically decompose some negative existential quantifier phrases (e.g., no employees) into two components: (a) an existential quantification over numbers, and (b) a quantification over sums of individuals. We show how these two components have an effect on truth conditions just in case the scope positions of the two components are separated by certain other quantificational expressions, including modal verbs. We argue that this approach is consistent with the proposals about the syntax and semantics of negation and quantifier scope in Collins and Postal (2014) and, moreover, yields an analysis of no employees type split scope cases requiring no special assumptions about negation or quantifier scope beyond those in that work.


Key Words: split scope, pseudo-partitives, existential quantifiers

## 1. Introduction

There is now a significant literature on the so-called split scope phenomenon, originally driven by Dutch and German facts, which we do not discuss; see such works as the following and references therein: Abels and Martí (2010), Iatridou and Sichel (2011), Jacobs (1980), Penka
(2012), de Swart (2000), Zeijlstra (2011). The term refers to cases where a single quantifier expression has a semantic structure in which distinct components of the expression have distinct scopes, one above and one below some other expression. We focus here exclusively on English split scope data.

An English split scope case studied in Potts (2000) involved multiple possible interpretations of (1):
(1) The company need fire no employees
a. $\neq \quad$ The company is obligated to fire no employees.
b. $=\quad$ There are no employees x such that the company is obligated to fire x.
c. $=\quad$ It is not the case that the company is obligated to fire employees.

The key element here is evidently the phrase no employees, which seems to contribute to the meaning of the expressions both negation and an existential quantifier.

Potts notes that (1) lacks interpretation (1a), which would be the low scope (of no employees) or de dicto reading, but has the other two. Interpretation (1b) is the high scope or de $r e$ reading, while that in (1c) is the split reading. To see the independence of (1b) and (1c), consider the following scenarios:
(2) Scenario 1: The company needs to fire some employees, because there are financial problems. However, there is no particular employee that the company needs to fire. For example, Carol, the CEO, does not need to be fired, neither does Joe the salesman.

In this scenario, (1c) is false, but (1b) is true. So (1b) does not entail (1c). Now consider the following scenario:
(3) Scenario 2: Suppose two particular employees need to be fired because they have stolen some of the company's money. Under these circumstances, it is true to say that the company needs to fire employees.

In this scenario, (1b) is false, which entails that (1c) is false. Therefore, (1c) entails (1b). So there are the following entailment relations. In other words, (1c) is stronger than (1b).
(4) a. $\quad(1 b) \nvdash(1 \mathrm{c})$
b. $\quad(1 c) \vDash(1 b)$
(4) shows why it is sometimes hard to distinguish between de re and split scope interpretations, since they are not independent.

The facts in (1) illustrate how the term split scope describes situations where a quantificational phrase Q , in (1) no employees, occurring with some other predicational or quantificational element $T$, in (1) modal need, is seemingly interpreted in such a way that part of Q scopes over T and part under T. In (1), the higher scoping part of no employees is interpreted as negation, while on the split reading, the lower scoping element represents existential quantification.

The lack of a low scope interpretation (1a), in which no employees scopes under modal need, is not mysterious since need is a negative polarity item (NPI). Accepting the now general wisdom that NPIs need to occur under the scope of a decreasing operator, such a condition cannot be met in a low scope structure for NPI need. Therefore, the missing reading of (1a) is blocked by general constraints on NPIs. Following an earlier account by Rullman (1995), this pattern of explanation was given by de Swart (2000: 110) for the Dutch equivalent of need, hoeven, also an NPI.

The de re reading of (1) can be taken to represent a standard high scope representation of a quantificational phrase located superficially in an embedded clause. The initially puzzling case is the split reading. About this, Potts concludes:
(5) "...why does no, normally so preserving of its integrity, allow itself to come unglued only in the presence of a higher need?"

We observe that paradigms like (1) can be duplicated in their essentials for mass noun DPs:
(6) That bakery need sell no strudel.
a. $\neq$ It is necessary that the bakery sell no strudel.
b. = There is no strudel x such that it is necessary that the bakery sell x .
c. = It is not the case that the bakery need sell some strudel.

It turns out that the analysis we provide of split scope readings like (1c) carries over with quite minor differences to the treatment of the mass noun cases. For simplicity, we thus build most of the treatment on count noun cases like (1).

In considering the analysis of quantificational DPs like no employees and no strudel in (1) and (6), two distinct analytic questions always arise logically:
(7) a. What are the scopes of the denoted quantifiers?
b. What are the internal structures of the relevant DPs themselves?

Potts (2000) and much other research on split scope deal principally with question (7a). While Potts (2000) does not consider the internal structure of phrases like no employees, his study, consistent with much other work, clearly implies about question (7b) at least this:
(8) The internal structure of no employees is identical on both the de re and split readings.

The shared structure would be one which determines quantification over individuals, in (1)
employees. In the present paper, we reject this identity of structure view and claim that the two different extant interpretations of no employees in (1) and of no strudel in (6) represent partially distinct syntactic structures of the expression no employees.

In particular, we argue that when no employees takes split scope its syntactic structure is parallel to that of the pseudo-partitive no number of employees, while when no amount of strudel takes split scope its syntactic structure is parallel to that of the pseudo-partitive no amount of strudel.

## 2. Background Assumptions

Before advancing our analysis, we briefly discuss the background assumptions about negation and scope in Collins and Postal (2014) (hereafter: CP2014), which fundamentally underlie the present proposals. CP2014 analyzes negative existential quantifiers as in (9):
no person $\quad=\quad[[\mathrm{NEG}<$ SOME $>]$ person $]$
Taking the symbol NEG to be our representation of syntactic negation and angled brackets < ...> to systematically indicate covert occurrences, NEG in (9) modifies a covert instance of SOME, which is our syntactic representation of the existential quantifier.

Given those assumptions, consider:
(10) a. I saw no person.
b. I didn't see any person.

In the framework of CP2014, these sentences have the following partial representations.
Fuller structures with scope indication for no/any person are touched on presently.
(11) a. I saw [ $\mathrm{DP}\left[\mathrm{D}_{\mathrm{D}} \mathrm{NEG}_{1}[\mathrm{D}<\right.$ SOME $>]$ person $]$
b. I did $\mathrm{NEG}_{1}$ see [ $\mathrm{DP}_{\mathrm{D}}<\mathrm{NEG}_{1}>$ [D $<$ SOME $>$ ]] person]

That is, the object DP any person in (10b), like no person in (10a), is taken to be an inherently negative DP expression. The difference is that in (10b) the original DP-internal NEG has raised to the post-aux position of the immediately containing clause.

CP2014 assumes $\mathrm{NEG}_{1}$ in (10b) is interpreted in its original position (indicated by $<\mathrm{NEG}_{1}>$ ) and not the post-aux position. Examples (10a,b) then have the same truth conditions because both involve the same negative quantificational DP object interpreted in the same position.

The analysis in CP2014 requires the following spell out rules for SOME:
(12) The SOME/any Mapping
a. SOME $\rightarrow$ any, in the context $[<$ NEG $>\ldots] \quad$ (NEG unpronounced)
b. SOME $\rightarrow$ null, in the context [NEG __] (NEG pronounced)
c. SOME $\rightarrow$ some, otherwise

Principle (12a) states that if NEG raises away from SOME, SOME is spelled out as any. (12b) indicates states that if non-raised NEG modifies SOME, SOME is covert.

CP2014 also claims that no and not are variant forms of NEG, governed by the following condition:
(13) The NEG Mapping
a. $\mathrm{NEG} \rightarrow$ no in the context $[\mathrm{D} \ldots \ldots[\mathrm{D}<\mathrm{SOME}>]]$
b. NEG $\rightarrow$ not, otherwise

Principle (13a) determines that if a non-raised NEG modifies SOME, then NEG is realized as $n o$. Otherwise, NEG is realized as not.

Fundamental to the present proposal about split scope are the CP2014 assumptions about the representation of scope. Our view of the de re readings of cases like (1) and (6) differs in no
significant way from past treatments. It involves quantification over individuals. We assume that all quantifier DPs, hence no employees, have at least two occurrences: one defining their scope, the other their 'ordinary' or 'argument' syntactic positions (see CP2014: 13-14 for details). In these terms a fuller relevant structure of the de re reading of (1) would be:

## [DP1 [ $[\text { NEG SOME] employees }]_{1}$ [the company need fire $\left.\mathrm{DP}_{1}\right]$ ]

Here the leftmost representation of the object, $\mathrm{DP}_{1}$, gives its scope position occurrence, while the abbreviated rightmost representation of that DP specifies its surface object occurrence. The former occurrence specifies indirectly (via its semantic interpretation) the scope of the quantifier $\mathrm{DP}_{1}$, while the latter is interpreted as a variable. The fact that only the 'argument' position yields morphophonological consequences is a function of a general principle from (CP2014: 14):
(15) Scope position occurrences of quantifier DPs are always covert.

But (14) is and can only be our account of the de re reading of (1). This holds since the split reading requires the negation to be higher than need, while the existential quantifier phrase must occur as part of the complement clause of need. And it is clear that no alternation of the position of $\mathrm{DP}_{1}$ in (14) will yield the split reading. This seems at first to present a difficult challenge for the conception of scope developed in CP2014.

## 3. The Decomposition of Negative Existential Quantifier Phrases

We propose that the proper approach to the challenge just noted is to recognize the following: Despite the surface identity of the object of fire under the de re and split readings, the phrase no employees under the split reading realizes a syntactic structure distinct from that of $\mathrm{DP}_{1}$ in (14); and parallel claims hold for the no strudel type examples. The crucial feature of this distinct syntactic structure is that it yields quantification over numbers for (1) and over amounts
for (6). Such structures will also involve positive existential quantification over (sums of) individuals. We argue that such a decomposition allows a simple analysis of the split scope interpretations such as those in $(1 \mathrm{c}) /(6 \mathrm{c})$. The relevant structure is entirely compatible with the CP2014 representation of scope and requires no appeal to any additional mechanisms.

As a first approximation, our proposal for the split reading structure of (1) is as follows:
(16) Decomposition of Negative Existentials (First Version)
no employees $=[$ DP1 [DP2 [D NEG [d SOME $]$ ] NUM] employees $]$ ]
In this structure, the restriction of [NEG SOME] is the denotation of the covert noun NUM (which denotes the positive integers). In this structure, the symbol NUM is intended as a covert instance of what overt number represents. We note that recognition of a covert element representing numerals is neither original nor recent. For instance, Kayne (2005: chapter 8) argues in great detail for the occurrence of such an element in a wide variety of English contexts, albeit not those relevant to the present discussion.

In effect, structure (16) represents one quantifier phrase ([[NEG SOME] NUM]) embedded in another. Section 5 adds details to this structure in order to bring out the parallels to pseudo-partitives and to ground the compositional semantics. In particular, section 5 will introduce an existential determiner SOME in $\mathrm{DP}_{1}$. This first version is only intended to illustrate how our analysis works.

Given representation (16), there are six possible scopal representations of (1):
(17) a. [[NEG SOME $]$ NUM $]_{2}\left[\mathrm{DP}_{2} \text { employees }\right]_{1}$ The company need fire $\mathrm{DP}_{1}$
b. $\quad *\left[\mathrm{DP}_{2} \text { employees }\right]_{1}[[\mathrm{NEG} \text { SOME }] \mathrm{NUM}]_{2}$ The company need fire $\mathrm{DP}_{1}$
c. $\quad$ The company need $[[\mathrm{NEG} \text { SOME }] \mathrm{NUM}]_{2}\left[\mathrm{DP}_{2} \text { employees }\right]_{1}$ fire $\mathrm{DP}_{1}$
d. *The company need $\left[\mathrm{DP}_{2} \text { employees }\right]_{1}[[\mathrm{NEG} \text { SOME }] \text { NUM }]_{2}$ fire $\mathrm{DP}_{1}$
e. $\quad[[\mathrm{NEG} \text { SOME }] \mathrm{NUM}]_{2}$ The company need $\left[\mathrm{DP}_{2} \text { employees }\right]_{1}$ fire $\mathrm{DP}_{1}$
f. $\quad *\left[\mathrm{DP}_{2} \text { employees }\right]_{1}$ The company need $[[\mathrm{NEG} \text { SOME }] \text { NUM }]_{2}$ fire $\mathrm{DP}_{1}$

That is, where both quantifier phrases $\left(\mathrm{DP}_{1}\right.$ and $\left.\mathrm{DP}_{2}\right)$ have scope over the verb need, there are two possibilities. Similarly, there are two possibilities where both have scope under need, and two possibilities where one scope position is higher but the other lower than need.

Representations (17a) and (17c) are truth conditionally equivalent to (18a,b) respectively:
$\begin{array}{lll}\text { a. } \quad[[\mathrm{NEG} \mathrm{SOME}] \text { employees }]_{1}[\text { The company }]_{3} \text { need fire } \mathrm{DP}_{1} & \text { (de re) } \\ \text { b. } & \left.*[\text { The company }]_{3} \text { need }[[\mathrm{NEG} \text { SOME }] \text { employees }]_{1} \text { fire } \mathrm{DP}_{1}\right] & \text { (de dicto) }\end{array}$ That is, the complex, dual quantifier DPs with covert NUM in (17a) and (17c) are equivalent respectively to the de re and de dicto readings with simple quantifier phrases. So in effect under our analysis, the structures yielding de re and de dicto interpretations are both structurally ambiguous. But the de dicto reading with modal need is blocked by NPI considerations, as noted earlier.

We assume that (17b,d,f) are ruled out by the Proper Binding Condition (May 1977: 21):
(19) A variable is properly bound by binding phrase A iff it is c-commanded by A .

For example, in (17b), the quantifier phrase [dp [D NEG [d SOME] NUM] ${ }_{2}$ does not c-command the $\mathrm{DP}_{2}$ occurrence that is interpreted as a variable.

Consider now (17e), which we claim to represent the split reading. The resulting semantics can be paraphrased as either of (20a,b):
(20) a. There is no number n such that the company need fire n employees.
b. For every number n , it is not the case that the company need fire n employees.

But these are equivalent to the conjunction of following unbounded sequence of statements (recall that NUM contains only positive integers):
(21) a. It is not the case that the company need fire 1 employee.
b. It is not the case that the company need fire 2 employees.
c. etc.

If the company needs to fire 2 or more employees, then logic determines that it needs to fire 1or more employees. Then (21a) entails (21b), so (21a) is the strongest statement of (21). But (21a) is equivalent to the split scope reading:
(22) It is not the case that the company need fire employees.

In other words, we have shown that (17e) accurately represents the split reading as given in sources like Potts (2000), where negation alone scopes over the matrix modal auxiliary.

Given the equivalence of (17e) and (22), the obvious question is why de re negative existential quantification over numerals is equivalent to the split scope reading (that is, (17e) is equivalent to (22)), whereas de re negative existential quantification over individuals is not. In the two paraphrases of (1) above, replace employees with number of employees:
(23) b. There is no number of employees that the company is obligated to fire.
c. It is not the case that the company is obligated to fire some number of employees.

Whereas (1b,c) were not equivalent, $(23 b, c)$ do seem equivalent. One way we can check this is by looking at Scenario 1 above, which distinguished de re and split scope readings. We can check Scenario 1 for (23):
(24) Scenario 1: The company needs to fire some number of employees because there are financial problems. However, there is no particular number of employees that the company needs to fire.

Unlike the situation in the scenario in (2), that in (24) is a contradiction. If the company needs to fire some number of employees, then the company needs to fire one or more employees, and therefore there is a particular number (one) of employees that the company needs to fire.

We observe that our novel account of split scope explains a constraint on the phenomenon, stated as follows by Iatridou and Sichel (2011: 621) (see also De Swart 2000: 115): (25) "Crucially, there is no reading in which the indefinite scopes above the predicate and the negation below."

Our analysis of split scope accounts for this constraint since the relevant structure in (17f) is ruled out by the Proper Binding Condition in (19).

## 4. Pseudo-Paritive number

The postulation in analysis (16) of a null noun NUM draws a parallel between simple negative quantifier phrases having split scope readings and more complex pseudo-partitive constructions like those in (26a,b):
a. Some number of athletes would be able to do that.
b. No number of athletes would be able to do that.

While cases like (26a) seem quite natural, instances of those like (26b) are sometimes strained. Nonetheless, the internet provides multiple citations of numerical cases as well as of analogous amount ones:
(27) a. Ferrous alloys and titanium alloys[2] have a distinct limit, an amplitude below which there appears to be no number of cycles that will cause failure.
(https://en.wikipedia.org/wiki/Fatigue_limit)
b. No number of sweeteners can sugar-coat it.
(https://twitter.com/SenJeffMerkley/status/941846478313664512)
c. No number of sightings of white swans can prove the theory that all swans are white. (https://www.theguardian.com/books/2002/apr/27/artsandhumanities.highereducation1)
(28) a. Umar Ibn Al-Khattaab - 'No amount of guilt can change the past, and no amount of worrying can change the future. Go easy on yourself, for the outcome of ...
(https://www.goodreads.com/.../855080-no-amount-of-guilt-can-change-the-past-and-...)
b. No amount of hype about Dubai's cool gadgetry can hide the rotten police state in the Emirates.
(https://www.alaraby.co.uk/.../no-amount-of-dubai-gadgetry-can-coolwash-police-state...)
c. There's no amount of rice that will help remove these as they need to be physically cleaned.
(gocellphonerepair.com/../how-does-rice-remove-moisture-from-cell-phones-tmot-dec...)
Significantly then, there are examples like (29b), which has a split reading equivalent to that of (29a):
(29) a. The company need hire no new employees.
b. The company need hire no number of new employees.

Like (29a), (29b) also has a de re reading. We claim that (29a) on its split scope reading has a structure parallel to the pseudo-partitive in (29b). The common structure of both no employees (on the split analysis) and no number of employees is the following:
(30) Decomposition of Negative Existentials (Second Version)
[DP1 [DP2 [D NEG SOME] [NP2 NUM/number]] (of) [NP1 employees]]
Example (29a) contains the covert noun NUM while (29b) contains the overt noun number. An important aspect of structure (30) is that [DP2 [D NEG SOME] [NP2 NUM/number]]
acts as a modifier of employees. So the structure is very different from noun complement constructions such as the carving of Ceasar:
(31) [ DP the [ NP carving [pp of Ceasar]]]

In (31), the noun carving has a complement of Ceasar. In (30), the noun number/NUM has no complement.

Some support for (30) comes from not even continuations. We assume that there is a condition (not formalized here) that a not even continuation must quantify over a set of entities that is included in the restriction of the preceding negative quantifier phrase. Consider then:
(32) a. *We need bake no bread today, not even one.
b. *No number of employees would be enough, not even Carl and Louise.

Example (32a) is unacceptable since no bread in (32) in our terms involves a null <amount> noun and hence quantifies over amounts, while not even one quantifies over individuals. Example (32b) is unacceptable since no number of employees quantifies over numbers, but not even Carl and Louise quantifies over sums of individuals.

Strikingly, both $(33 \mathrm{a}, \mathrm{b})$ can have the continuation not even one:
(33) a. The company need hire no new employees, not even one.
b. The company need hire no number of new employees, not even one.

The continuation not even one in (33b) is acceptable, since, in our terms, the antecedent represents quantification over numbers. And on our analysis, exactly the same holds of the overall DP no new employees in (33a). It also contains a negative DP whose NP is NUM, which denotes numbers.

There are further parallels between (29a,b). Iatridou and Sichel (2011) cite two context types which force split readings. First, Iatridou and Sichel (2011: 606) note that "...de re
readings presuppose the existence of entities satisfying the descriptive content of NegDP..." By choosing predicates which preclude existence presuppositions, the de re readings are blocked, as in (34):
(34) The company need bake no cancer-curing cakes.

Here, a de re reading combines with the future orientation of bake to yield a commitment to the existence at the time of utterance of cancer-curing cakes while claiming incoherently such cakes are to be baked subsequently. But (34) has a coherent reading, its split reading ('It is not the case that they need to bake cancer-curing cakes.'), which involves no cake existence presupposition.

Consider then:
(35) The company need bake no number of cancer-curing cakes.

A de re reading of (35), that is, a reading corresponding to a structure in which both the elements corresponding to $\mathrm{DP}_{1}$ and $\mathrm{DP}_{2}$ of (30) take high scope (with $\mathrm{DP}_{2}$ higher than $\mathrm{DP}_{1}$ ) commits to the existence at the time of utterance of positive integers, which is unproblematic. But $\mathrm{DP}_{1}$ also takes high scope (over need), so that that (35) also incoherently commits to the existence at the time of utterance of cancer-curing cakes only to be produced in the future. But no such commitment holds for its split reading. Both (34) and (35) have unexceptional split readings despite their lack of coherent de re readings. This fact reveals a rather subtle property which covert number cases like (29a), share with overt number cases like (29b).

The second diagnostic advanced by Iatridou and Sichel for split readings involves the existential there construction. Consider:
(36) a. There need be no articles written about that charlatan.
b. No articles about that charlatan are such that it is necessary that they be written.

There is no scope ambiguity in (36a) for the existential corresponding to articles because the there construction requires the existential to be low (below need). Thus a de re meaning (36b) of (36a) is not possible and the only sensible interpretation of (36a) is 'it is not necessary that articles about that charlatan be written', which is of course a split reading.

Strikingly, the same properties are seen in cases where overt number is present:
a. There need be no number of articles written about that charlatan.
b. 'No number of articles about that charlatan are such that it is necessary that they be written'.

The de re reading (37b) is as impossible as (36b). That is, the only coherent reading of (37a) is a split reading.

The fact that existential cases of split readings involving covert number are governed by the same relatively subtle conditions as those of cases with overt number provides a good initial support of the idea that on the split reading, no employees is a pseudo-partitive with covert number.

Moreover, further parallels between no employees and no number of employees can be cited, particularly involving selectional properties. Consider (38):
(38) No employees can be interrogated.

This has a split reading 'it is not possible for some employees to be interrogated'. It does not of course have a split reading 'it is not possible for some number which gives the cardinality of employees to be interrogated', which would involve a selectional violation. But the same facts hold for:
(39) No number of employees can be interrogated.

The possible and impossible split readings of (39) are just those of (38). That is, (39) can also not mean 'it is not possible for some number which gives the cardinality of employees to be interrogated'.

We can account for these facts in terms of the structure in (30). In (30), [DP2 [d NEG SOME] [NP2 NUM/number]] is a modifier of the $\mathrm{NP}_{1}$ employees. Another way to put it is that $\mathrm{NP}_{1}$ is the highest NP in $\mathrm{DP}_{1}$. We propose that the selectional properties of a subject DP are determined by the highest NP of that DP.

Second, consider verb agreement in grammatical number in cases where a DP like no number of employees is subject of a finite clause:
(40) a. No employees are/*is being considered for a promotion.
b. No number of employees are/*is being considered for a promotion.

From this point of view, the fact that our treatment of superficial DPs like no employees on split readings involves a structure like (30) does not create a special problem about verb agreement, given that parallel cases with the overt noun number have the same property. Both facts would follow from a principle that a finite verb agrees in number with the highest NP of the DP forming its subject.

Third, DPs like those under study can serve as antecedents for various sorts of anaphoric pronominals. Phrases like that in (1) analyzed as in (30) cannot be the antecedents for singular pronominal anaphora but only for plural pronominals. But exactly the same is true for cases with overt number.

## a. Reflexive Agreement

No (number of) employees are expected to criticize themselves/*is expected to criticize itself.
b. Regular Pronoun Agreement

No (number of) employees believe that they/*it will be promoted.
c. Tag Question Agreement

No (number of) employees will be promoted will they/*it?
Regardless of the overtness of number, the pronominal agreement is driven by the employees NP. A structure like (30) for both variants renders this comprehensible if it is claimed, parallel to the situation with selection restrictions and subject-verb agreement, that the highest NP in a $\mathrm{DP}_{\mathrm{x}}$ antecedent of a pronominal P determines agreement of P with $\mathrm{DP}_{\mathrm{x}}$.

There is further evidence showing similarities between covert number structures and those with overt instances of the noun number. Consider an example of the sort yielding split readings expanded to include an exceptive phrase.
(42) The authorities need analyze no cakes except the iced ones.

We find that this has a clear de re reading but lacks a split reading. This judgment can be supported by modifying the environment to one of those cited above which block de re readings.
(43) They need bake no cakes except the iced ones.

Since the exceptive seems to force a de re reading, while the future-oriented baking context precludes that, one expects (43) to have no coherent reading at all. As far as de re and split readings, we believe this is correct.

However, (43) nonetheless has a good reading, one of the sort not usually discussed in dealing with split readings. Namely, it has a type reading equivalent to:
(44) They need bake no type of cakes except iced ones.

In such a reading, cakes does not denote existing physical cakes, but rather combines, we speculate, with a covert noun type to denote an abstract object and the exceptive refers to cake types as well.

The question then is why the presence of exceptives renders structures like no cakes incapable of forming split readings. The answer under our quantitative analysis of the DPs underlying split readings is seen in (45):
(45) *They need analyze no number of cakes except the iced ones.

Under our analysis, whatever constraint determines the ill-formedness of (45) naturally extends to (43). Plausibly, given the widespread view that exceptives in general must associate with endpoint quantifier phrases, the ungrammaticality follows from the fact that in an analysis like (30), the only DP that forms an endpoint quantifier is $\mathrm{DP}_{2}=[\mathrm{DP}$ [d [d NEG] SOME] NUM]. The larger $\mathrm{DP}_{1}$ with NP head employees does not. And the endpoint quantifier condition precludes exceptives associating with such a DP.

## 6. Other Cases

We have sketched a novel approach to split scope cases by positing complex syntactic structures in which one quantifier DP is embedded as a modifier in another. The embedded DP involves an NP denoting a null instance of number in countable cases and a null instance of amount in uncountable ones. However, our exposition focused on only one type of quantificational case, that involving antiadditive existential forms like no employees/strudel.

But it is claimed in de Swart (2000: 110) and de Clercq (2011: 3-4) that split readings are found with all decreasing operators. A particular challenge for the present approach are cases where the relevant forms exhibit no overt instance of NEG. de Swart (2000:126-17;) cites cases
with few and at most. de Clercq (2011: 4) cites cases with zero and what we take to be a British determiner usage of the expression fuck all (unknown to our knowledge in the United States). We suspect that in any event it just means 'zero' so that the issues it raises for split scope do not differ from that of zero. The relevant examples are given below:
(46) He could buy fuck all/ zero/ no books
a. = No books are such that it was possible for him to buy them. (de re)
b. = It was not possible for him to buy any books. (split scope)
c. $\neq$ It was possible for him not to buy any books.

To deal with such cases, which arguably all involve existential quantification, internal to our general approach would require choosing and justifying the presence of some decreasing quantificational DP analogous to the [dp [d NEG [d SOME]] NUM] we have posited for the antiadditive existential cases like no employees/strudel. Actual working out of such analyses is, however, beyond the scope of the present study.

A different challenge involving universal rather than existential quantification is provided by an example from Sternfeld (2006: 333) cited in Penka (2011: 6), who comments about her example (14a) (Not many people attended the meeting.) as in (47):
(47) "The assumption that cases like (14a) involve constituent negation does not only introduce a distinction that is not supported by intuitions but faces problems too. One problem is that the assumption that the negative marker and the quantifier form a constituent does not always yield correct truth conditions. Consider the following example (from Sternfeld, 2006: 333)
(15) Not every boy can be above average height. $\neg>$ can $>$ every
'It is not possible that every boy is above average height.'

In the reading (15) intuitively has, negation takes scope above the modal verb can, but the quantifier every is interpreted in the scope of the modal..."

In this case, the split reading is the only one with any plausibility, since the de re reading is just false. As with the examples in (46), we have nothing to add about these cases here.

## 5. Sketch of Compositional Semantics

In this section, we articulate our analysis of quantifier phrases that allow split scope such as no employees. As we noted in the previous section, such quantifier phrases have many properties that are parallel to those of pseudo-partitives such as no number of employees/no amount of strudel. This parallelism supports the key point of our analysis that split scope involves quantification over numbers/amounts. Articulating the analysis that we gave in (30), we propose the following structure:
(48) Decomposition of Negative Existentials (Third and Final Version)


SOME
In this structure, NEG maps to the formative no, NUM is null, SOME is null and so the phonological realization of $\mathrm{DP}_{1}$ is no employees. QnP , Quantity Phrase, provides the proper semantic relations between number phrases and those based on the NP employees. The SOME
daughter of $\mathrm{DP}_{1}$ is needed to make $\mathrm{DP}_{1}$ an existential quantifier phrase. We note that KobuchiPhilip (2007) in particular provides an analysis of e.g. three birds landed in the pumpkin field which posits a null existential quantifier.

We have not dealt with the syntactic position of the preposition of when there is an overt noun number. One possibility is that of heads the QnP , so that it would appear directly under Qn in the diagram in (48). But there may be other possibilities as well. Any analysis of of in (48) will have to deal with the distribution of of in closely related constructions, such as those in (49). We cannot pursue these issues here.
(49) a. $\quad$ seven (*of) men
b. a hundred (*of) men
c. hundreds *(of) men
d. hundreds (of) the men
e. many (*of) woman
f. beaucoup *(de) femmes (French).

We turn to the semantic values of the relevant components of our analysis.
(50) a. $\quad \mathrm{SOME} \rrbracket \quad=\quad \lambda \mathrm{P} . \lambda \mathrm{Q} \cdot \exists \mathrm{x}[\mathrm{P}(\mathrm{x}) \wedge \mathrm{Q}(\mathrm{x})]$
b. $\llbracket \mathrm{NEG} \rrbracket \quad=\quad \lambda \mathrm{X} . \lambda \mathrm{P} . \lambda \mathrm{Q} . \neg \mathrm{X}(\mathrm{P})(\mathrm{Q})$
c. $\llbracket \mathrm{NUM} \rrbracket \quad=\quad \lambda \mathrm{x}$.positive-integer $(\mathrm{n})(=1,2,3 \ldots)$
d. $\llbracket \mathrm{Qn} \rrbracket \quad=\quad \lambda \mathrm{P} \cdot \lambda \mathrm{n} . \lambda \mathrm{x}[\mathrm{P}(\mathrm{x}) \wedge|\mathrm{x}| \geq \mathrm{n}]$

SOME in (50a) has the standard value for existential quantifiers. NEG modifies SOME. NUM is the set of positive integers. Qn is what relates the NP to a particular numeral. Basically, Qn takes the denotation of NP (a set of sums) and limits it to the set of sums of cardinality n or greater.

Given these semantic values, consider the following logical form:
(51) a. No students were there.
b. $\quad[[\mathrm{NEG} \text { SOME }] \text { NUM }]_{1}\left[\mathrm{SOME} \mathrm{DP}_{1} \text { Qn students }\right]_{2}\left[\mathrm{DP}_{2}\right.$ were there $]$

Here is the calculation of the truth conditions:
a. 【NEG SOME』 $\quad=\quad \lambda \mathrm{P} . \lambda \mathrm{Q} . \neg \exists \mathrm{x}[\mathrm{P}(\mathrm{x}) \wedge \mathrm{Q}(\mathrm{x})]$
b. $\llbracket[N E G S O M E] N U M \rrbracket \quad=\quad \lambda Q . \neg \exists x[\operatorname{pos}-i \operatorname{int}(x) \wedge Q(x)]$
c. $\llbracket \mathrm{DP}_{1}[\mathrm{Qn}$ students $] \rrbracket \quad=\quad \lambda x\left[\right.$ students $\left.(\mathrm{x}) \wedge|\mathrm{x}| \geq \llbracket \mathrm{DP}_{1} \rrbracket\right]$
d. $\quad$ SOME $\left[\mathrm{DP}_{1} \mathrm{Qn}\right.$ students $] \rrbracket=\quad \lambda \mathrm{Q} \cdot \exists \mathrm{x}\left[\mathrm{Q}(\mathrm{x}) \wedge\right.$ students $\left.(\mathrm{x}) \wedge|\mathrm{x}| \geq \llbracket \mathrm{DP}_{1} \rrbracket\right]$
e. $\quad \llbracket(51 b) \rrbracket=\quad \neg \exists x[\operatorname{pos}-\operatorname{int}(x) \wedge \exists y[w e r e-t h e r e(y) \wedge \operatorname{students}(y) \wedge|y| \geq x]$

To paraphrase in English: There is no positive integer n such that n or more many students were there. But this is equivalent to:
$\neg \exists y[$ students $(\mathrm{y}) \wedge$ were-there $(\mathrm{y})]$
Given the decomposition of negative existential quantifier phrases in (48), consider once again the example in (1). On the new analysis, (1) has the following logical form representing the split-scope interpretation:
(54) [[NEG SOME] NUM $]_{1}$ [the company need [SOME [DP ${ }_{1}$ employees $\left.]\right]_{2}$ fire $\left.\mathrm{DP}_{2}\right]$

Calculating the truth conditions based on the semantic values in (52) yields (55) (c stands for 'the company'):
$\neg \exists \mathrm{x}[\operatorname{positive-integer}(\mathrm{x}) \wedge$ need $(\mathrm{c}, \wedge \exists \mathrm{y}[\operatorname{employees}(\mathrm{y}) \wedge$ fire $(\mathrm{c}, \mathrm{y}) \wedge|\mathrm{y}| \geq \mathrm{x}])]$
Translated into English this reads: There is no positive integer n such that the company needs to fire n many employees. Now consider a particular number, the number 1.

$$
\begin{equation*}
\neg \operatorname{need}(\mathrm{c}, \wedge \exists \mathrm{y}[\operatorname{employees}(\mathrm{y}) \wedge \operatorname{fire}(\mathrm{c}, \mathrm{y}) \wedge|\mathrm{y}| \geq 1])] \tag{56}
\end{equation*}
$$

Clearly, (55) entails (56), since (56) is the result of substituting in a particular number. But also (56) entails (55). If (56) is true, then there will be no higher number $n$ such that the company need fire $n$ employees. In other words, if (56) is true, then (57) will also be true:
$\neg$ need $(\mathrm{c}, \wedge \exists \mathrm{y}[\operatorname{employees}(\mathrm{y}) \wedge$ fire $(\mathrm{c}, \mathrm{y}) \wedge|\mathrm{y}| \geq 2])]$
But (56) is equivalent to (58):
$\neg \operatorname{need}(\mathrm{c}, \wedge \exists \mathrm{y}[\operatorname{employees}(\mathrm{y}) \wedge$ fire $(\mathrm{c}, \mathrm{y}))]$
In English, this reads: The company does not need to fire employees. So we have shown that (53) represents the split scope reading of (1).

## 6. Conclusion

We decomposed negative existential quantifier phrases into two components: (a) negative existential quantification over numbers and (b) quantification over sums of individuals. We showed that in sentences with no other quantificational operators, such a decomposition makes no difference to truth conditions. If there is an additional quantification operator (such as the verb need), it can intervene between the two components. We proposed that just such an intervention is the reason for the so-called split scope of negative existential quantifier phrases of the type discussed in Potts (2000).

The next step in the program of research initiated here would be to show that other cases like those sketched in section 4 could be analyzed in similar terms, and furthermore, to show how other approaches to split scope compare to our proposals (see Penka 2012 for a survey).

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