

Factivity, presupposition projection, and the role of discrete knowledge in gradient inference judgments

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Abstract We investigate whether the factive presuppositions associated with some clause-embedding predicates are fundamentally discrete in nature—as classically assumed—or fundamentally gradient—as recently proposed (Tonhauser, Beaver, and Degen 2018). To carry out this investigation, we develop statistical models of presupposition projection that implement these two hypotheses, fit these models to existing inference judgment data aimed at measuring factive presuppositions (Degen and Tonhauser 2021), and compare the models’ fit to the data using standard statistical model comparison metrics. We find that models implementing the hypothesis that presupposition projection is fundamentally discrete fit the data better than models that implement the hypothesis that it is fundamentally gradient. To evaluate the robustness of this finding, we collect three additional datasets: a replication of the original dataset, as well as two datasets that modify the methodology of the original. Across each of these three datasets, we again find that models implementing the discreteness hypothesis fit the data better than models that implement the gradient hypothesis. Based on these results, we argue that classical semantic accounts of factive predicates can remain largely intact.

1 Introduction

Semantic theories aim to characterize the inferences that natural language expressions support and to account for at least a subset of those inferences that are necessary given the meanings of the expressions. Whether a particular inference is *necessary* or not is commonly assessed via native speaker judgments. Judgment data, however, tends to be influenced by a number of non-semantic factors, meaning that testing a theory against such data requires explicitly formulating a link between (some representation of) these factors and the theoretical constructs of interest. Such factors run the gamut: they include high-level factors, such as speakers’ prior beliefs about the likelihood that an inference is true or ambiguities about the expressions involved, as well as low-level factors, such as the strategies speakers use to map their judgments to a data collection instrument (e.g., a slider representing likelihood or certainty) or their skill in producing an accurate target response using such instruments.¹

The need to explicitly formulate linking assumptions is particularly pressing in light of the fact that recent theoretical developments within semantics have relied on fine-grained

¹This point is relevant *both* to judgments collected using informal methods and to judgments collected in formal experiments. Even judgments produced by trained semanticists are subject to the factors discussed here, including the low-level factors. Training merely helps to detect and control for them.

aspects of the distribution of such judgments. One area where these fine-grained aspects have been important in theory development is the domain of presupposition projection (in general) and factivity (in particular). In the domain of factivity, it is becoming increasingly common for researchers to collect judgments from native speakers in formal experiments, often in large quantities, in order to evaluate hypotheses about the semantic properties of expressions that drive inferences, as well as about how these semantic properties relate to the distributional properties of judgment data (Degen and Tonhauser 2021, 2022; Djärv and Bacovcin 2017; Djärv, Zehr, and Schwarz 2018; Jeong 2021; Kane, Gantt, and White 2022; Tonhauser 2016; White 2021; White and Rawlins 2018; White, Rudinger, et al. 2018).

Of particular importance in the experimental literature on factivity has been the observation that, in tasks aimed at measuring a predicate’s factivity, aggregate measures derived from inference judgment tasks show much more gradience than one might initially expect under a classical view of factivity as a discrete lexical category (White and Rawlins 2018). Some authors have gone so far as to claim that this gradience casts doubt on the very notion that there is a discrete category associated with factivity at all (Degen and Tonhauser 2022). Such doubt is consistent with the view that presupposition projection is fundamentally gradient in general (Tonhauser, Beaver, and Degen 2018). This *fundamental gradience hypothesis* contrasts with a *fundamental discreteness hypothesis*, which instead aims to retain the classical view of factivity as a discrete category by attributing a significant portion of the observed gradience to the sorts of non-semantic factors discussed above. We discuss these hypotheses and the data on which they are based in more detail in Section 2.

Our central aim in this paper is to quantitatively evaluate these two hypotheses by explicitly formulating the link between their respective construals of factivity and the way humans produce judgments that depend on these construals. To provide a foundation for our evaluation, we first describe a general framework in Section 3 which allows one to transparently relate the sorts of formal compositional analyses of expressions’ semantics that are common in the formal semantics literature to probabilistic models characterizing distributions over inference judgments. This framework, which builds on one proposed by Grove and Bernardy (2023), allows us to precisely specify the two hypotheses. At the same time, we are able to keep fixed both the formal analysis of the expressions of interest and the way that probability distributions over inference judgments are mapped onto a particular data collection instrument.

These aspects of the framework are important because, as we show in Sections 4 and 5, they allow us to carry out an apples-to-apples comparison of the two hypotheses that not only precisely targets where they make different predictions about the distribution of inference judgments across participants, but furthermore does so using standard statistical model comparison metrics which balance out a model’s fit to some inference judgment data against the model’s complexity. Using such metrics, we find that models that implement the fundamental discreteness hypothesis unambiguously outperform models that implement the fundamental gradience hypothesis across both existing datasets aimed at measuring factivity (Degen and Tonhauser 2021), as well as three new datasets: a replication of the existing dataset, along with two novel datasets. Based on these results, we argue in Section 6 that a classical semantic account of factive predicates can remain largely intact, and we discuss how the framework

we develop in this paper might be understood as providing a common view of classical theories of factivity and theories that attempt to reduce it to an entirely pragmatic process (Simons 2007; Simons, Beaver, et al. 2017; Simons, Tonhauser, et al. 2010).

2 Gradient inference patterns among factive predicates

The advent of large-scale inference judgment datasets—such as MegaVeridicality (White and Rawlins 2018; White, Rudinger, et al. 2018), VerbVeridicality (Ross and Pavlick 2019), and CommitmentBank (De Marneffe, Simons, and Tonhauser 2019)—has enabled fine-grained analyses of inference judgment patterns across the entire clause-embedding lexicon. A property of the distribution of inference judgments that has garnered sustained focus is the substantial gradience manifest in the aggregate judgments of multiple speakers. In the domain of veridicality and factivity, this gradience is noted by White and Rawlins (2018), who observe (p. 228) that “there are not necessarily clear dividing lines between... classes [expected in a standard classification of clause-embedding predicates]... suggesting that speakers’ inferences about veridicality are generally quite gradient and likely influenced by the fine-grained semantics of particular verbs.”²

In later work building on White and Rawlins’s observation, Degen and Tonhauser (2022) investigate the nature of this gradience in six experiments, arguing that its persistence across experiments militates against the hypothesis that there is a coherent class of factive predicates. Our own modeling work will use data collected under the same experimental paradigm that Degen and Tonhauser employ, and so we describe their data and arguments in detail in Section 2.1.

In Section 2.2, we turn to the broad question of which factors are responsible for the gradience in inference judgment tasks; we discuss evidence that, when one appropriately accounts for these factors, a small number of clear, inferentially defined classes of predicates are brought into relief (Kane, Gantt, and White 2022), thus casting doubt on Degen and Tonhauser’s argument that there is no coherent class of factive predicates.

Nonetheless, as we discuss in Section 2.3, there is apparent gradience internal to each of these classes, as well as among them, which may be compatible with the program, laid out by Tonhauser, Beaver, and Degen (2018), of viewing all presupposition projection as fundamentally gradient in nature. It is this latter hypothesis that we address in this paper.

2.1 Measuring veridicality and factivity

In each of their experiments, Degen and Tonhauser (2022) focus on the set of twenty clause-embedding predicates listed in (1).

- (1) Twenty clause-embedding predicates (Degen and Tonhauser 2022, p. 559, ex. 13)

²This gradience is not White and Rawlins’s main focus, since they are interested in the relationship between inference and predicate distribution, rather than the semantic classification of predicates. They thus make no particular claims about its importance.

- a. canonically factive: *be annoyed, discover, know, reveal, see*
- b. non-factive
 - (i) non-veridical non-factive: *pretend, say, suggest, think*
 - (ii) veridical non-factive: *be right, demonstrate*
- c. optionally factive: *acknowledge, admit, announce, confess, confirm, establish, hear, inform, prove*

Degen and Tonhauser draw these classifications from previous literature, grouping predicates according to the categories they are typically taken to fall into (see Abrusán 2011, 2016; Abusch 2002, 2010; Anand and Hacquard 2014; Givón 1973; Hooper 1975; Hooper and Thompson 1973; Karttunen 1971; Kiparsky and Kiparsky 1970, i.a.).

In their discussion of the relationship between the traditional classification of these predicates and the experimental data involving projective inferences which they go on to collect, they say, "... we expect to see a categorical difference in projection between canonically factive predicates on the one hand, and optionally factive and nonfactive predicates on the other" (p. 569). To assess projection, Degen and Tonhauser provide participants with a scenario in which someone asks a polar question whose main verb is one of the factive predicates of interest, e.g., (2).

(2) **Helen asks:** Did Amanda discover that Danny ate the last cupcake?

They then ask participants to provide a rating on a continuous scale from *no* to *yes* in answer to a prompt of the form in (3), in order to assess the extent to which participants believe that the embedded clause is presupposed.

(3) Is Helen certain that Danny ate the last cupcake?

In another experiment, Degen and Tonhauser give participants a variant of this task in which their answer is provided as a binary forced choice between *no* and *yes*.

Degen and Tonhauser also claim that categories of predicates ought to emerge when analyzing judgments of veridicality: "we expect the [contents of the complements of] canonically factive and veridical nonfactive predicates to be entailed" (p. 569). They assess veridicality inferences using two methods. First, they provide participants with a scenario in which a sentence containing one of the predicates of interest is assumed to be true, as in (4).

(4) **What is true:** Edward proved that Grace visited her sister.

They then prompt participants using a question of the form in (5).

(5) Does it follow that Grace visited her sister?

Depending on the experiment, either participants answer on a sliding scale from *no* to *yes*, or they are asked to make a binary forced choice between *no* and *yes*.

Second, Degen and Tonhauser provide participants with a scenario in which someone makes an utterance which should be contradictory if the relevant complement clause is entailed, as in (6).

(6) **Margeret:** “Edward heard that Mary is pregnant, but she isn’t.”

Participants are then prompted to answer a question of the form in (7) either on a sliding scale or by making a binary forced choice, depending on the experiment.

(7) “Is Margaret’s utterance contradictory?”

Degen and Tonhauser’s main finding across the six experiments is that the patterns of inference across predicates are gradient in nature for both projection and veridicality, consistent with White and Rawlins’s original observation. The degree to which predicates display projective inferences appears to evolve continuously from the least projective predicate (*pretend*) to the most projective (*be annoyed*) when predicates are compared in terms of their mean ratings. Such gradience is manifest in both of the experiments assessing projection—the one which collects sliding scale judgments and the one which collects binary judgments. A similar pattern emerges in the experiments assessing veridicality inferences. Crucially, no predicate patterns with the entailing control items consistently across all four of their experiments assessing veridicality.

2.2 Gradience in inference datasets

Degen and Tonhauser’s results are consistent, not only with White and Rawlins’s original observation, but with findings from adjacent domains. An and White (2020) observe similar gradience in neg-raising inferences captured in their MegaNegRaising dataset; and Kane, Gantt, and White (2022) note an analogous pattern among belief and desire inferences captured in their MegaIntensionality dataset.

Kane, Gantt, and White note that, in the face of such gradience, it is reasonable to entertain two kinds of hypotheses. One possibility is that “apparent gradience indicates that no formally represented lexical property controls whether a particular inference is triggered” (p. 572). Another is that “apparent gradience [may be] partly or wholly a product of the methods often used to collect inference judgments, and that there are discrete, formally represented lexical properties that are [nevertheless] active in triggering... inferences” (p. 572). To pursue this question, they ask whether clear *patterns* of inference emerge across the inference judgment datasets discussed above—MegaVeridicality, MegaNegRaising, and MegaIntensionality—by clustering predicates into classes according to the responses from those datasets so as to optimize their ability to predict predicates’ syntactic distributions, as measured in the MegaAcceptability dataset (White and Rawlins 2016).³ They uncover fifteen classes of predicates that correspond extremely closely to those that one would expect from prior work on clause-embedding predicates. These classes include a variety of factive subclasses that differ from each other principally in the pattern of belief and desire inferences they support, though not the veridicality inferences they trigger.

Indeed, Kane, Gantt, and White’s findings establish that there *is* a coherent class of factive predicates (which are, in turn, subclassed by the belief and desire inferences which they give

³The idea behind optimizing the predictability of predicates’ syntactic distribution is that, insofar as the classes to which a predicate belongs are predictive of its syntactic distribution, there is preliminary evidence that that class is associated with some distributionally active lexical representation.

rise to). But they also find that there are a variety of classes associated with weaker veridicality inferences than one might expect from a truly factive class. These classes tend to include the semifactives, as one might expect. Thus while it is not correct to say that there is no class of factive predicates, as Degen and Tonhauser argue (Section 4.1, Objection 3), one must still explain the source of the apparent gradience among classes. Inter-class gradience of this kind is unlikely to be—as Kane, Gantt, and White put it—“partly or wholly a product of the methods often used to collect inference judgments”, since their analysis expressly accounts for the relevant task effects.

One way to account for this gradience is to adapt Tonhauser, Beaver, and Degen’s hypothesis that projection is fundamentally gradient to an explanation of class-level gradience; for example, by admitting predicate classes that may be associated with particular amounts of gradience in the degree to which a predicate’s complement projects. Alternatively, one might take seriously a hypothesis which Degen and Tonhauser discuss—namely, that “the observed gradience in projection [is] compatible with a binary factivity category in combination with two assumptions: first, that predicates may be ambiguous between a factive lexical entry... and a nonfactive lexical entry... and, second, that interpreters may be uncertain about which lexical entry a speaker intended in their utterance” (p. 583). Our task in this paper is to formalize these two possibilities so that we may quantitatively compare them.

2.3 The role of world knowledge in gradient inference judgments

Our comparison will rely crucially on a paradigm used by Degen and Tonhauser (2021), who aim to characterize the influence of world knowledge on projection inferences, focusing on the same twenty clause-embedding predicates in (1). Similar to the experiment reported above, in which Degen and Tonhauser (2022) measure presupposition projection out of the complement of a predicate placed inside of a polar question, Degen and Tonhauser (2021) measure such projective inferences in the presence a background fact whose content they manipulate. The following experimental trial (from their experiment 2b), for example, features the predicate *pretend*:

Fact (which Elizabeth knows): Zoe is a math major.

Elizabeth asks: “Did Tim pretend that Zoe calculated the tip?”

Is Elizabeth certain that Zoe calculated the tip?

no yes

Next

The same twenty complement clauses as from Degen and Tonhauser 2022 are also featured

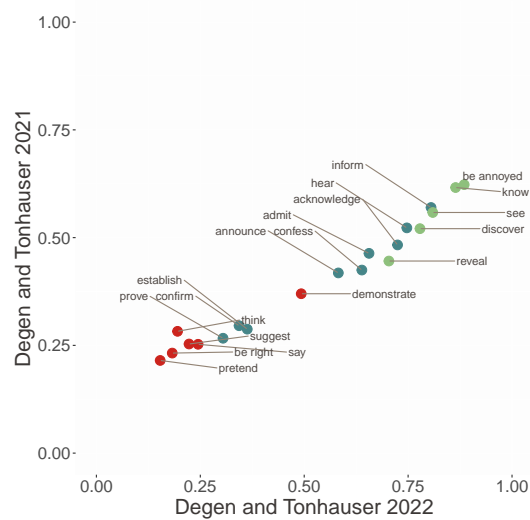


Figure 1: Verb means from Degen and Tonhauser’s (2022) experiment 1a, in which there was no background fact, versus those from their 2021 experiment 2b (Spearman’s $r = 0.98^{***}$). “Non-factive” verbs are in red, “optionally factive” verbs are in teal, and “canonically factive” verbs are in green.

in this experiment, but now each clause is paired with one of two facts: either a fact intended to make the clause likely to be true (as in the example above), or a fact intended to make the clause unlikely to be true. Each participant in this experiment sees twenty items (along with six control items). On each experimental trial, a predicate is placed in the context of one of the twenty clauses, along with one of the two background facts constructed for that clause. The results Degen and Tonhauser (2021) obtain in this setting mirror those of Degen and Tonhauser (2022). In particular, the mean projection ratings for the twenty predicates show a similar gradient pattern, as can be seen in Figure 1.⁴


In addition to the assessment of projective inferences given background facts, Degen and Tonhauser (2021) conduct a norming experiment (their experiment 2a), in which the prior certainties about the truth of the complement clauses featured in their projection experiment are assessed independently, given the same background facts. Trials in this experiment ask participants to judge how likely the relevant clause is to be true, given one of the two background facts constructed for it. For example, the following trial features the same clause as in the example given above, but with the alternate low-probability fact:

⁴Following standard convention, we use three asterisks to indicate that $p < 0.001$ when reporting correlation coefficients.

Fact: Zoe is 5 years old.

How likely is it that Zoe calculated the tip?

impossible definitely



Degen and Tonhauser find that the by-item means for the forty pairs of complement clauses and background facts, as assessed in their norming experiment, are a good linear predictor of the inference ratings for items featuring the same complement clauses and facts which they obtain in their experiment investigating projection inferences.⁵ Thus at least one source of variation among the projective inferences associated with clause-embedding predicates is the *context* in which these predicates are placed; in particular, the prior certainties that people associate with these contexts. This, of course, cannot be the whole story, as Degen and Tonhauser observe: the mean projection ratings for predicates display substantial gradience even after collapsing across the contexts in which they occur, as Figure 1 shows. So, what explains the remaining variation?

2.4 Two accounts of gradience

We consider two hypotheses about the source of variation in projective inference judgments among clause-embedding predicates:

- (8) a. Hypothesis 1. There are at least two major grammatical classes of clause-embedding predicates: a class of predicates which trigger discrete projective inferences at least some of the time, and a class of predicates which do not trigger such projective inferences. Among the predicates triggering projective inferences at least some of the time, there may be further grammatically important subdivisions, such as those observed by Kane, Gantt, and White, that condition the frequency with which a predicate's complement projects.
- b. Hypothesis 2. There are no grammatical classes distinguishing potentially factive from non-factive clause-embedding predicates. Rather, the gradient distinctions among predicates (and classes thereof) reflect the different gradient contributions specific predicates make to the inferences about the truth of their complement clauses.

According to Hypothesis 1, the gradience Degen and Tonhauser observe among the predicates they investigate is metalinguistic in nature. Individual occasions on which a predicate is used may be associated with uncertainty about whether or not the expression containing the predicate triggers projection, but that uncertainty is about which of the alternative interpretations of the expression should be selected. Alternative interpretations may be available because the predicate has multiple senses—at least one that is implicated in triggering projection and at least one that is not; or because the predicate may occur in multiple structures—at

⁵They find a similar effect when the type of prior fact, whether “low” or “high”, is coded as a categorical variable.

least one that is implicated in triggering projection and at least one that is not.⁶ The gradient associated with particular classes of predicates that Kane, Gantt, and White observe might then indicate a sort of regular polysemy among predicates within a class.⁷

According to Hypothesis 2, the variation in projective inferences among clause-embedding predicates is gradient because the inferences that the predicates trigger are themselves gradient (Tonhauser, Beaver, and Degen 2018). In this respect, such inferences may be on a par with those contributed by prior world knowledge: the use of a given predicate boosts the likelihood that its complement clause is true, but this boost is not conditioned by a discrete, formal aspect of the predicate's semantic representation that produces a presupposition or an entailment. Crucially, there is no selection among alternative interpretations on particular occasions of use, under this hypothesis. One way to think about it is that it analogizes clause-embedding predicates, like *know*, to vague predicates, like *tall*.

The difference between these hypotheses cuts to the core of what factivity is, how it is grammatically encoded, and whether or not there is a useful notion of grammatical *class* of predicate that underlies its presence. If the gradient observed among the inferences triggered by clause-embedding predicates is metalinguistic in nature, then the fundamental differences among predicates have to do with the *frequency* with which they receive a factive interpretation versus a non-factive one. Gradient differences among the frequencies with which predicates are intended and understood as factive are fully compatible with a semantic category of factivity, along with a basic distinction between predicates which may be understood with a factive interpretation and those which may not.

To draw a comparison to a different domain, if the differences in the frequencies with which different quantifiers take exceptional scope out of finite clauses were found to be gradient, this would not compromise the status of exceptional scope as a semantically important category; nor would it hinder an investigation of the semantic properties of expressions that allow them to take (or prevent them from taking) exceptional scope in the first place. Similarly, if clause-embedding predicates display a gradient pattern of differences in the frequencies with which they trigger factive inferences, factivity as a semantic category is not thereby comprised.

3 Probabilistic semantics

To state a theory of gradient precisely, it will be useful to have a general method for integrating probabilistic reasoning into a compositional semantics. Here, we rely on the broad

⁶The second option is a live possibility in light of a substantial amount of cross-linguistic evidence that functional items surrounding a predicate—e.g., nominal morphology attached to verbs or their clausal complements—can modulate veridicality inferences (see Abrusán 2011; Farudi 2007; Giannakidou 1998, 1999, 2009; Kastner 2015; Ozyildiz 2017; Roussou 2010; Varlokosta 1994; see White 2019 for discussion).

⁷Alternatively, one could posit that there is no indeterminacy in the interpretations for a string and, rather, that there is uncertainty over possible questions under discussion (QUD) against which the string could be interpreted, which would be compatible with, e.g., the proposal in Simons, Beaver, et al. 2017. While this possibility is live, it is not clear how to reconcile it with the observation that *classes of* predicates are associated with particular levels of gradient without saying that lexical semantic knowledge somehow conditions QUD choice. This move would violate the spirit of such *conversationalists* proposals, which generally attempt to do away with heavy conditioning on lexical information (see White 2019 for discussion).

framework provided by Grove and Bernardy (2023), which supplies an interface for performing Bayesian reasoning in the simply typed λ -calculus (with products) using *monads* (see Bergen, Levy, and Goodman 2016; Monroe 2018; Potts et al. 2016 for a collection of similar approaches, as well as the monadic approaches of Asudeh and Giorgolo 2020; Giorgolo and Asudeh 2014 and Bernardy, Blanck, Chatzikyriakidis, and Lappin 2018, which have different aims from Grove and Bernardy, which are reflected in the distinct interfaces they supply). The main upshot of this framework is that it allows one to transparently relate the sorts of compositional analyses of expressions’ semantics common in the formal semantics literature to probabilistic models characterizing distributions over inference judgments.

As we will show, the framework allows for a precise specification of the two hypotheses laid out above, while keeping fixed both the formal analysis of the expressions of interest and the way in which probability distributions over inference judgments are mapped onto a particular data collection instrument. These aspects of the framework are important because, as we show in Sections 4 and 5, they allow us to conduct an apples-to-apples comparison of the two hypotheses that not only precisely targets where they make different predictions about the distribution of inference judgments across participants, but furthermore does so using standard statistical model comparison metrics which balance out a model’s fit to inference judgment data against the model’s complexity.

We begin in Section 3.1 with necessary formal background on Grove and Bernardy’s framework before turning in Section 3.2 to our extension of their framework, which allows us to finely delineate uncertainty that is core to the semantic value of an expression—giving rise to phenomena such as vagueness—from uncertainty about which interpretation should be associated with a particular string—giving rise to metalinguistic uncertainty. To highlight the distinction between these two forms of uncertainty, we first walk through an analysis of gradable adjectives in the Grove and Bernardy setting, since it allows us to highlight how their framework approaches vague predicates; we then give a minimalistic analysis of factivity in Section 3.3.

3.1 Denotations as probabilistic programs

The main idea behind Grove and Bernardy’s framework is that we can map any type α to a type $P\alpha$ of *probabilistic programs* computing values of type α associated with *probabilistic effects*. The framework thereby provides a means of assimilating the *probabilistic* component of a probabilistic semantics to other notions of *effect* that have been studied in the formal semantics literature using monads—e.g., Shan’s (2002) first introduction of monads into the formal semantics literature, with illustrations from focus, question semantics, anaphora, and quantification; Unger’s (2012) and Charlow’s (2014) approaches to anaphora using the State monad (and, in the latter case, the State transformer of Liang, Hudak, and Jones 1995); and various other phenomena, including conventional implicature (Giorgolo and Asudeh 2012), intensionality (Charlow 2020; Elliott 2022), and presupposition (Grove 2022).

To take an example familiar from probabilistic semantics settings, consider the meaning of the gradable adjective *tall*. Modeling only its role as a descriptor of individuals, one might regard *tall* as a predicate of type $e \rightarrow t$. To capture the contribution of *tall* to the entailments of expressions that contain it, one might then model its denotation as contributing the

entailment that the height of the predicated individual x is greater than some contextually determined threshold d . Doing this, however, might lead to a semantic representation like the following, which involves an unbound degree variable d :

$$\lambda x.\text{height}(x) \geq d$$

There are different ways of remedying this situation. One approach assumes that the degree variable is existentially quantified—e.g., in virtue of the presence of an unpronounced morpheme which binds it—and that its value is constrained by some property made available by the context (see, e.g., Kennedy and McNally 2005). Another—and the one we build on here—leaves the variable unbound and relies on the context to directly fix its value (see, e.g., Barker 2002; Kennedy 2007).⁸ Among approaches that implement the second possibility, many rely on probabilistic knowledge to constrain how the value is fixed (Bernardy, Blanck, Chatzikyriakidis, and Lappin 2018; Bernardy, Blanck, Chatzikyriakidis, Lappin, and Maskharashvili 2019a,b; Bernardy, Blanck, Chatzikyriakidis, and Maskharashvili 2022; Goodman and Lassiter 2015; Lassiter 2011; Lassiter and Goodman 2017, i.a.).

Grove and Bernardy’s framework is one such probabilistic implementation, which uses a monad (P) to constrain the interpretation of the degree variable without tampering with the underlying compositional semantics. P maps types, such as e , t , $e \rightarrow t$, $e \times t$, etc., to types Pe , Pt , $P(e \rightarrow t)$, $P(e \times t)$, etc., which are inhabited by probabilistic programs. Because it is a monad, P comes with two monadic operators: $(\sim) : P\alpha \rightarrow (\alpha \rightarrow P\beta) \rightarrow P\beta$ (*‘bind’*) and $(\boxed{\cdot}) : \alpha \rightarrow P\alpha$ (*‘return’*), which we describe in turn.

3.1.1 The ‘bind’ operator

The bind operator can be used to characterize the interpretation of contextually regulated parameters, like d above, by sequencing one probabilistic program with another that depends on a variable. This sequencing—notated $m \sim \lambda x.k(x)$ —can be understood as sampling a value $x : \alpha$ from a probabilistic program $m : P\alpha$, and then using that value to construct the new probabilistic program $k(x) : P\beta$ (which is now parameterized by x). Following standard convention, $m \sim \lambda x.k(x)$ can be written in the following “imperative style”:

$$\begin{array}{l} x \sim m \\ k(x) \end{array}$$

We use this notation throughout the remainder of the paper. It is important to note that these two lines together describe the probabilistic program $m \sim \lambda x.k(x)$ and that similar multi-line descriptions below will also describe a single complex probabilistic program.

3.1.2 The ‘return’ operator

The return operator allows ordinary logical meanings to be *lifted* to probabilistic programs associated with a *trivial effect*.

$$\boxed{\cdot} : \alpha \rightarrow P\alpha$$

⁸Our use of the term ‘variable’ here is a bit metaphorical: we mean to include any approach that values the standard of the relevant gradable adjective through contextual means.

$$\begin{array}{ccc}
\textit{Left identity} & \textit{Right identity} & \textit{Associativity} \\
x \sim \boxed{v} = k(v) & x \sim m = \boxed{x} & y \sim \left(\begin{array}{c} x \sim m \\ n(x) \end{array} \right) = \begin{array}{c} x \sim m \\ y \sim n(x) \\ o(y) \end{array}
\end{array}$$

Figure 2: The monad laws

The effect associated with the resulting program is trivial in the sense that it always returns the same thing. (Indeed, as we will discuss shortly, this behavior is part-and-parcel of what it means to be a monad.) For instance, sampling from $\boxed{\llbracket \mathcal{J}o \rrbracket} : Pe$ will always result in $\llbracket \mathcal{J}o \rrbracket : e$. In the parlance of probability theory, such programs describe *degenerate distributions*.

3.1.3 The semantic value of *tall* as a probabilistic program

To model gradable adjectives like *tall*, Grove and Bernardy assume that $\llbracket \textit{tall} \rrbracket$ is a probabilistic program of type $P(e \rightarrow t)$. Their analysis uses the two monadic operators described above to model the interpretation of such adjectives in terms of probabilistic like the following one:

$$d \sim \textit{thresholdPrior} \\
\boxed{\lambda x. \textit{height}(x) \geq d}$$

This program first samples a random degree value $d : r$, where r is the type of real numbers, from *thresholdPrior*—a program of type Pr —and then uses it inside the program $\boxed{\lambda x. \textit{height}(x) \geq d}$ of type $P(e \rightarrow t)$, thus providing a function of type $e \rightarrow t$ which depends on a probability distribution over degrees of height.

Importantly, *thresholdPrior* can be anything, as long as it is of the right type (Pr). Its main function is to represent the constraints that the context—including comprehenders’ prior beliefs—imposes on d . For instance, one could assume that d is normally distributed with some mean μ and standard deviation σ , in which case the meaning of *tall* would be:

$$d \sim \mathcal{N}(\mu, \sigma) \\
\boxed{\lambda x. \textit{height}(x) \geq d}$$

Under this assumption, the height threshold is sampled from—that is, *bound by*—the program $\mathcal{N}(\mu, \sigma) : Pr$ that computes a normal distribution.

3.1.4 Why it matters that P is a monad

Because P , together with $\boxed{\cdot}$ and (\sim) , is assumed to be a monad, it must satisfy the laws in Figure 2. Among these laws, Left identity guarantees that transforming a value v via $\boxed{\cdot}$ creates a “pure” probabilistic program that just returns v ; that is, v is the only value which may be sampled. Right identity guarantees that returning a value randomly sampled from m is just the same as computing a value from m . Associativity provides a syntactic convenience by allowing probabilistic programs to be re-bracketed: if one samples y from a complex probabilistic program that contains a use of (\sim) , one may also pull out the parts composing

the program and, instead sample y from the last one. Together, these guarantees ensure that \mathbb{P} never tampers with the underlying compositional semantics (see Charlow 2014, 2020 for extensive discussion).

3.1.5 Extracting probabilities from probabilistic programs

Because Grove and Bernardy are interested in modeling how participants form and report judgments about sentences containing gradable adjectives (*qua* vague predicates), they require not only a way of describing how sampling from a particular program works, but also a way of computing the probability of a particular value—for example, the probability that a sentence containing the gradable adjective is true. Thus they require a method of going from programs m of type $\mathbb{P}\alpha$ to values of type r (real values). To satisfy this requirement, they (implicitly) use an *expected value* operator:

$$\mathbb{E}_{(\cdot)} : \mathbb{P}\alpha \rightarrow (\alpha \rightarrow r) \rightarrow r$$

Given a function f from values of type α to real numbers, $\mathbb{E}_{x \sim m} [f(x)]$ is the expected value of f , given the probability distribution over values of type α represented by m . If m returns truth values—i.e., if it is of type $\mathbb{P}t$ —it can be associated with a probability by taking the expected value of the indicator function $\mathbb{1} : t \rightarrow r$, which maps \top (‘true’) to 1 and \perp (‘false’) to 0:

$$\begin{aligned} \mathbb{P} &: \mathbb{P}t \rightarrow r \\ \mathbb{P}(m) &= \mathbb{E}_{\tau \sim m} [\mathbb{1}(\tau)] \end{aligned}$$

For illustration, suppose we want to find the probability that the sentence *Jo is tall* is true. Taking the denotation of this sentence to be

$$\begin{aligned} \llbracket \text{Jo is tall} \rrbracket &: \mathbb{P}t \\ \llbracket \text{Jo is tall} \rrbracket &= d \sim \mathcal{N}(\mu, \sigma) \\ &\quad \text{height}(j) \geq d \end{aligned}$$

we use the probability operator \mathbb{P} to compute the probability

$$\begin{aligned} &\mathbb{P} \left(\begin{array}{c} d \sim \mathcal{N}(\mu, \sigma) \\ \text{height}(j) \geq d \end{array} \right) \\ &= \mathbb{E}_{\tau \sim \left(\begin{array}{c} d \sim \mathcal{N}(\mu, \sigma) \\ \text{height}(j) \geq d \end{array} \right)} [\mathbb{1}(\tau)] \\ &= \mathbb{E}_{d \sim \mathcal{N}(\mu, \sigma)} [\mathbb{1}(\text{height}(j) \geq d)] \end{aligned}$$

Thus the probability that Jo is tall is the expected value of $\mathbb{1}(\text{height}(j) \geq d)$, where d is normally distributed with mean μ and standard deviation σ . Since this formula will be valued at 1 when Jo’s height exceeds the threshold, and 0 otherwise, the resulting probability should just be the mass of $\mathcal{N}(\mu, \sigma)$ contained below Jo’s height.

3.1.6 Contexts in a probabilistic semantics

To model clause-embedding predicates, we need some way of representing the denotations of declarative clauses, which are standardly taken to be propositions. Following Grove and Bernardy (2023), we implement this representation by allowing the meanings of expressions to depend on *contexts*. Contexts, in our setting, are finite tuples of parameters that determine the semantic values of expressions. Thus, they are akin to models, possible worlds (see von Stechow and Heim 2021 and references therein), or situations (Barwise and Perry 1983). In addition to providing parameters that determine the denotations of expressions, contexts provide values for contextual parameters—for example, the height threshold relevant to evaluating the meaning of gradable adjectives like *tall*. Taking κ to be the type of contexts (i.e., κ is an n -ary product, for some n), we may use the following notation to provide a new meaning for *tall*:

$$\begin{aligned} \llbracket tall \rrbracket &: e \rightarrow \kappa \rightarrow t \\ \llbracket tall \rrbracket &= \lambda x, c. \text{height}(c)(x) \geq d_{tall}(c) \end{aligned}$$

$\text{height}(c)$ selects whichever component of c maps individuals to their heights, and $d_{tall}(c)$ selects whichever component of c provides the contextual degree threshold relevant to determining the truth of the gradable adjective *tall*. In addition to settling facts about how the world is—e.g., people’s heights—contexts settle matters of vagueness and metalinguistic uncertainty—e.g., how tall one must be in order to be considered tall. They are thus also akin to the “counterstances” of Kennedy and Willer (2016, 2022) or the “outlooks” of Coppock (2018).

Propositions in the current setting can now be conveniently viewed as sets of contexts, or functions of type $\kappa \rightarrow t$. Further, following Grove and Bernardy (2023), the common ground may be viewed as a *distribution* over contexts, or a probabilistic program of type $P\kappa$. To update the common ground with a proposition, we make use of a function *observe*, which is defined, in turn, using a more primitive operation *factor*, whose role is to scale the distribution represented by the probabilistic program which follows it by some scalar value:⁹

$$\begin{aligned} \text{factor} &: r \rightarrow P\diamond \\ \text{observe} &: t \rightarrow P\diamond \\ \text{observe}(\phi) &= \text{factor}(\mathbb{1}(\phi)) \end{aligned}$$

Given a common ground $cg : P\kappa$, one can update it with the proposition $\phi : \kappa \rightarrow t$ by turning

⁹In the continuation-based setting of Grove and Bernardy 2023, *factor* is defined as

$$\text{factor}(x) = \lambda k. x * k(\diamond)$$

so that it scales its continuation by the relevant factor. For current purposes, we maintain a relatively abstract interface so that our main points aren’t obscured by implementation details.

ϕ from a static into a dynamic proposition:

$$\begin{aligned} \text{update} &: (\kappa \rightarrow t) \rightarrow P\kappa \rightarrow P\kappa \\ \text{update}(\phi)(cg) &= c \sim cg \\ &\quad \text{observe}(\phi(c)) \\ &\quad \boxed{\kappa} \end{aligned}$$

Dynamizing propositions is thus a matter of *observing* them in the context provided by the relevant common ground.

To foreshadow our analyses a bit, each of the models we consider in this paper provides a representation of the common ground: at their heart, our models characterize distributions over contexts. The ways in which they differ from one another has to do with how the distributions over certain relevant parameters of a given context are evaluated, and in turn, how these distributions contribute to the predicted behavior of someone who makes an inference.

3.2 Our contribution: two levels of uncertainty

Our main contribution comes in how we model the common ground. Rather than represent the common ground as a probability distribution over contexts—i.e., of type $P\kappa$ —we represent it as a probability distribution *over* probability distributions over contexts. That is, by representing it as a program of type $P(P\kappa)$. By invoking the map P twice, we are effectively providing two layers, or levels, of probabilistic uncertainty.

We use the “inner” P to represent the uncertainty that is manifest on particular occasions of use and interpretation. Such uncertainty may, in principle, be caused by linguistic expressions which are vague or subjective, or it may be uncertainty related to world knowledge. As an umbrella term, we refer to any of these sources of uncertainty as *contextual* uncertainty.

We use the “outer” P to represent *metalinguistic* uncertainty. Although there may, in general, be uncertainty about the values of linguistic parameters that govern the meanings of expressions, by regulating them on the outer layer, we take those values to be fixed on particular occasions of language use and interpretation. Thus one may regard the outer P as providing a distribution over possible *kinds* of occasions of use and interpretation—that is, which fix the values of parameters which are metalinguistically uncertain—while the inner P may be considered to be residual uncertainty that arises on particular occasions of use and interpretation, once the relevant type of occasion is fixed. Such residual uncertainty may relate to linguistic phenomena such as vagueness and subjectivity, as well as non-linguistic world knowledge.

Which phenomena should be tethered to which layer of uncertainty is, importantly, up for debate and should ultimately be settled empirically. Our attempt to study the source of the gradience induced by factive predicates aims to help resolve this question in one of its manifestations. To sharpen the discussion in Section 2.2, we ask whether the uncertainty that gives rise to gradience among judgments of presupposition projection is (a) uncertainty that is settled as the occasion of use is fixed, or (b) an inherent property of particular uses and interpretations, so that presuppositions might project gradiently.

We note two important properties of the layering described above. First, the composition of P with itself has a certain formal license: because P is a monad, it is also a *functor*. This

$$\begin{array}{ll}
 \textit{Identity} & \textit{Composition} \\
 id^{\Downarrow} = id & (f \circ g)^{\Downarrow} = f^{\Downarrow} \circ g^{\Downarrow}
 \end{array}$$

Figure 3: The functor laws

means that it comes with an operation $(\cdot)^{\Downarrow}$ (*map*) allowing one to perform pure operations on the values returned by probabilistic programs, while keeping their probabilistic effects intact. $(\cdot)^{\Downarrow}$ may be defined in terms of the monadic $\boxed{(\cdot)}$ and (\sim) , as follows:

$$\begin{aligned}
 (\cdot)^{\Downarrow} &: (\alpha \rightarrow \beta) \rightarrow P\alpha \rightarrow P\beta \\
 f^{\Downarrow} &= \lambda m. x \sim m \\
 &\quad \boxed{f(x)}
 \end{aligned}$$

The two laws regulating functors are given in Figure 3.¹⁰

Crucially, functors are *composable*, meaning that we can take the composition of the functor P with itself to obtain the new functor $P(P\alpha)$ (whose $(\cdot)^{\Downarrow}$ may be defined simply as $(\cdot)^{\Downarrow\Downarrow}$).¹¹ Old operations are easily recast in the current setting involving structured uncertainty, that is, by *mapping them over* higher-order probabilistic programs. Updates to the common ground, for instance, may be presented as follows:

$$\begin{aligned}
 \text{update}_2 &: (\kappa \rightarrow t) \rightarrow P(P\kappa) \rightarrow P(P\kappa) \\
 \text{update}_2(\phi) &= \text{update}(\phi)^{\Downarrow}
 \end{aligned}$$

The second property of note is that, because $P(P\alpha)$ is obtained as the composition of functors, it provides a tight constraint on the way information may flow from one level to another; the flow is unidirectional, going from the outer level that regulates metalinguistic uncertainty to the inner level that regulates contextual uncertainty. As a result, it is possible for contextual uncertainty to remain even after questions of metalinguistic uncertainty have been settled—e.g., whether a semantically ambiguous expression has one interpretation versus another. But by necessity, settling contextual uncertainty also settles metalinguistic uncertainty.

This asymmetry is motivated by the general behavior of the two sources of uncertainty being modeled. To illustrate this, say someone makes the utterance *Jo is tall* in a noisy environment, rendering it ambiguous between *Jo is tall* and *Jo is small*. Moreover, say that, from the interlocutor’s perspective, the probability that *Jo is tall* was uttered is 0.7 and the probability that *Jo is small* was uttered is 0.3. Then (setting aside our commitment to employing

¹⁰Note that both laws may be proved from the monad laws of Figure 2.

¹¹Indeed, because P is a monad, it is not only a functor, but an *applicative functor* (McBride and Paterson 2008), meaning that it comes with an operation

$$(\otimes) : P(\alpha \rightarrow \beta) \rightarrow P\alpha \rightarrow P\beta$$

called *sequential application*, which can apply an effectful function to an effectful argument, in order to sequence the effects. Applicatives also enjoy composability (so that $P(P\alpha)$ is also applicative), but we suppress this fact in the discussion for now, since applicatives provide a somewhat more powerful interface than we require.

contexts, momentarily), the metalinguistically uncertain $\mathcal{J}o$ is X can be assigned the following interpretation:

$$\begin{aligned} \llbracket \mathcal{J}o \text{ is } X \rrbracket &: P(Pt) \\ \llbracket \mathcal{J}o \text{ is } X \rrbracket &= \tau \sim \text{Bernoulli}(0.7) \\ &\begin{cases} \left(\begin{array}{l} d \sim \mathcal{N}(\mu_t, \sigma_t) \\ \text{height}(j) \geq d \end{array} \right) & \tau \\ \left(\begin{array}{l} d \sim \mathcal{N}(\mu_s, \sigma_s) \\ \text{size}(j) \leq d \end{array} \right) & \neg\tau \end{cases} \end{aligned}$$

According to this interpretation, the meaning of $\mathcal{J}o$ is X depends on the Bernoulli-distributed variable $\tau : t$. If τ is \top (which occurs with a probability of 0.7), then the interpretation is the returned program which encodes the meaning of $\mathcal{J}o$ is *tall*; whereas, if τ is \perp (which occurs with a probability of 0.3), then it is the returned program which encodes the meaning of $\mathcal{J}o$ is *small*. Crucially, once the value of the random variable τ , which represents the metalinguistic uncertainty about what was uttered, is settled, one obtains a meaning having contextual uncertainty, encoded by a normal distribution over degrees of height or size, respectively. Thus the probabilistic effects encoding contextual uncertainty depend on those encoding metalinguistic uncertainty (about the value of τ , in particular). But the former cannot, in turn, influence the latter, simply because they are part of the program which is *returned*; any parameters introduced by such effects are not in scope early enough.

We now turn to an account of factivity within this two-layered probabilistic setting.

3.3 The meaning of factivity

In general, we assume that clause-selecting predicates entail the complement clauses they select with some probability.¹² For example, we may represent the meaning of *know* as follows, where τ_{know} selects from the context c a truth value determining whether to instantiate the meaning of *know* with a factive or a non-factive meaning:

$$\begin{aligned} \llbracket \text{know} \rrbracket &: (\kappa \rightarrow t) \rightarrow e \rightarrow (\kappa \rightarrow t) \\ \llbracket \text{know} \rrbracket &= \lambda\phi, x, c. \begin{cases} \text{know}_f(\phi)(x)(c) & \tau_{\text{know}}(c) \\ \text{know}_{nf}(\phi)(x)(c) & \neg\tau_{\text{know}}(c) \end{cases} \end{aligned}$$

To aid the analysis, we will assume the following postulate, guaranteeing that the factive meaning of *know* entails its complement clause:

$$\forall p, x, c : \text{know}_f(p)(x)(c) \rightarrow p(c)$$

And likewise for all clause-selecting predicates. Those verbs which are always factive will have the meaning verb_f with probability 1, and those verbs which are never factive will have the meaning verb_{nf} with probability 1.

¹²We do not distinguish between factivity and veridicality for current purposes. This approach bears a resemblance to the general approach in Simons 2007 and Simons, Tonhauser, et al. 2010.

Two points should be mentioned. First, the kind of analysis we present here is *ad hoc*, since it provides no explanation of factivity beyond the postulates associated with each predicate’s interpretation. Our current purpose, however, is not to provide an explanation of factivity, but to discover properties of its behavior; that is, whether the gradience it exhibits is a manifestation of contextual uncertainty (supporting the fundamental gradience hypothesis) or metalinguistic uncertainty (supporting the fundamental discreteness hypothesis).

Second, while the entry provided above for *know* may appear to render it semantically ambiguous, we stress that it does not. On our account, ambiguity is a potential cause of metalinguistic uncertainty, but not of contextual uncertainty (ambiguities are resolved in context). Thus whether the above entry for *know* renders it ambiguous versus, say, *vague* is a matter of how the parameter τ_{know} is regulated; that is, whether its distribution is determined by metalinguistic uncertainty or contextual uncertainty.

4 Modeling

To investigate the above theories of factivity and world knowledge, we implemented Bayesian models in the Stan programming language ({Stan Development Team} 2023), via the CmdStanR interface provided by the R programming language (Gabry and Češnovar 2023). Models were fit using Degen and Tonhauser’s (2021) experimental data and then compared in terms of their expected log pointwise predictive densities, as provided by the widely applicable information criterion (Gelman, Hwang, and Vehtari 2014; Watanabe 2013), using R’s loo package (Vehtari et al. 2023).

To construct the four models, we used a *pipelined* approach. That is, we first fit a model of Degen and Tonhauser’s norming data, in order to obtain posterior distributions for parameters associated with the forty pairs of complement clauses and facts, which we refer to as *contexts*. We then used (approximations of) these posterior distributions as prior distributions for the corresponding parameters in our four models of Degen and Tonhauser’s projection experiment data. Here, we describe each of our models of factivity, which were fit to Degen and Tonhauser’s projection experiment data. For further details concerning the modeling pipeline, see Appendix A.

4.1 Linking to response behavior

Before we describe the models, we should make explicit our assumptions about the link between semantic knowledge and response behavior. Specifically, we need a linking hypotheses that relates this knowledge to the inference judgments participants produce on a slider scale between ‘no’ and ‘yes’, which can be modeled as a distribution of values on the unit interval.

Our main theoretical focus is the structure of the probabilistic program that characterizes the common ground. Taking κ to be the type of contexts, this program will invariably be of type $P(P\kappa)$ —it provides a probability distribution over probability distributions over contexts. If we fix the parameters regulated by the “outer” layer of probabilistic uncertainty, we fix, in turn, an “inner” probability distribution over contexts. The parameters regulated by the outer layer resolve metalinguistic uncertainty; for example, semantic ambiguities. Meanwhile, the parameters regulated by the inner layer resolve contextual uncertainty. These parameters

might relate to world knowledge or vagueness, but, crucially, not to parameters which are fixed on specific occasions of making or interpreting an utterance.

Given all of these assumptions, the interesting question is whether uncertainty about the components of the context that fix world knowledge and the presupposition projection behavior of factive verbs is best understood as metalinguistic uncertainty, which may be fixed on individual occasions of utterance production and interpretation, or contextual uncertainty, which remains even after metalinguistic parameters have been fixed. Four possibilities arise, depending on how the sources of uncertainty about world knowledge and presupposition projection are each understood. The remainder of this section presents the models that implement these possibilities. But first, we provide our assumptions about the link between contextual uncertainty and one’s response behavior when making a gradient inference judgment.

To recap, we assume that some amount of uncertainty is responsible for the gradient inferences one draws from an utterance. If one accepts the truth of the sentence *Jo is tall*, one may still be uncertain whether or not *Jo is at least six feet tall* is also true. If asked to evaluate how likely the second sentence is to be true, given that the first sentence is true, one might assess it to be, say, 50% likely.

If the slider responses people provide in an experimental setting reflect this uncertainty, we may link these responses to our models of the common ground by taking them to measure the uncertainty directly, given some likelihood function. To do so, we apply a function *respond*, which, given a distribution over contexts m of type $\text{Pr}\kappa$, along with a possible inference ϕ of type $\kappa \rightarrow t$, associates it with a distribution over slider responses on the unit interval:

$$\begin{aligned} \text{respond}_{(2)}^{(1)} : r \rightarrow \text{Pr}\kappa \rightarrow (\kappa \rightarrow t) \rightarrow \text{Pr} \\ \text{respond}_{c \sim m}^{\sigma}(\phi(c)) = \mathcal{N}(x, \sigma) \upharpoonright [0, 1] \\ \text{where } x = \mathbb{P} \left(\begin{array}{c} c \sim m \\ \boxed{\phi(c)} \end{array} \right) \end{aligned}$$

This implementation of response behavior assumes the possibility of error, encoded in the likelihood; that is, given an intended response x , an experimental participant’s physical response is normally distributed (with standard deviation σ) around x and truncated to $[0, 1]$, the bounds of the slider scale.¹³ x is just the contextual certainty of the relevant inference, given the common ground.

¹³The likelihood assumed here is known as a truncated normal distribution. This assumption is analogous to the one that Degen and Tonhauser (2021) make in using a linear mixed model—though, in using a *truncated* normal, we additionally capture the boundedness of the response scale. An alternative likelihood sometimes used with bounded response scales is a Beta distribution (see, e.g., Degen and Tonhauser 2022). This assumption is not strictly appropriate for bounded response scales that include their endpoints—e.g., a response scale on the closed interval $[0, 1]$ rather than the open interval $(0, 1)$ —because Beta distributions only have support on the open interval—i.e., they exclude $\{0, 1\}$ (see Liu and Eugenio 2018 and references therein). It is particularly problematic in the current context, where endpoint responses are meaningful by hypothesis. Truncated normals do not have this problem because they can have support on the closed interval. Zero-one inflated Beta distributions are another option (again, see Liu and Eugenio 2018), but they bring their own conceptual challenges in the current context because they effectively require the assumption that all models assume some amount of discreteness.

To give a schematic example, let’s say that the common ground of interest is characterized by a probabilistic program `commonGround` of type $P(\text{Pk})$. Then the following program of type Pr characterizes the distribution of slider responses on the unit interval, where a response reflects a judgment of certainty about the truth of the sentence *Grace visited her sister*, given the information *Susan knows that Grace visited her sister*:

$$\begin{aligned}
& m \sim \text{commonGround} \\
& \text{respond}_{c \sim m'}^{\sigma}(\llbracket \text{Grace visited her sister} \rrbracket^c) \\
& \quad \text{where } m' = \text{update}(\llbracket \text{Susan knows that Grace visited her sister} \rrbracket)(m) \\
= & m \sim \text{commonGround} \\
& \mathcal{N}(x, \sigma) \upharpoonright [0, 1] \\
& \quad \text{where } x = \mathbb{P} \left(\begin{array}{l} c \sim m \\ \text{observe}(\llbracket \text{Susan knows that Grace visited her sister} \rrbracket^c) \\ \llbracket \text{Grace visited her sister} \rrbracket^c \end{array} \right)
\end{aligned}$$

This probabilistic program first samples a distribution over contexts m of type Pk , in which the parameters regulating metalinguistic knowledge have been *fixed*; though, in which the parameters regulating contextual uncertainty still remain indeterminate. It then computes a distribution over responses by doing a couple of things inside the scope of the \mathbb{P} operator: it samples a context from m , which it uses to perform Bayesian update—that is, by observing that *Susan knows that Grace visited her sister* is true—before returning \top or \perp , depending on which is the interpretation of *Grace visited her sister*, given that context. The program, therefore, results in a normal distribution centered at the probability that *Grace visited her sister* is true, given m under the update, truncated to the unit interval.

The general set-up described here will remain invariant under the theories considered in the rest of this section. What varies is the structure of `commonGround`, and, crucially, whether the parameters regulating world knowledge and factivity are understood as being governed by metalinguistic uncertainty or contextual uncertainty.

4.2 Models of factivity

We now provide our four models of factivity and prior world knowledge, which we fit to Degen and Tonhauser’s projection experiment data. Our presentation of these models here is abstract, but see Appendix A for the full model specifications.

4.3 Factivity as a fundamentally discrete phenomenon

The first theory we consider is consistent with the traditional view of factivity, implicit in Karttunen (1974) and Kiparsky and Kiparsky (1970), *inter alia*. It regards clause-embedding predicates as either triggering or not triggering factive inferences on particular occasions of use, as according to their interpretations. Thus the uncertainty about whether or not a given predicate’s complement clause projects is seen as metalinguistic in nature. Meanwhile, it allows for uncertainty related to world knowledge to manifest itself as contextual uncertainty, which may, in turn, make individual judgments of truth uncertain. We refer to this theory as

the *discrete-factivity* theory, emphasizing that it regards the contribution factivity makes as fundamentally discrete in nature. It gives rise to the following common ground:

$$\begin{aligned}
 &\text{discrete-factivity} : P(P\kappa) \\
 \text{discrete-factivity} = & \mathbf{v} \sim \text{factivityPriors} \\
 & \mathbf{w} \sim \text{worldPriors} \\
 & \boldsymbol{\tau}_v \sim \text{Bernoulli}(\mathbf{v}) \\
 & \boldsymbol{\tau}_w \sim \text{Bernoulli}(\mathbf{w}) \\
 & \langle \boldsymbol{\tau}_v, \boldsymbol{\tau}_w \rangle
 \end{aligned}$$

For the purpose of specifying our models abstractly, the type κ of contexts is assumed to be a product $t^m \times t^n$, where the inhabitants of t^m are m -tuples of truth values $\boldsymbol{\tau}_v$ determining whether or not the complement of each predicate under consideration indeed *projects*, and the inhabitants of t^n are n -tuples of truth values $\boldsymbol{\tau}_w$ determining whether or not each fact under consideration related to world knowledge is true or false.

Each truth value, moreover, is sampled from a Bernoulli distribution which is parameterized by some probability of returning either \top or \perp . The probability parameterizing each of these Bernoullis is, in turn, sampled from some prior distribution over probabilities—one for each predicate or fact. (Thus $\text{factivityPriors} : Pr^m$, and $\text{worldPriors} : Pr^n$.) For example, the probability that the complement of *know* (a purported semi-factive) should project might take a distribution whose mean is around 0.5, while the probability that the complement of *think* (a purported non-factive) should project might take a distribution whose mean is close to zero.

The aspect of this model crucial to the way in which it regards factivity is the location of the sampling statement ‘ $\boldsymbol{\tau}_v \sim \text{Bernoulli}(\mathbf{v})$ ’; in particular, it is crucial that this statement occurs *prior* to returning the probabilistic program of type $P\kappa$ that characterizes contextual uncertainty—that is, outside of the orange boxes. As a result, whether or not the complement of a given predicate projects is fixed in individual utterance contexts. By contrast, the sampling statement ‘ $\boldsymbol{\tau}_w \sim \text{Bernoulli}(\mathbf{w})$ ’ is part of the returned program, rendering world knowledge contextually uncertain.

Updating this representation of the common ground with a proposition and predicting the distribution of judgments generated by an inference is a matter of following the procedure outlined Section 4.1. Using the same example, we would obtain the following characterization

of this distribution:

$$\begin{aligned}
& m \sim \text{discrete-factivity} \\
& \mathcal{N}(x, \sigma) \top[0, 1] \\
& \text{where } x = \mathbb{P} \left(\begin{array}{l} c \sim m \\ \text{observe}(\llbracket \textit{Susan knows that Grace visited her sister} \rrbracket^c) \\ \llbracket \textit{Grace visited her sister} \rrbracket^c \end{array} \right) \\
= & \mathbf{v} \sim \text{factivityPriors} \\
& \mathbf{w} \sim \text{worldPriors} \\
& \boldsymbol{\tau}_v \sim \text{Bernoulli}(\mathbf{v}) \\
& \mathcal{N}(x, \sigma) \top[0, 1] \\
& \text{where } x = \mathbb{P} \left(\begin{array}{l} \boldsymbol{\tau}_w \sim \text{Bernoulli}(\mathbf{w}) \\ \text{observe}(\llbracket \textit{Susan knows that Grace visited her sister} \rrbracket^{\langle \boldsymbol{\tau}_v, \boldsymbol{\tau}_w \rangle}) \\ \llbracket \textit{Grace visited her sister} \rrbracket^{\langle \boldsymbol{\tau}_v, \boldsymbol{\tau}_w \rangle} \end{array} \right) \\
& \hspace{15em} \text{(by Assoc., Left id.)}
\end{aligned}$$

If we take $\text{know}(\boldsymbol{\tau}_v)$ to be the component of $\boldsymbol{\tau}_v$ that says whether or not the complement of *know* projects, and $\text{grace}(\boldsymbol{\tau}_w)$ to be the component of $\boldsymbol{\tau}_w$ that says whether or not Grace visited her sister, then the program above can be rephrased as follows:

$$\begin{aligned}
& \mathbf{v} \sim \text{factivityPriors} \\
& \mathbf{w} \sim \text{worldPriors} \\
& \boldsymbol{\tau}_v \sim \text{Bernoulli}(\mathbf{v}) \\
& \mathcal{N}(x, \sigma) \top[0, 1] \\
& \text{where } x = \mathbb{P} \left(\begin{array}{l} \boldsymbol{\tau}_w \sim \text{Bernoulli}(\mathbf{w}) \\ \text{know}(\boldsymbol{\tau}_v) \vee \text{grace}(\boldsymbol{\tau}_w) \end{array} \right)
\end{aligned}$$

That is, because the characterization of the common ground assumes that the tuples sampled from $\text{Bernoulli}(\mathbf{v})$ and $\text{Bernoulli}(\mathbf{w})$ are independently distributed, and because all that is required for *Grace visited her sister* to be entailed by the common ground is for the complement of *know* to project (i.e., for $\text{know}(\boldsymbol{\tau}_v)$ to be \top) or for it to be entailed by prior knowledge (i.e., for $\text{grace}(\boldsymbol{\tau}_w)$ to be \top), its semantic value is equivalent to a disjunction. The full model specifications presented in Appendix A make crucial use of this fact.

The next theory we consider shows how factivity itself may be regarded as contextually uncertain.

4.4 Factivity as a fundamentally gradient phenomenon

The theory which regards factivity as fundamentally gradient in nature does so by pushing what would otherwise be metalinguistic uncertainty about factivity onto the contextual uncertainty level. We refer to this theory as the *wholly-gradient* theory, since it regards both factivity and world knowledge as contextually uncertain. We obtain the corresponding model by making a small modification to the discrete-factivity model—that is, by changing the loca-

tion of the relevant sampling statement:

$$\begin{aligned}
 &\text{wholly-gradient} : P(P\kappa) \\
 \text{wholly-gradient} = & \ v \sim \text{factivityPriors} \\
 & \ w \sim \text{worldPriors} \\
 & \ \tau_v \sim \text{Bernoulli}(v) \\
 & \ \tau_w \sim \text{Bernoulli}(w) \\
 & \ \langle \tau_v, \tau_w \rangle
 \end{aligned}$$

4.5 Alternative theories

The above two theories, which regard world knowledge as contextually uncertain, imply the existence of the two alternatives which regard it on a par with metalinguistic uncertainty—that is, by giving rise to discrete inferences in individual contexts of language use. These alternatives may be *prima facie* implausible, but we include them in our comparisons to provide a better test of the first two theories.

4.5.1 Certainty about world knowledge as a discrete phenomenon

One of these alternatives understands uncertainty about world knowledge, but not factivity, to be resolved in context. We refer to this alternative as the *discrete-world* theory, since it regards world knowledge as discrete and factivity as gradient in nature. Thus the locations of the sampling statements which were used to encode the discrete-factivity model are switched:

$$\begin{aligned}
 &\text{discrete-world} : P(P\kappa) \\
 \text{discrete-world} = & \ v \sim \text{factivityPriors} \\
 & \ w \sim \text{worldPriors} \\
 & \ \tau_w \sim \text{Bernoulli}(w) \\
 & \ \tau_v \sim \text{Bernoulli}(v) \\
 & \ \langle \tau_v, \tau_w \rangle
 \end{aligned}$$

4.5.2 Everything as a discrete phenomenon

The other alternative understands both factivity and world knowledge as discrete in nature. Its corresponding model is specified by moving both sampling statements outside of the returned program:

$$\begin{aligned}
 &\text{wholly-discrete} : P(P\kappa) \\
 \text{wholly-discrete} = & \ v \sim \text{factivityPriors} \\
 & \ w \sim \text{worldPriors} \\
 & \ \tau_v \sim \text{Bernoulli}(v) \\
 & \ \tau_w \sim \text{Bernoulli}(w) \\
 & \ \langle \tau_v, \tau_w \rangle
 \end{aligned}$$

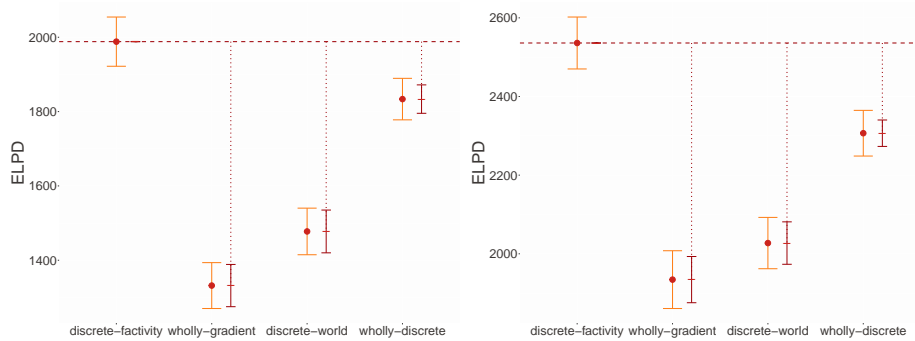


Figure 4: Left: expected log pointwise predictive densities for the four models. Right: expected log pointwise predictive densities for the four model evaluations on our replication experiment data. Dotted lines indicate estimated differences between each model and the discrete-factivity model. Error bars indicate standard errors.

This *wholly-discrete* theory effectively hypothesizes that no inferences display uncertainty in context: any gradience displayed in people’s measured inferences must therefore be due to response error.

4.6 Comparisons

Figure 4 (left plot) provides expected log pointwise predictive densities estimated for the four models, based on log-likelihoods computed from Degen and Tonhauser’s experimental data. As we see, the discrete-factivity model captures the data the best, while the wholly-discrete model trails behind it; meanwhile, the wholly-gradient and discrete-world models perform the worst. Thus we have evidence that the best model of Degen and Tonhauser’s data treats factive presupposition projection as a discrete phenomenon and the inferences contributed by world knowledge as gradient. Meanwhile, by simply modifying the discrete-factivity model so that it treats factivity as gradient, one goes from the *best*-performing model to the *worst*-performing one.

To give a sense of the performance of the models as assessed against empirical data, the posterior predictive distributions for each model are plotted for six predicates, given three different contexts, in Figures 5 to 7 (see Figures 18 to 20 of Appendix B.2 for all predicates).¹⁴

5 Evaluations

Of the four models considered, the discrete-factivity model provides the best fit to Degen and Tonhauser’s experimental data. This section further evaluates these models in two ways.

¹⁴Notably, the canonically non-projective predicates *think* and *pretend* produce empirical distributions which all four of our models appear to have difficulty capturing, at least by visual inspection. This may be because of an inference associated with these predicates that the complement clause is *not* true; this inference likely results from a conversational implicature in the case of *think*, while it may be due to an aspect of the lexical semantics of *pretend*.

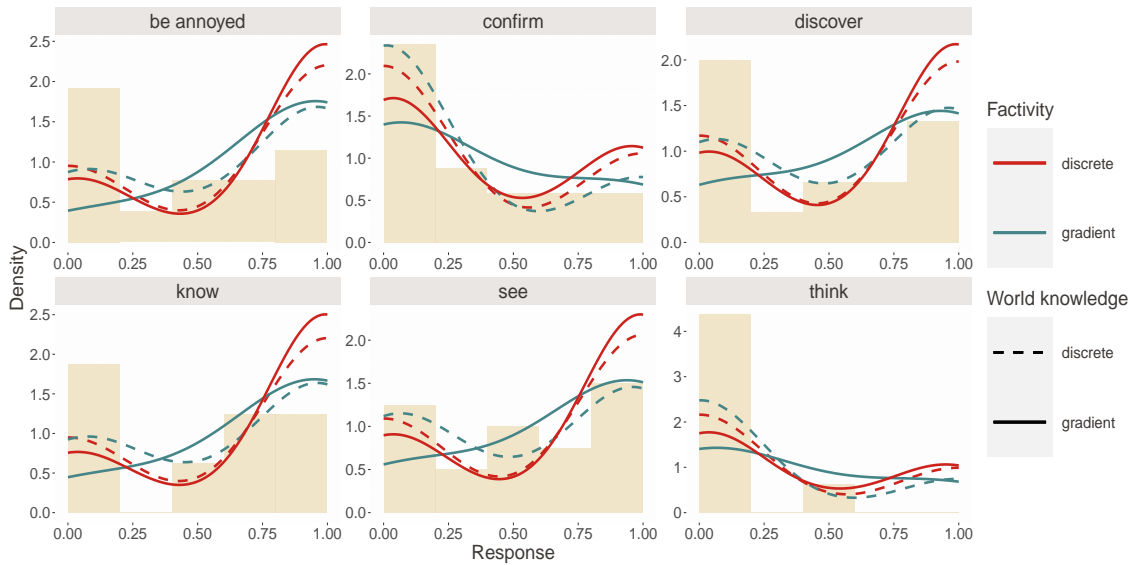


Figure 5: Posterior predictive distributions (with simulated participant intercepts) of all four models for six predicates from Degen and Tonhauser’s (2021) projection experiment. Complement clause: *Grace visited her sister*; background fact: *Grace hates her sister*. Empirical distributions are represented by density histograms of data pooled from Degen and Tonhauser 2021 and our replication study.

First, we test the inferred posterior distributions over certainties and probabilities of projection associated with each model on held-out data from an experiment that replicates Degen and Tonhauser’s. A replication allows us to assess whether the relative model performances reported above might have arisen from accidental properties of the original data set, rather than genuine differences among the models as accounts of people’s behavior in the given experimental task.

Second, we test the inferred posterior distributions over probabilities of projection on held-out data from two experiments in which the context of each predicate is stripped of the rich lexical content that partially governs the inferences produced in the original experiment. We obtain contexts for this evaluation in two ways. In our first experiment, we *bleach* each predicate’s complement clause so that it is just *a particular thing happened*. In the second experiment, we use a *templatic* complement clause of the form *X happened*. (We describe these methods in more detail in Sections 5.2.1 and 5.2.2, respectively.) These manipulations serve two purposes. First, they allow us to assess the performance of the four models when the source of variance among inferences contributed by prior world knowledge is removed. On the other hand, they put the predicates in environments in which knowledge about the context is minimal; as a result, they may produce a great deal of uncertainty in people’s inferences. Hence, they help to assess the robustness of the discrete contribution factive predicates make to inference: in contexts producing great prior uncertainty about the relevant inference, people might resort to inference strategies that reflect this uncertainty; for example,

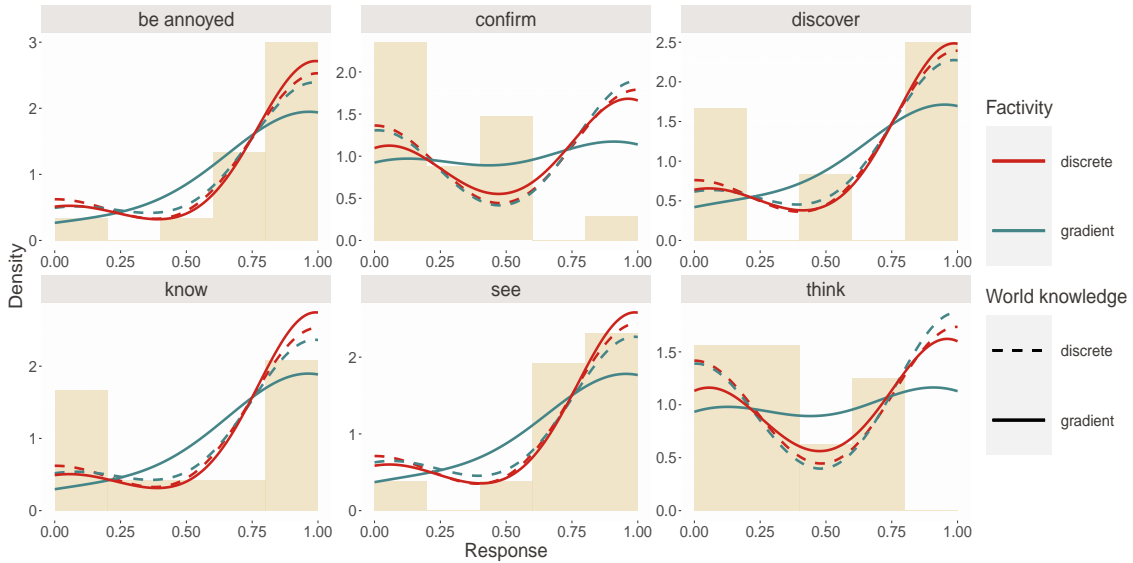


Figure 6: Posterior predictive distributions (with simulated participant intercepts) of all four models for six predicates from Degen and Tonhauser’s (2021) projection experiment. Complement clause: *Sophia got a tattoo*; background fact: *Sophia is a hipster*. Empirical distributions are represented by density histograms of data pooled from Degen and Tonhauser 2021 and our replication study.

by tending toward responses in the middle of the scale, rather than at the endpoints. Such strategies may confer an *a priori* advantage to the wholly-gradient model, which considers all inferences, even those triggered by projective predicates, to be beset with some uncertainty. We thus consider these evaluations to be a somewhat stronger test of the discrete-factivity model’s edge over the wholly-gradient model.

5.1 Experiment 1: held-out projection experiment data

Our materials and methods were identical to those of Degen and Tonhauser (2021). We collected data from 300 participants using Amazon Mechanical Turk, paying each participant one dollar. Each participant was required to pass the qualification test described in White, Hacquard, and Lidz 2018, in order to ensure that they were a native speaker of American English. We removed two participants’ data who claimed to have technical difficulties completing the experiment, and ten more whose performance was more than two standard deviations below the mean on the six control items, leaving us with data from a total of 288 participants. Figure 8 shows the item means from our replication experiment plotted against Degen and Tonhauser’s original experiment, along with the means gotten by collapsing across predicates.

To evaluate the four models using this data, we obtained, from each model, means μ_v and standard deviations σ_v of the marginal posterior distribution of the log-odds of projection for each predicate, as well as means μ_ω and standard deviations σ_ω of the marginal posterior

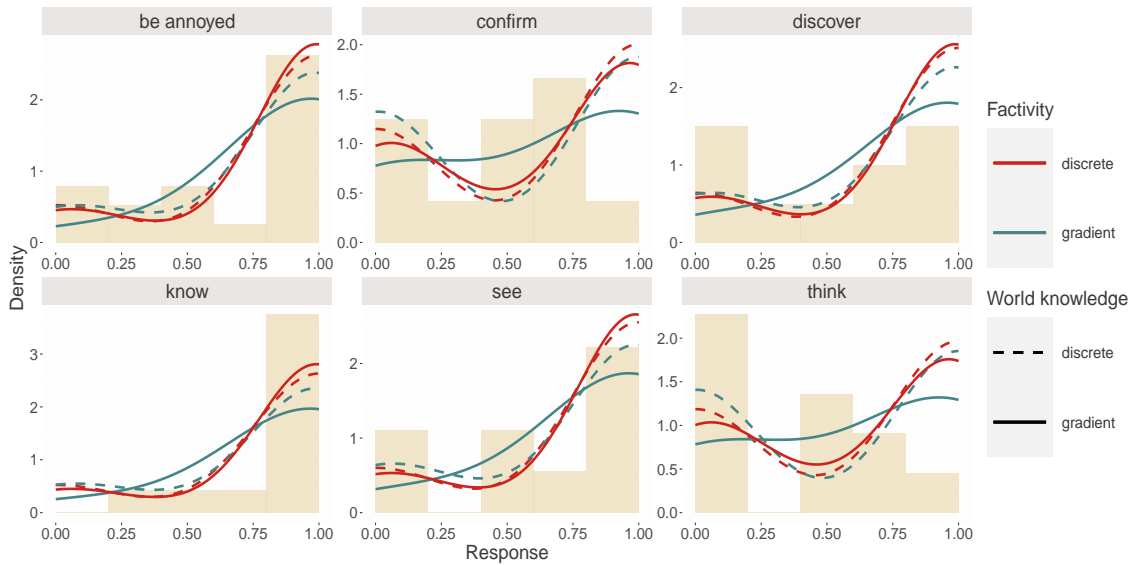


Figure 7: Posterior predictive distributions (with simulated participant intercepts) of all four models for six predicates from Degen and Tonhauser’s (2021) projection experiment. Complement clause: *Grace visited her sister*; background fact: *Grace loves her sister*. Empirical distributions are represented by density histograms of data pooled from Degen and Tonhauser 2021 and our replication study.

distribution of the log-odds certainty for each context. We then used normal distributions with these means and standard deviations as prior distributions for the corresponding parameters in the models constructed for the evaluations. (See Appendix A.3 for further details concerning these models.)

Figure 4 (right plot) provides expected log pointwise predictive densities estimated for the four model evaluations, based on log-likelihoods computed from the data obtained in our replication experiment. The pattern of goodness-of-fit witnessed here is apparently identical to that exhibited by the original model comparison on the left in Figure 4: the discrete-activity model fares the best, followed by the wholly-discrete model; meanwhile, the wholly-gradient and discrete-world models fare the worst.

Thus we have evidence that the differences in performance among these models that was reported in the previous section are quite robust, at least when assessed using data from Degen and Tonhauser’s experimental task. The next two experiments provide a test of the models in a somewhat different setting—one in which the uncertainty contributed by the linguistic contexts of the predicates of interest is sent to an extreme.

5.2 Experiments 2 and 3: non-contentful contexts

For each of the bleached and templatic experiments, we collected data from 50 new participants using Amazon Mechanical Turk, paying each participant one dollar. Each participant was, again, required to pass the qualification test described in White, Hacquard, and Lidz 2018.

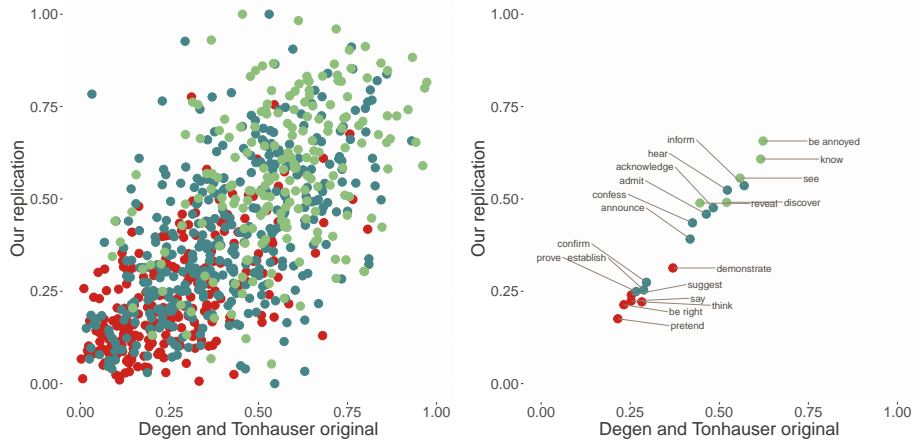


Figure 8: Degen and Tonhauser’s (2021) projection data versus our replication. Left: item means (Spearman’s $r = 0.68^{***}$). Right: verb means (Spearman’s $r = 0.98^{***}$). For both, “non-factive” verbs are in red, “optionally factive” verbs are in teal, and “canonically factive” verbs are in green.

Both experiments were modeled after Degen and Tonhauser’s original task, with two crucial differences: (1) the complement clause was modified to either a bleached or templatic variant; and (2) no background fact was provided. In addition, six control items were constructed for each experiment with an intended answer of 1. The details pertinent to each experimental set-up are provided in the next two subsections.

To evaluate the four models using both the bleached and templatic data, we used the same means μ_v and standard deviations σ_v of the marginal posterior distributions of the log-odds of projection that we used for the evaluations on the replication experiment data. As before, we used normal distributions with these means and standard deviations as prior distributions for the corresponding parameters in the models constructed for the evaluations. Then, in each evaluation, we inferred a distribution over the parameters σ_ω and ω that regulate the certainty associated with either the bleached or the templatic context. (See Appendix A.4 for further details concerning these models.)

5.2.1 Experiment 2: bleached items

To construct the bleached items, each of the twenty predicates investigated in Degen and Tonhauser’s experiment was placed in a context in which its subject was one of the proper names from the original experiment, and in which its complement clause was just *a particular thing happened*. On each trial, experimental participants were provided with a background context that was intended to make the prompt as natural as possible. The only thing that varied in this background context from one trial to the next was the name of the individual who makes the relevant utterance. Finally (taking that individual’s name to be P), participants were prompted to answer the question *Is P certain that that thing happened?* on a sliding scale with ‘no’ on the left and ‘yes’ on the right. The following experimental trial, for example,

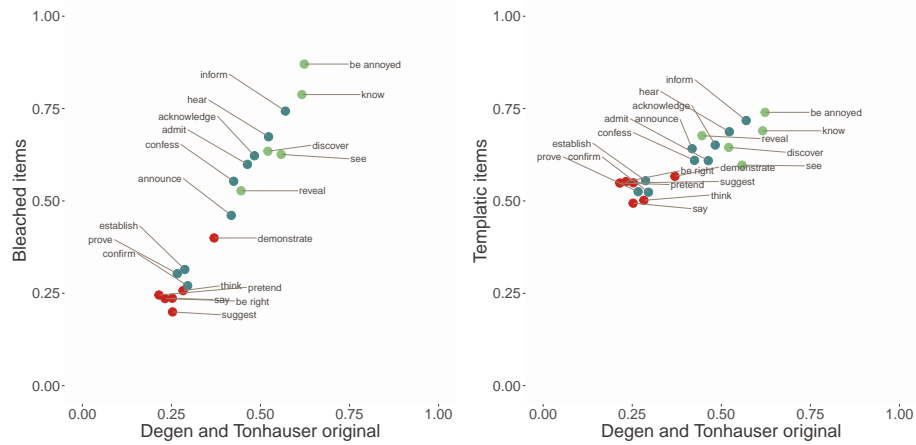


Figure 9: Degen and Tonhauser’s (2021) projection data versus data from contexts with minimal lexical content. Left: bleached data (Spearman’s $r = 0.97^{***}$). Right: templatic data (Spearman’s $r = 0.87^{***}$). For both, “non-factive” verbs are in red, “optionally factive” verbs are in teal, and “canonically factive” verbs are in green.

involves the predicate *pretend*:

You are at a party. You walk into the kitchen and overhear Linda ask somebody else a question. Linda doesn’t know you and wants to be secretive, so speaks in somewhat coded language.

Linda asks: “Did Tim pretend that a particular thing happened?”

Is Linda certain that that thing happened?

no yes

Next

In addition to the twenty bleached items, participants saw six control items which were constructed in order to somehow incorporate a bleached subordinate clause; for example, *Did Madison have a baby, despite the fact that a particular thing happened?* All six control items had an intended response of 1, and any participant whose average score on these items did not fall within two standard deviations below the mean of all participant’s responses was excluded from the analysis. Using this criterion, three participants data was excluded, leaving 47 participants for analysis.

It can be seen by inspecting the left plot in Figure 9 that the responses elicited by the bleached items track the gradient knowledge about factivity that people deploy in the typical contentful setting extremely well. Not only is the same type of gradience observed among

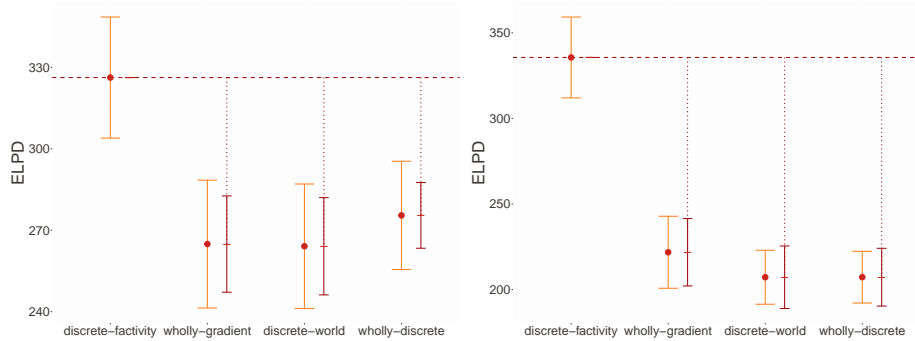


Figure 10: Expected log pointwise predictive densities for the four model evaluations on the bleached data (left) and the templatic data (right). Dotted lines indicate estimated differences between each model and the discrete-factivity model. Error bars indicate standard errors.

predicates when they are placed in bleached contexts, but the ranking among predicates is maintained almost entirely.

Finally, the left plot of Figure 10 provides expected log pointwise predictive densities for all four model evaluations, given the bleached data. We see, here, that the discrete-factivity model fares the best, while the other three models fare comparably with each other. It is somewhat remarkable that the more fine-grained differences among models observable from both the original fits and the evaluation on the replication data does not appear to hold up under the current evaluation; for example, the wholly-discrete model no longer appears distinguished from the wholly-gradient and discrete-world models by its better performance. Rather, the discrete-factivity model seems to have a unique advantage.

To give a sense of the differences among the model evaluations on the bleached data, Figure 11 shows the posterior predictive distributions of the evaluations for six predicates (see Figure 21 of Appendix B.2 for all predicates).

5.2.2 Experiment 3: templatic items

To construct the templatic items, each of the same twenty predicates was placed in a context in which its subject was, again, a proper name from the original Degen and Tonhauser experiment, and in which its complement clause was *X happened*. A background context was provided on each trial, so that the prompt was natural. Background contexts, again, only differed from one another in the name of the individual who makes the relevant utterance. Given a trial on which the individual *P* was the speaker, participants were prompted with the question *Is P certain that X happened?*, which they answered on a sliding scale with ‘no’ on the left and ‘yes’ on the right. The following example trial features the predicate *pretend*:

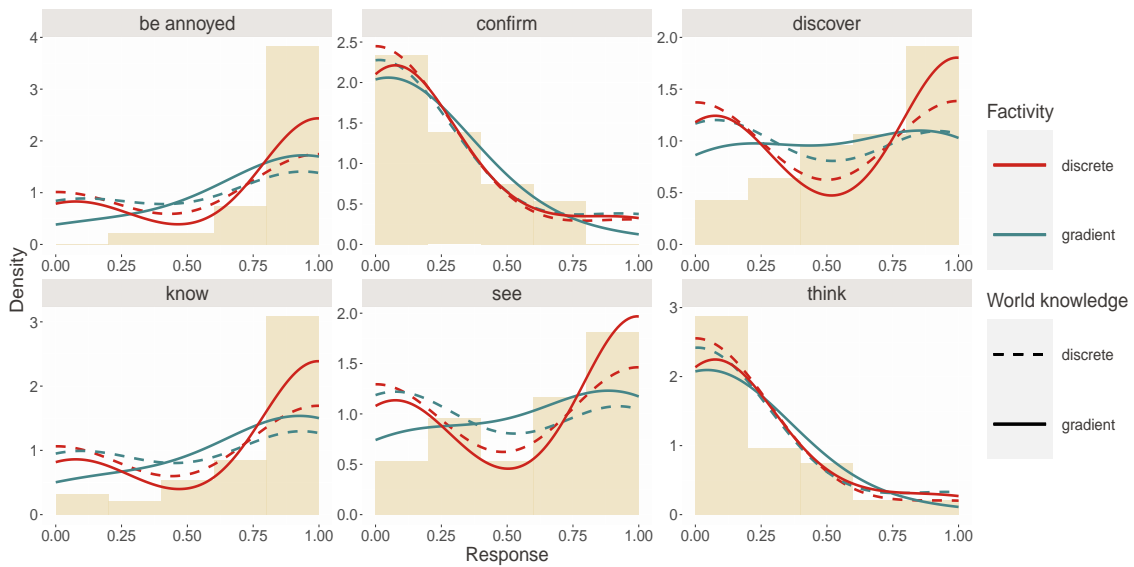


Figure 11: Posterior predictive distributions (with simulated participant intercepts) of all four model evaluations for six predicates from Degen and Tonhauser’s (2021) projection experiment. Complement clause: *a particular thing happened*. Empirical distributions are represented by density histograms.

You are at a party. You walk into the kitchen and overhear William ask somebody else a question. The party is very noisy, and you only hear part of what is said. The part you don't hear is represented by the 'X'.

William asks: "Did Ray pretend that X happened?"

Is William certain that X happened?

no yes

Participants, again, saw six control items which were constructed in order to incorporate a templatic subordinate clause; for example, *Did Madison have a baby, despite the fact that X happened?*. Controls, again, had an intended response of 1, and the same data exclusion criterion was used for the current experiment as was used previously. Using this criterion, one participant was excluded, leaving 49 participants for analysis.

The right plot of Figure 9 shows that the responses elicited by the templatic items track the gradient knowledge about factivity that people deploy in the contentful setting fairly well. Notably, the range of average ratings for predicates is not as wide as exhibited in the previous

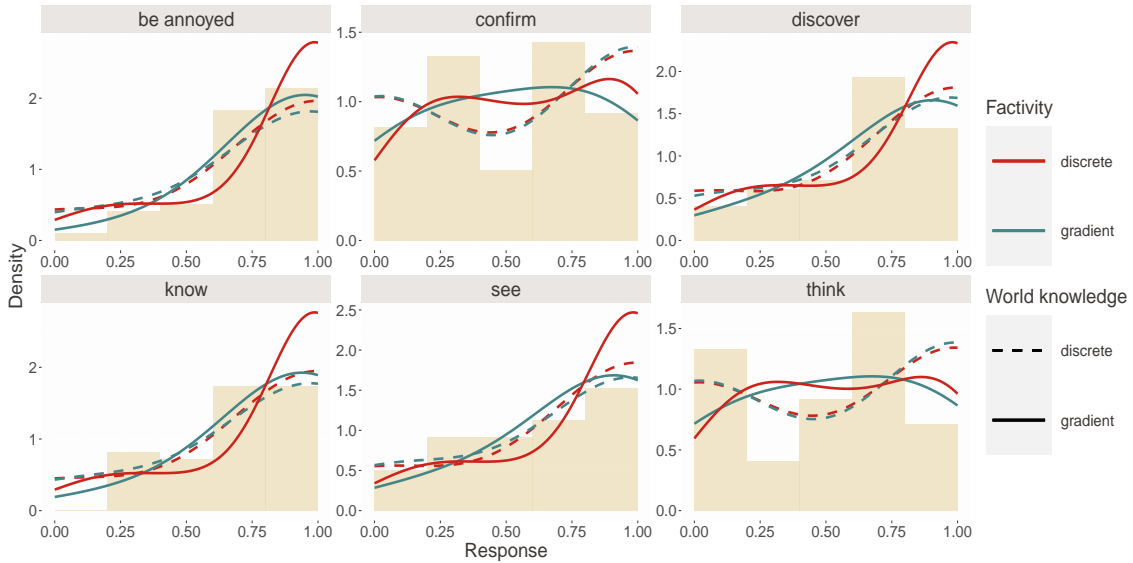


Figure 12: Posterior predictive distributions (with simulated participant intercepts) of all four model evaluations for six predicates from Degen and Tonhauser’s (2021) projection experiment. Complement clause: *X happened*. Empirical distributions are represented by density histograms.

experiments, with most falling between 0.5 and 0.75, suggesting that there may have been a great deal of uncertainty governing the inferences produced from this task.

Finally, the right plot of Figure 10 gives expected log pointwise predictive densities for all four model evaluations, given the templatic data. We see that the discrete-factivity model again fares the best, and that the other three models are, again, comparable with one another. This difference is especially notable, given the somewhat squashed average responses seen across verbs in Figure 9. That is, despite the high amount of uncertainty which this task may have produced, such uncertainty seems to have been filtered through the discrete behavior of factive inferences. The uncertainty about such inferences appears to relate to the interpretation of a given predicate, rather than the contribution which that predicate makes to an inference.

To give a sense of the differences among the model evaluations on the templatic data, Figure 12 shows the posterior predictive distributions of the evaluations for six predicates (see Figure 22 of Appendix B.2 for all predicates).

6 Consequences for theories of factivity

We have compared four models of the task reported in Degen and Tonhauser 2021, each fit to their experimental data. These models differ from one another along two axes: (a) whether they consider the contribution of prior world knowledge to inferences about the truth of the clause embedded by a predicate to be gradient or discrete; and (b) whether they consider the

contribution of the relevant predicate itself to these inferences to be gradient or discrete. The contribution of a given factor to an inference is “gradient” if varying that factor produces a continuous effect on the magnitude of the judgment associated with the inference; and it is “discrete” if varying the factor affects the probability with which the inference is judged as certain, versus remains unaffected.

Our initial comparison of the four models found that the discrete-factivity model best accounts for the distributions of judgments in Degen and Tonhauser’s experimental data. That is, the model which regards the contribution of prior world knowledge to such inferences as gradient and the contribution of a given predicate to such inferences as discrete (as assessed by expected log pointwise predictive density, plotted in Figure 4, left). Moreover, follow-up evaluations of the four models confirmed the initial comparison: the same model best accounts for held-out data from a replication of Degen and Tonhauser’s experiment, for which distributions over the parameters of interest are extracted from the posteriors of the initial models (Figure 4, right plot). The discrete-factivity model also best accounts for data from two tasks in which predicates are placed in contexts with minimal lexical content (Figure 10). Taken together, these results provide strong evidence that the observed gradience among the clause-embedding predicates studied by Degen and Tonhauser is *metalinguistic*. Different clause-embedding predicates differ in the frequencies with which they trigger projective inferences, but the contribution a predicate makes on particular occasions of use and interpretation is *discrete*, either producing the relevant inference or not producing it at all.

What do these conclusions mean for the notion of ‘factive predicate’ as a class? Are there factive predicates? We propose that our results support an account of factivity whereon it is a semantically live property of expressions, but a property that may be observed on only some uses of those expressions. Hence, many predicates which have traditionally been considered factive may, in fact, be systematically ambiguous. We take this finding to largely confirm what prior work stretching back to Karttunen 1971 has (at least implicitly) assumed in discussing ‘semifactive predicates’. These predicates may support factive readings in certain contexts, and with some proclivity which varies by individual predicate, or by class of predicate (Kane, Gantt, and White 2022). Hence, our results are consistent with a fairly conservative picture of factivity, according to which it is an optional property of at least a subset of the predicates that Degen and Tonhauser investigate.

6.1 Factivity as an epiphenomenon

Our proposal is crucially a proposal about semantic properties. Yet, it is consistent with an account of factivity on which projection is intimately tied to properties of the discourse in which the expressions of interest are embedded.¹⁵ A prime example of such an account can be found in Simons, Beaver, et al. 2017, who rely crucially on the notion of a question under discussion (QUD; Roberts 2012). On their account, whether or not the complement of a clause-embedding predicate projects varies according to prosodic and contextual factors associated with the QUD. Simons, Beaver, et al.’s main aim in giving this account is to do away with

¹⁵See Qing, Goodman, and Lassiter 2016 for an approach along these lines within the Rational Speech-Act framework (Frank and Goodman 2012; Goodman and Stuhlmüller 2013).

factivity as a semantic property of expressions, arguing rather that projective inferences are simply those which are backgrounded by the QUD, while non-projective inferences are those that are at-issue (and generally, entailed).

We do not believe that completely doing away with the notion of factivity as a semantic property will be possible for reasons that we have already mentioned in passing (Footnote 7): insofar as one is willing to countenance that discrete choices are made about the identity of the QUD on individual occasions of interpretation (*pace* Tonhauser, Beaver, and Degen 2018), one could posit that there is no indeterminacy in the interpretations for the relevant string and, rather, that there is uncertainty over possible questions under discussion (QUD) against which the string is interpreted; but it is not clear how to reconcile such an account with the observation that *classes of* predicates are associated with particular levels of gradience without saying that lexical knowledge somehow conditions QUD choice. Such lexical knowledge could be knowledge one has about the kinds of discourses in which a (class of) predicates is used—and therefore not semantic in nature—but this assumption raises a further question about why such knowledge would predict predicates’ syntactic distributions, as Kane, Gantt, and White (2022) show that it does. It seems much more plausible that this knowledge is at least partly semantic in nature.

So how could a semantic notion of factivity be integrated with accounts that intimately tie projection to properties of the discourse in which the expressions of interest are embedded? In answering this question, we believe it will be fruitful to combine standard dynamic accounts of presupposition projection (Heim 1992 *et seq*) with a probabilistic framework like Grove and Bernardy’s (2023) and our extension.

In broad strokes, dynamic accounts of presupposition projection in the Heimian tradition associate factive predicates with constraints on the contexts they can be used in—generally, requiring that the common ground entails the content of the factive’s embedded clause. Such constraints could be stated in Grove and Bernardy’s framework (coupled with our extension) as constraints imposed by the predicate on the distribution over contexts associated with the common ground. Uncertainty about whether a predicate is factive or not thus implies uncertainty about what constraints to impose on that distribution over contexts.

Under such an account, properties of the QUD correlate with projection because possible QUDs are constrained by the common ground in at least the sense that the QUD cannot be trivial—i.e., the common ground cannot entail an answer to the QUD. Hence, insofar as a factive variant of a predicate is more probable, QUDs that are trivial under common grounds that are made more probable by that variant will have lower probability.

6.2 Which predicates are factive?

Which clause-embedding predicates *do*, in fact, belong in the class of factives is not a question whose answer we have formally pursued here. But to retrace the discussion of Section 2, we note that Kane, Gantt, and White (2022) have already done relevant work on this front by investigating how best to cluster the predicates of interest (and many other predicates) into semantic classes that are predictive of their syntactic distributions. Among other data sets, Kane, Gantt, and White rely on the MegaVeridicality data set (White and Rawlins 2018), which Degen and Tonhauser (2022) also use to support their findings of gradience among predicates’

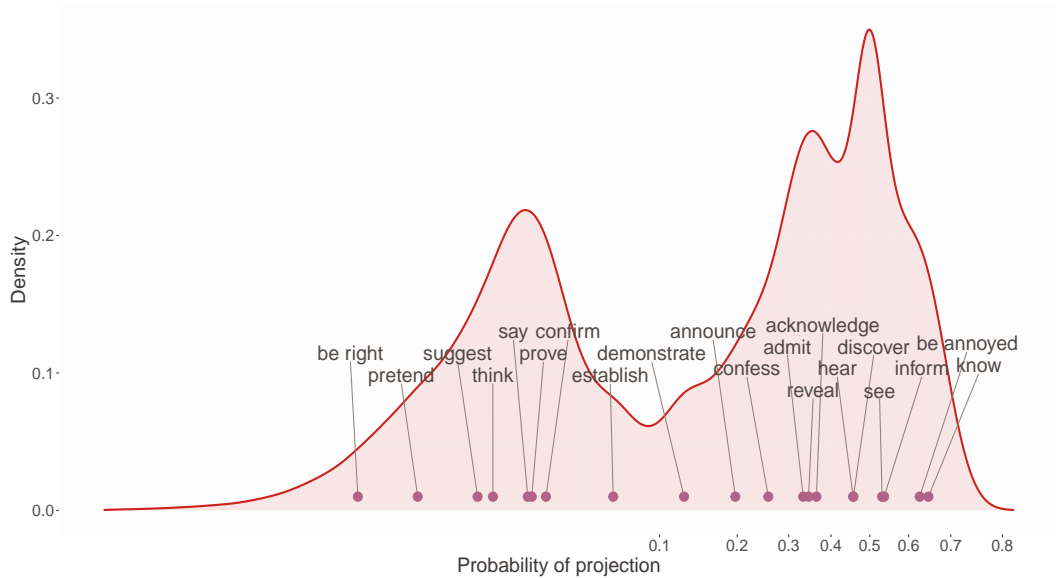


Figure 13: Density plot of the posterior probability of projection (with participant intercepts zeroed out) for the discrete-factivity model, for all predicates combined, scaled to log-odds space. Points represent the posterior mean log-odds associated with individual predicates (Spearman’s $r = 0.98^{***}$, when compared with the empirical means).

veridicality inferences. Kane, Gantt, and White find that *emotive* predicates, such as *love* and *be pleased*, yield the most strongly factive inferences, and that such predicates are followed by *discourse commitment* predicates, which include *know*. Thus we are optimistic that clusterings of predicates based solely on diagnostics of factivity will also give rise to semantically potent classes.

In this vein, restricting attention only to the predicates that Degen and Tonhauser study yields a promising outlook. Figure 13 plots the posterior probability of projection associated with the discrete-factivity model for all predicates combined, with the mean log-odds for individual predicates represented toward the bottom. One can see that there are roughly two modes underneath which the means associated with individual predicates cluster. The left mode is around 0.018, or near zero. The right mode is close to a probability of around half. Indeed, one could make a cut between the predicates whose means appear to fall under the left versus the right mode. Such a cut would classify all of the predicates under Degen and Tonhauser’s “non-factive” category, except for *demonstrate*, as non-factive, along with *confirm* and *establish*; it would then classify the remaining predicates as optionally factive. Moreover, the contours visible in the right mode might suggest that more than one semantic class is active, insofar as such classes govern the frequency with which a given predicate triggers presupposition projection.

These observations are merely suggestive, however. We leave a detailed investigation of the semantic classes organizing the lexical knowledge of factivity for the future, noting merely that the extension of Grove and Bernardy’s framework proposed in the current paper

provides a natural way to integrate uncertainty over predicate classes and their inferential effects into a model that connects the compositional semantics to experimental data in an unbroken chain.

7 Conclusion

As a whole, the results presented here can be taken to motivate a fairly traditional view of factivity, of the kind originally advocated by Kiparsky and Kiparsky (1970), Karttunen (1971), *inter alia*. Some predicates may be understood to trigger a presupposition that the clause they select is true. The key departure from this tradition we would advocate, based on our results (and following Degen and Tonhauser), is in the particular classification of predicates which researchers ought to appeal to. Indeed, *none* of the predicates which Degen and Tonhauser investigate appear to be assigned a factive interpretation in all of their uses; rather, all seem to be associated with some degree of metalinguistic uncertainty about their status as factive. For many predicates, such as *think*, the degree of uncertainty is fairly trivial, fixing a near-zero probability of being factive. This is natural: if people are Bayesian reasoners about the knowledge they maintain about the world, including its linguistic conventions, some uncertainty about the semantic properties of linguistic expressions will be an essential feature of that knowledge.

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A Full model specifications

Each model was fit using Stan’s Hamiltonian Markov chain Monte Carlo sampling algorithm. For each model of Degen and Tonhauser’s data, we obtained 6,000 posterior samples of the model parameters, following 6,000 burn-in samples, on four chains each. For each model of our replication experiment data, we obtained 24,000 posterior samples of the model parameters, following 24,000 burn-in samples, on four chains each. For each model of either the bleached or templatc experiment data, we obtained 6,000 posterior samples of the model parameters, following 6,000 burn-in samples, on four chains each.

A.1 The norming model

We characterize our model of the norming data as a probabilistic program, with the following structure, given data $\mathbf{y}_{\text{norming}} : r^{\frac{n_{\text{context}}}{2} * n_{\text{participant}}}$, where n_{context} and $n_{\text{participant}}$ are the number of contexts and participants, respectively, featured in the experiment. That is, each participant saw half of the available contexts, where each complement clause from the projection experiment was rated in conjunction with either a low-prior fact or a high-prior fact. We use $\mathbf{y}_{\text{norming},i,j}$ to denote participant j ’s response, given context i .

We encode the certainties for contexts as parameters ω on a log-odds scale, with participant random intercepts ϵ added to these parameters before they are mapped to transformed parameters w for certainty on the unit interval. Normal priors centered at zero are placed on the participant intercepts, as well as the log-odds parameters for contexts; the standard deviations (σ_{ϵ} and σ_{ω}) of these normal distributions are, in turn, given exponential hyper-priors. Finally, the likelihood for our model is given by a normal distribution centered at the certainty, whose standard deviation σ_e is parameterized with a prior uniform on the unit interval, truncated to the unit interval. We use $w_{i,j}$ to denote the parameter encoding the certainty for participant j , given context i .

We point out an important notational convention, which pertains to all of the model specifications we give here. We use the operator

$$D_{(\cdot)} : \mathbb{P}\alpha \rightarrow \alpha \rightarrow r$$

to obtain a density (or mass, as the case may be) function on α ’s from a probabilistic program that returns α ’s as values. Thus if m returns, say, tuples of real numbers, then we may obtain the density (or mass) that m assigns to the tuple x as $D_m(x)$.

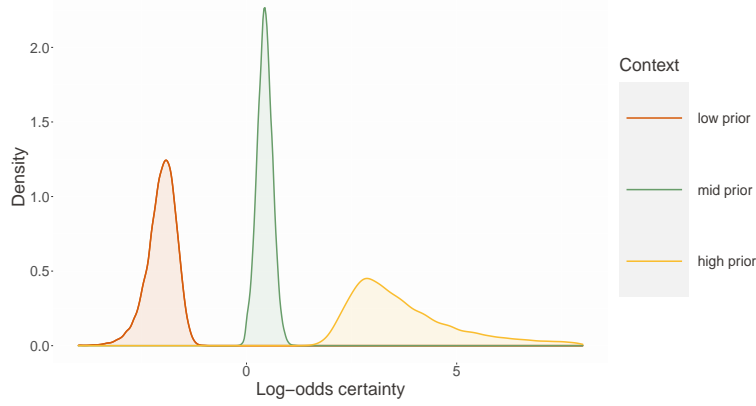


Figure 14: Density plots of the posterior log-odds certainty (with participant intercepts zeroed out) for three items in Degen and Tonhauser’s (2021) norming task. Low and high priors are for *Grace visited her sister*, given the facts *Grace hates her sister* and *Grace loves her sister*, respectively. Mid prior is for *Sophia got a tattoo*, given the fact *Sophia is a hipster*.

The model of the norming data is the following:

$$\begin{aligned}
 \text{norming} &: P(r^{\text{context}} \times r^{\text{participant}} \times r^{\text{context}} \times r^2) \\
 \text{norming} &= \sigma_\omega \sim \text{Exponential}(1) \\
 &\quad \sigma_\epsilon \sim \text{Exponential}(1) \\
 &\quad \sigma_e \sim \text{Uniform}(0, 1) \\
 &\quad \omega \sim \mathcal{N}(0, \sigma_\omega) \\
 &\quad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon) \\
 &\quad \text{factor}(D_{\mathcal{N}(\omega, \sigma_\omega) \top [0, 1]}(\mathbf{y}_{\text{norming}})) \\
 &\quad \langle \omega, \epsilon, \sigma_\omega, \sigma_\epsilon, \sigma_e \rangle \\
 &\quad \text{where } \mathbf{w}_{i,j} = \text{logit}^{-1}(\omega_i + \epsilon_j)
 \end{aligned}$$

The parameters ω encode a log-odds certainty rating for each item. We obtain prior distributions for these parameters in our models of factivity by extracting their marginal posterior distributions from our norming model and, for each item (i.e., each parameter of ω), taking a normal distribution with mean and variance equal to that of the posterior distribution. Density plots for three items are given in Figure 14 (see Figure 16 of Appendix B for all items).

A.2 The factivity models

We now provide our four models of factivity and prior world knowledge, which we fit to Degen and Tonhauser’s projection experiment data. In specifying each one, we use μ_ω and σ_ω to denote the means and standard deviations, respectively, of the marginal posterior distributions of the log-odds certainty ratings for the contexts assessed in the norming experiment. In each model specification, $\mathbf{y}_{\text{projection}} : r^{\text{verb} * \text{participant}}$ encodes the experimental data, since each participant saw each verb exactly once. We use $\mathbf{y}_{\text{projection}_{i,j,k}}$ to denote participant k ’s response, given verb i and context j .

For each model, we encode the log-odds of projection for verbs, along with the log-odds certainties for contexts, as parameters ν and ω . Participant random intercepts ϵ_1 and ϵ_2 are added to these parameters, respectively, before they are mapped to transformed parameters v and w on the unit interval. Normal priors centered at zero are placed on the participant intercepts, as well as the log-odds parameters for verbs; the standard deviations (σ_{ϵ_1} , σ_{ϵ_2} , and σ_ν) of these normals are, in turn, given exponential hyper-priors. Finally, the likelihoods for our models are again given by normal distributions truncated to the unit interval, and whose standard deviation σ_e is parameterized by a prior uniform on the unit interval. The mean θ of this truncated normal likelihood varies by model, as we show next. In general, we use $v_{i,j,k}$ and $w_{i,j,k}$ to denote the parameters encoding the probability of projection and certainty, respectively, for participant k , given verb i and context j .

A.2.1 The discrete-factivity model

The discrete-factivity model defines the parameters θ as either 1 or the certainty determined by world knowledge, depending on whether or not the relevant predicate’s complement clause projects. This definition of θ is justified by the following fact, given some fixed τ_1 :

Fact 1.

$$Pr \left(\begin{array}{l} \tau_2 \sim \text{Bernoulli}(p) \\ \tau_1 \vee \tau_2 \end{array} \right) = \mathbb{1}(\tau_1) + \mathbb{1}(\neg\tau_1) * p$$

In other words, a given predicate’s complement projects *or* it doesn’t project; if it doesn’t, then the prior certainty determined by world knowledge takes the reins. This yields the following model specification:

$$\text{discrete-factivity} : P(r^{n_{\text{verb}}} \times r^{n_{\text{context}}} \times r^{n_{\text{participant}}} \times r^{n_{\text{participant}}} \times r^{n_{\text{verb}}} \times r^3)$$

$$\text{discrete-factivity} = \sigma_\nu \sim \text{Exponential}(1)$$

$$\sigma_{\epsilon_1} \sim \text{Exponential}(1)$$

$$\sigma_{\epsilon_2} \sim \text{Exponential}(1)$$

$$\sigma_e \sim \text{Uniform}(0, 1)$$

$$\nu \sim \mathcal{N}(0, \sigma_\nu)$$

$$\omega \sim \mathcal{N}(\mu_\omega, \sigma_\omega)$$

$$\epsilon_1 \sim \mathcal{N}(0, \sigma_{\epsilon_1})$$

$$\epsilon_2 \sim \mathcal{N}(0, \sigma_{\epsilon_2})$$

$$\tau_v \sim \text{Bernoulli}(v)$$

$$\text{factor}(D_{\mathcal{N}(\theta, \sigma_e) \uparrow [0,1]}(\mathbf{y}_{\text{projection}}))$$

$$\langle \nu, \omega, \epsilon_1, \epsilon_2, \sigma_\nu, \sigma_{\epsilon_1}, \sigma_{\epsilon_2}, \sigma_e \rangle$$

$$\text{where } v_{i,j,k} = \text{logit}^{-1}(\nu_i + \epsilon_{1k})$$

$$w_{i,j,k} = \text{logit}^{-1}(\omega_j + \epsilon_{2k})$$

$$\theta_{i,j,k} = \mathbb{1}(\tau_{v_{i,j,k}}) + \mathbb{1}(\neg\tau_{v_{i,j,k}}) * w_{i,j,k}$$

A.2.2 The wholly-gradient model

The wholly-gradient model sets each parameter $\theta_{i,j,k}$ equal to $v_{i,j,k} + (1 - v_{i,j,k}) * w_{i,j,k}$, an encoding justified by the following fact:

Fact 2.

$$Pr \left(\begin{array}{l} \tau_1 \sim \text{Bernoulli}(p) \\ \tau_2 \sim \text{Bernoulli}(q) \\ \tau_1 \vee \tau_2 \end{array} \right) = p + (1 - p) * q$$

Under this model, presupposition projection is genuinely gradient, since it adds directly to the certainty that the relevant complement clause is true, giving it a *boost* (albeit not all the way to 1).

$$\text{wholly-gradient} : P(r^{\text{verb}} \times r^{\text{context}} \times r^{\text{participant}} \times r^{\text{participant}} \times r^{\text{verb}} \times r^3)$$

$$\text{wholly-gradient} = \sigma_{\nu} \sim \text{Exponential}(1)$$

$$\sigma_{\epsilon_1} \sim \text{Exponential}(1)$$

$$\sigma_{\epsilon_2} \sim \text{Exponential}(1)$$

$$\sigma_e \sim \text{Uniform}(0, 1)$$

$$\nu \sim \mathcal{N}(0, \sigma_{\nu})$$

$$\omega \sim \mathcal{N}(\mu_{\omega}, \sigma_{\omega})$$

$$\epsilon_1 \sim \mathcal{N}(0, \sigma_{\epsilon_1})$$

$$\epsilon_2 \sim \mathcal{N}(0, \sigma_{\epsilon_2})$$

$$\text{factor}(D_{\mathcal{N}(\theta, \sigma_e) \uparrow [0,1]}(\mathbf{y}_{\text{projection}}))$$

$$\langle \nu, \omega, \epsilon_1, \epsilon_2, \sigma_{\nu}, \sigma_{\epsilon_1}, \sigma_{\epsilon_2}, \sigma_e \rangle$$

$$\text{where } v_{i,j,k} = \text{logit}^{-1}(\nu_i + \epsilon_{1k})$$

$$w_{i,j,k} = \text{logit}^{-1}(\omega_j + \epsilon_{2k})$$

$$\theta_{i,j,k} = v_{i,j,k} + (1 - v_{i,j,k}) * w_{i,j,k}$$

A.2.3 The discrete-world model

The discrete-world model is defined similarly to the discrete-factivity model, except by alternating which parameters are taken to make discrete versus gradient contributions to the response. Now, world knowledge affects the certainty discretely, producing values of either 0 or 1. Meanwhile, if the certainty is 0, the factivity of the relevant predicate makes a gradient

contribution to the response.

$$\begin{aligned}
&\text{discrete-world} : \mathbb{P}(r^{n_{\text{verb}}} \times r^{n_{\text{context}}} \times r^{n_{\text{participant}}} \times r^{n_{\text{participant}}} \times r^{n_{\text{verb}}} \times r^3) \\
&\text{discrete-world} = \sigma_{\nu} \sim \text{Exponential}(1) \\
&\quad \sigma_{\epsilon_1} \sim \text{Exponential}(1) \\
&\quad \sigma_{\epsilon_2} \sim \text{Exponential}(1) \\
&\quad \sigma_e \sim \text{Uniform}(0, 1) \\
&\quad \nu \sim \mathcal{N}(0, \sigma_{\nu}) \\
&\quad \omega \sim \mathcal{N}(\mu_{\omega}, \sigma_{\omega}) \\
&\quad \epsilon_1 \sim \mathcal{N}(0, \sigma_{\epsilon_1}) \\
&\quad \epsilon_2 \sim \mathcal{N}(0, \sigma_{\epsilon_2}) \\
&\quad \tau_w \sim \text{Bernoulli}(\mathbf{w}) \\
&\quad \text{factor}(D_{\mathcal{N}(\theta, \sigma_e) \uparrow [0,1]}(\mathbf{y}_{\text{projection}})) \\
&\quad \langle \nu, \omega, \epsilon_1, \epsilon_2, \sigma_{\nu}, \sigma_{\epsilon_1}, \sigma_{\epsilon_2}, \sigma_e \rangle \\
&\quad \text{where } v_{i,j,k} = \text{logit}^{-1}(\nu_i + \epsilon_{1k}) \\
&\quad \quad w_{i,j,k} = \text{logit}^{-1}(\omega_j + \epsilon_{2k}) \\
&\quad \quad \theta_{i,j,k} = \mathbb{1}(\tau_{w_{i,j,k}}) + \mathbb{1}(\neg \tau_{w_{i,j,k}}) * v_{i,j,k}
\end{aligned}$$

A.2.4 The wholly-discrete model

Finally, the wholly-discrete model generates parameters θ which are either 0 or 1, depending on two Bernoullis parameterized by the probabilities of projection and world-knowledge-derived certainties, respectively. Each parameter $\theta_{i,j,k}$ is thus 0 with probability $p = (1 - v_{i,j,k}) * (1 - w_{i,j,k})$, and 1 with probability $1 - p$.

$$\begin{aligned}
&\text{wholly-discrete} : \mathbb{P}(r^{n_{\text{verb}}} \times r^{n_{\text{context}}} \times r^{n_{\text{participant}}} \times r^{n_{\text{participant}}} \times r^{n_{\text{verb}}} \times r^3) \\
&\text{wholly-discrete} = \sigma_{\nu} \sim \text{Exponential}(1) \\
&\quad \sigma_{\epsilon_1} \sim \text{Exponential}(1) \\
&\quad \sigma_{\epsilon_2} \sim \text{Exponential}(1) \\
&\quad \sigma_e \sim \text{Uniform}(0, 1) \\
&\quad \nu \sim \mathcal{N}(0, \sigma_{\nu}) \\
&\quad \omega \sim \mathcal{N}(\mu_{\omega}, \sigma_{\omega}) \\
&\quad \epsilon_1 \sim \mathcal{N}(0, \sigma_{\epsilon_1}) \\
&\quad \epsilon_2 \sim \mathcal{N}(0, \sigma_{\epsilon_2}) \\
&\quad \tau_v \sim \text{Bernoulli}(\mathbf{v}) \\
&\quad \tau_w \sim \text{Bernoulli}(\mathbf{w}) \\
&\quad \text{factor}(D_{\mathcal{N}(\theta, \sigma_e) \uparrow [0,1]}(\mathbf{y}_{\text{projection}})) \\
&\quad \langle \nu, \omega, \epsilon_1, \epsilon_2, \sigma_{\nu}, \sigma_{\epsilon_1}, \sigma_{\epsilon_2}, \sigma_e \rangle \\
&\quad \text{where } v_{i,j,k} = \text{logit}^{-1}(\nu_i + \epsilon_{1k}) \\
&\quad \quad w_{i,j,k} = \text{logit}^{-1}(\omega_j + \epsilon_{2k}) \\
&\quad \quad \theta_{i,j,k} = \mathbb{1}(\tau_{v_{i,j,k}} \vee \tau_{w_{i,j,k}})
\end{aligned}$$

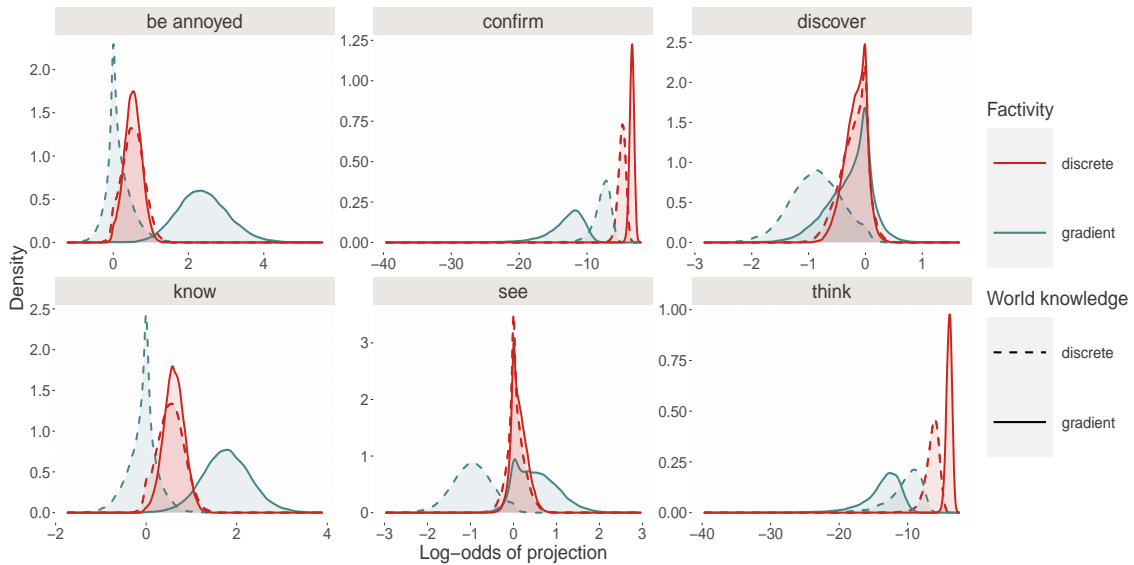


Figure 15: Density plots of the posterior log-odds of projection (with participant intercepts zeroed out) for all four models for six predicates from Degen and Tonhauser’s (2021) projection experiment.

A.3 The contentful evaluation model

To evaluate the four models using this data, we obtained, from each model, means μ_v and standard deviations σ_v of the marginal posterior log-odds of projection distributions for predicates, as well as means μ_ω and standard deviations σ_ω of the marginal posterior log-odds certainty distributions for contexts. We then used normal distributions with these means and standard deviations as prior distributions for the corresponding parameters in the models constructed for the evaluations. (See Figure 15 for density plots of these posterior distributions for six predicates, and Figure 17 of Appendix B for density plots of the posterior distributions for all predicates.)

Each evaluation has the following structure:

$$\begin{aligned}
 \text{replication-evaluation} &: P(r^{n_{\text{verb}}} \times r^{n_{\text{context}}} \times r^{n_{\text{participant}}} \times r^{n_{\text{participant}}} \times r^3) \\
 \text{replication-evaluation} &= \sigma_{\epsilon_1} \sim \text{Exponential}(1) \\
 &\quad \sigma_{\epsilon_2} \sim \text{Exponential}(1) \\
 &\quad \sigma_e \sim \text{Uniform}(0, 1) \\
 &\quad \boldsymbol{v} \sim \mathcal{N}(\boldsymbol{\mu}_v, \boldsymbol{\sigma}_v) \\
 &\quad \boldsymbol{\omega} \sim \mathcal{N}(\boldsymbol{\mu}_\omega, \boldsymbol{\sigma}_\omega) \\
 &\quad \epsilon_1 \sim \mathcal{N}(0, \sigma_{\epsilon_1}) \\
 &\quad \epsilon_2 \sim \mathcal{N}(0, \sigma_{\epsilon_2}) \\
 &\quad \vdots \\
 &\quad \text{factor}(D_{\mathcal{N}(\boldsymbol{\theta}, \boldsymbol{\sigma}_e) \top [0,1]}(\boldsymbol{y}_{\text{replication}})) \\
 &\quad \langle \boldsymbol{v}, \boldsymbol{\omega}, \boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2, \sigma_{\epsilon_1}, \sigma_{\epsilon_2}, \sigma_e \rangle \\
 &\quad \text{where } \boldsymbol{v}_{i,j,k} = \text{logit}^{-1}(\boldsymbol{v}_i + \boldsymbol{\epsilon}_{1k}) \\
 &\quad \quad \boldsymbol{w}_{i,j,k} = \text{logit}^{-1}(\boldsymbol{\omega}_j + \boldsymbol{\epsilon}_{2k}) \\
 &\quad \quad \boldsymbol{\theta}_{i,j,k} = \dots
 \end{aligned}$$

The ellipsis are used to represent the parts of any given evaluation that are model-specific. For example, the line above factor would be ' $\boldsymbol{\tau}_v \sim \text{Bernoulli}(\boldsymbol{v})$ ' for the evaluation of the discrete-factivity model, and the definition of $\boldsymbol{\theta}_{i,j,k}$ would be $\mathbb{1}(\boldsymbol{\tau}_{v_{i,j,k}}) + \mathbb{1}(\neg \boldsymbol{\tau}_{v_{i,j,k}}) * \boldsymbol{w}_{i,j,k}$.

A.4 The non-contentful evaluation models

To evaluate the four models using both the bleached and templatic data, we used the means $\boldsymbol{\mu}_v$ and standard deviations $\boldsymbol{\sigma}_v$ of the marginal posterior log-odds of projection that we used for the evaluations on the replication experiment data. As before, we use normal distributions with these means and standard deviations as prior distributions for the corresponding parameters in the models constructed for the evaluations. Then, in each evaluation, we inferred a distribution over the parameters σ_ω and ω that regulate the certainty associated with either the bleached or the templatic context.

In particular, both the bleached and the templatic evaluations have the following struc-

ture, where ellipses, as above, indicate the unique aspects of each of the four models:

$$\begin{aligned}
 \text{non-contentful-evaluation} &: P(r^{n_{\text{verb}}} \times r \times r^{n_{\text{participant}}} \times r^{n_{\text{participant}}} \times r^3) \\
 \text{non-contentful-evaluation} &= \sigma_{\omega} \sim \text{Exponential}(1) \\
 &\quad \sigma_{\epsilon_1} \sim \text{Exponential}(1) \\
 &\quad \sigma_{\epsilon_2} \sim \text{Exponential}(1) \\
 &\quad \sigma_e \sim \text{Uniform}(0, 1) \\
 &\quad \mathbf{v} \sim \mathcal{N}(\boldsymbol{\mu}_v, \sigma_v) \\
 &\quad \omega \sim \mathcal{N}(0, \sigma_{\omega}) \\
 &\quad \boldsymbol{\epsilon}_1 \sim \mathcal{N}(0, \sigma_{\epsilon_1}) \\
 &\quad \boldsymbol{\epsilon}_2 \sim \mathcal{N}(0, \sigma_{\epsilon_2}) \\
 &\quad \vdots \\
 &\quad \text{factor}(D_{\mathcal{N}(\boldsymbol{\theta}, \sigma_e) \top [0,1]}(\mathbf{y}_{\text{non-contentful}})) \\
 &\quad \langle \mathbf{v}, \omega, \boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2, \sigma_{\omega}, \sigma_{\epsilon_1}, \sigma_{\epsilon_2}, \sigma_e \rangle \\
 &\quad \text{where } \mathbf{v}_{i,j} = \text{logit}^{-1}(\mathbf{v}_i + \boldsymbol{\epsilon}_{1j}) \\
 &\quad \quad \mathbf{w}_{i,j} = \text{logit}^{-1}(\omega + \boldsymbol{\epsilon}_{2j}) \\
 &\quad \quad \boldsymbol{\theta}_{i,j} = \dots
 \end{aligned}$$

The data tuple $\mathbf{y}_{\text{non-contentful}}$ should be understood as either $\mathbf{y}_{\text{bleached}}$ or $\mathbf{y}_{\text{templatic}}$, depending on the evaluation performed.

B Plots

B.1 Posterior parameter distributions

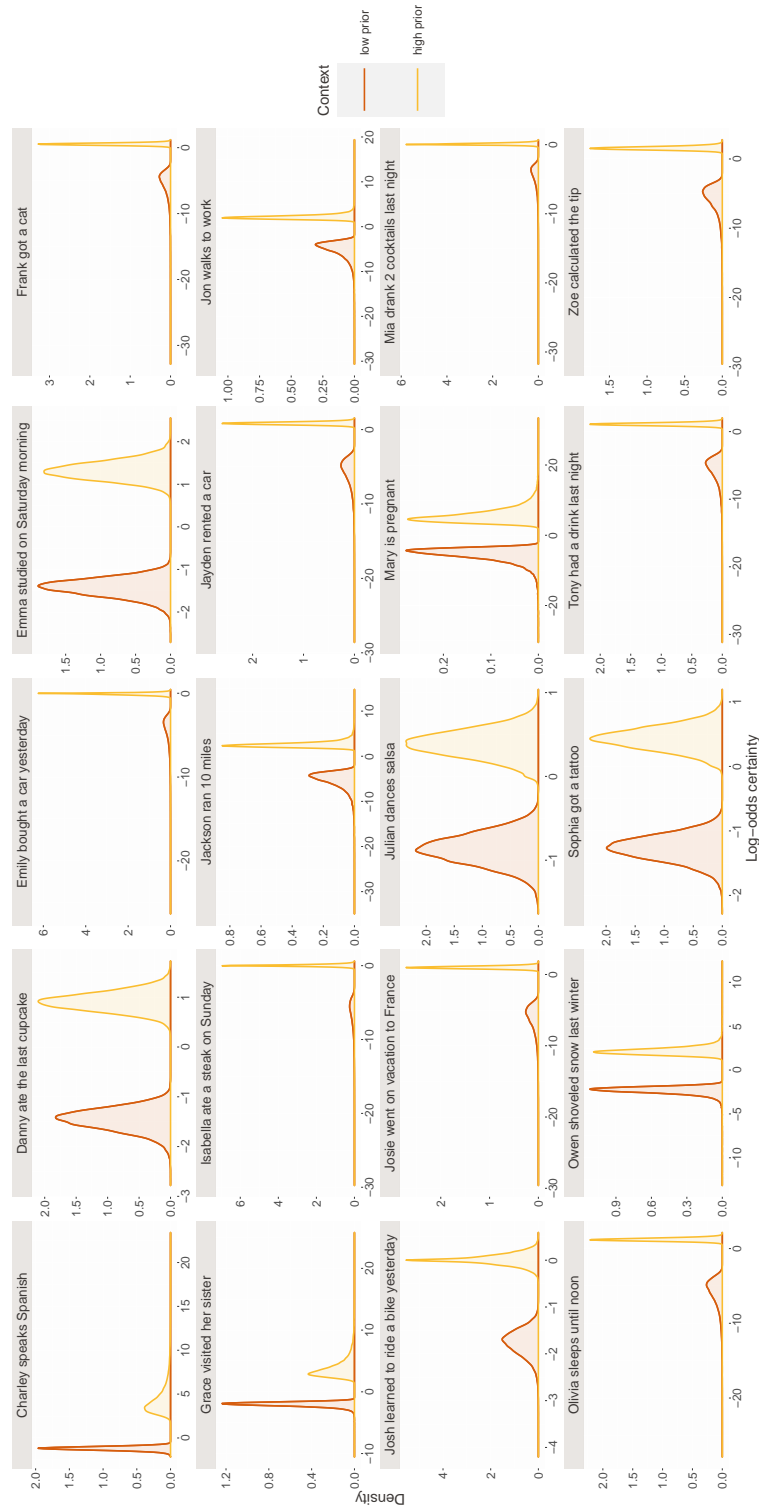


Figure 16: Density plots of the posterior log-odds certainty (with participant intercepts zeroed out) for all items in Degen and Tonhauser's (2021) norming task.

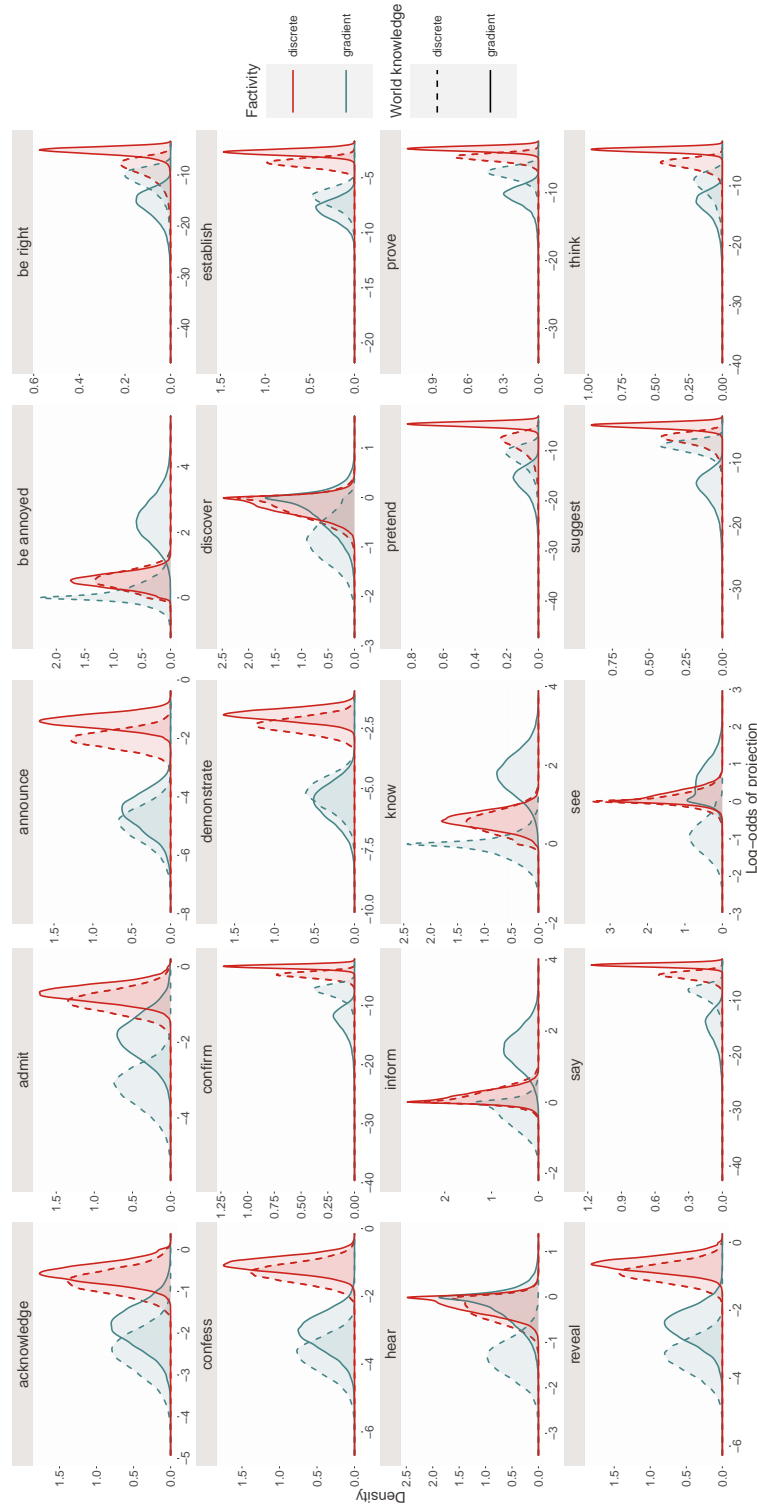


Figure 17: Density plots of the posterior log-odds of projection (with participant intercepts zeroed out) for all four models for all predicates in Degen and Tonhauser’s (2021) projection experiment.

B.2 Posterior predictive distributions

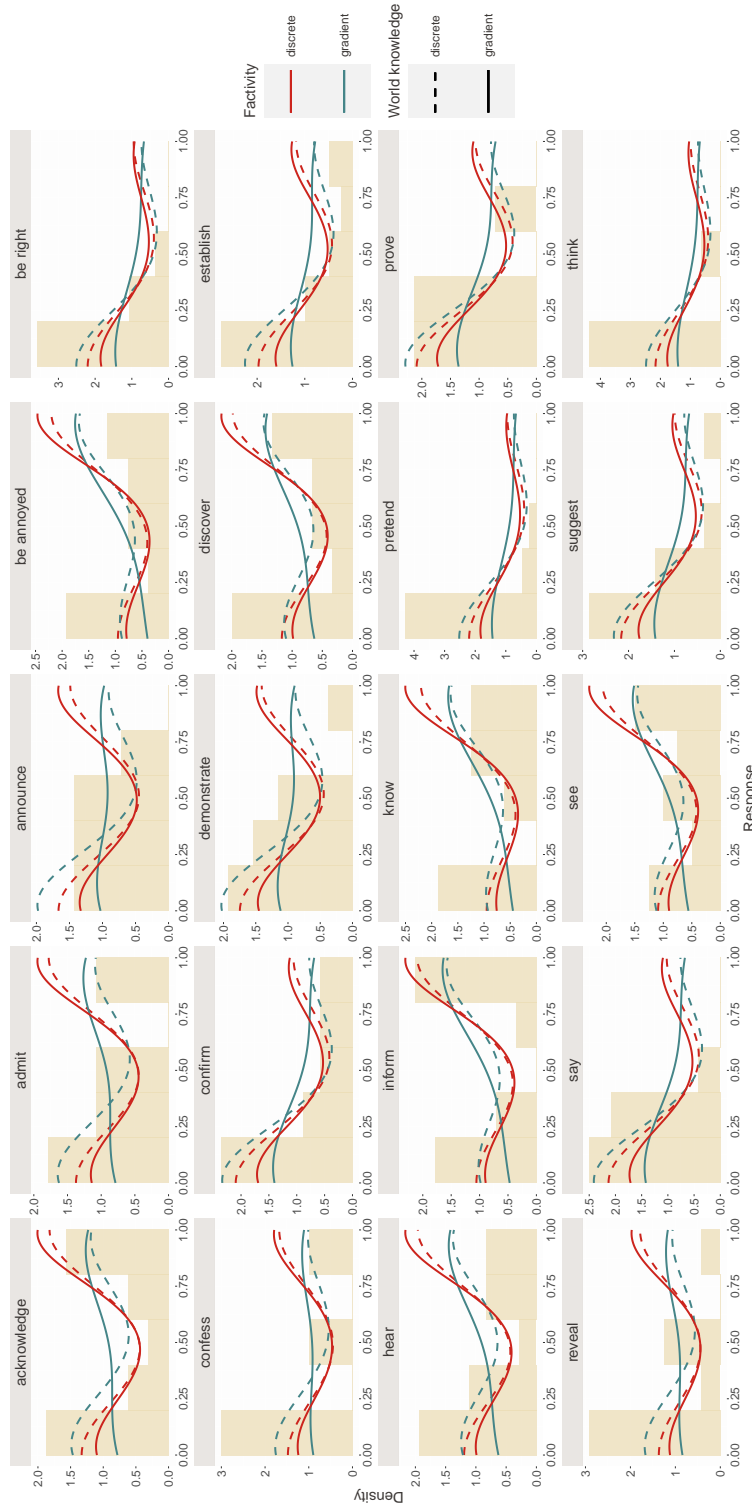


Figure 18: Posterior predictive distributions (with simulated participant intercepts) of all four models for all predicates in Degen and Tonhauser’s (2021) projection experiment. Complement clause: *Grace visited her sister*; background fact: *Grace hates her sister*. Empirical distributions are represented by density histograms of data pooled from Degen and Tonhauser 2021 and our replication study.

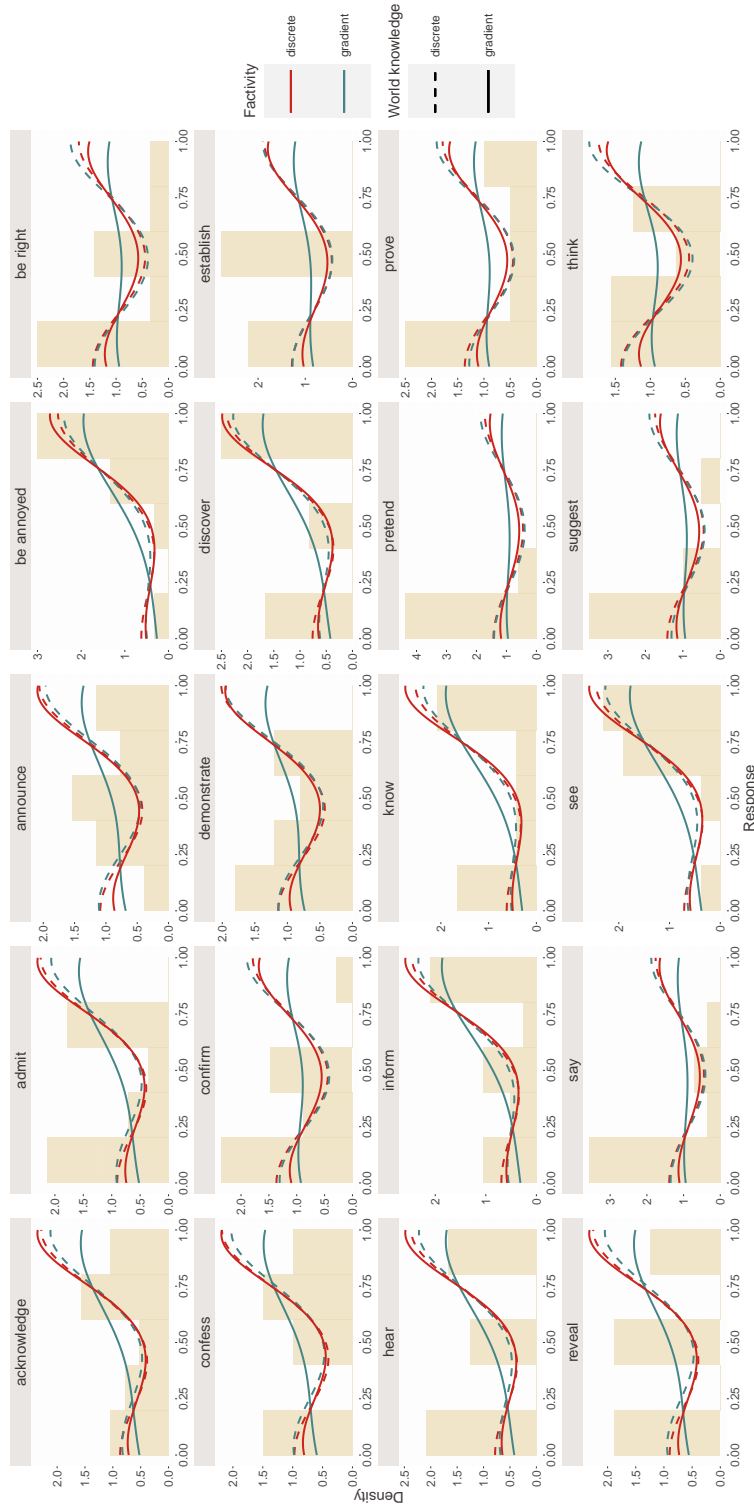


Figure 19: Posterior predictive distributions (with simulated participant intercepts) of all four models for all predicates in Degen and Tonhauser's (2021) projection experiment. Complement clause: *Sophia got a tattoo*; background fact: *Sophia is a hipster*. Empirical distributions are represented by density histograms of data pooled from Degen and Tonhauser 2021 and our replication study.

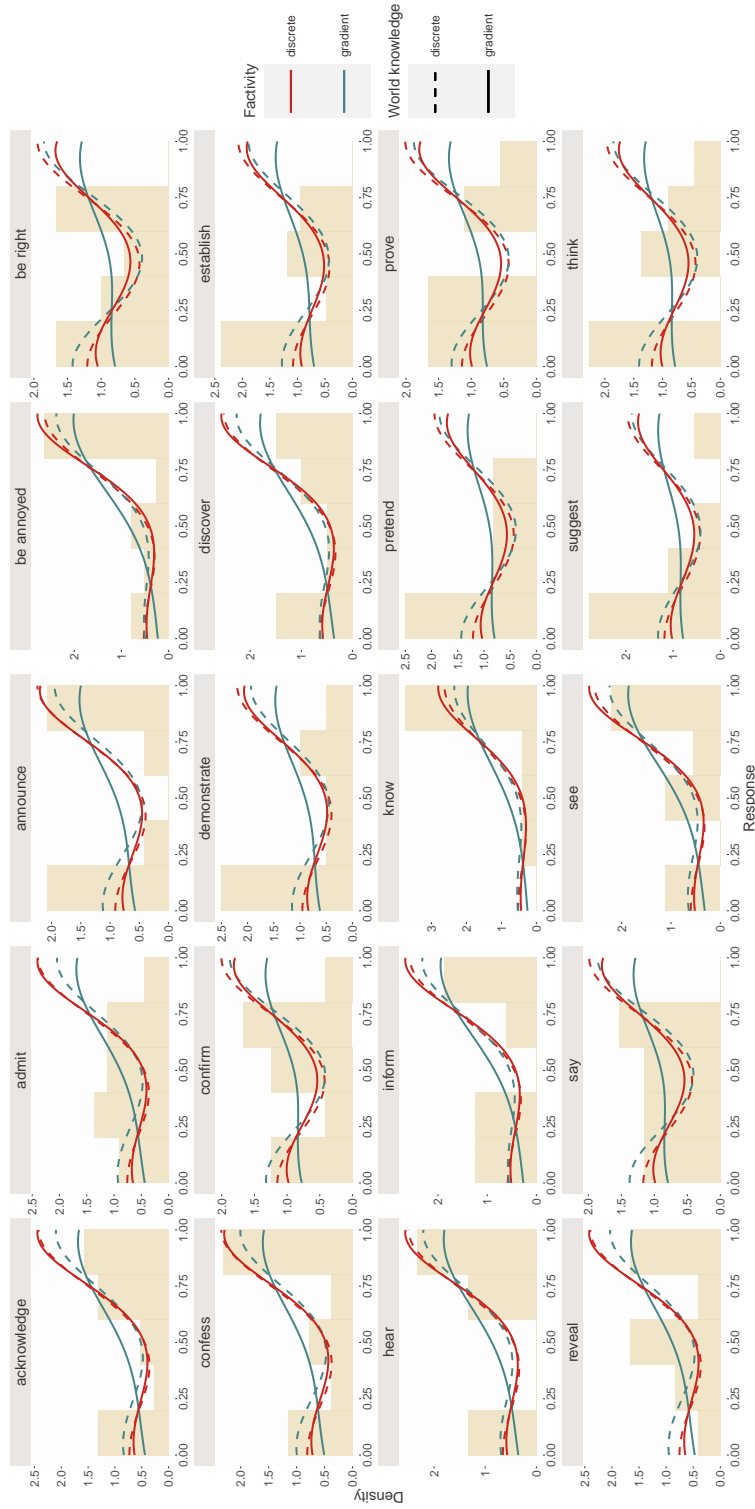


Figure 20: Posterior predictive distributions (with simulated participant intercepts) of all four models for all predicates in Degen and Tonhauser’s (2021) projection experiment. Complement clause: *Grace visited her sister*; background fact: *Grace loves her sister*. Empirical distributions are represented by density histograms of data pooled from Degen and Tonhauser 2021 and our replication study.

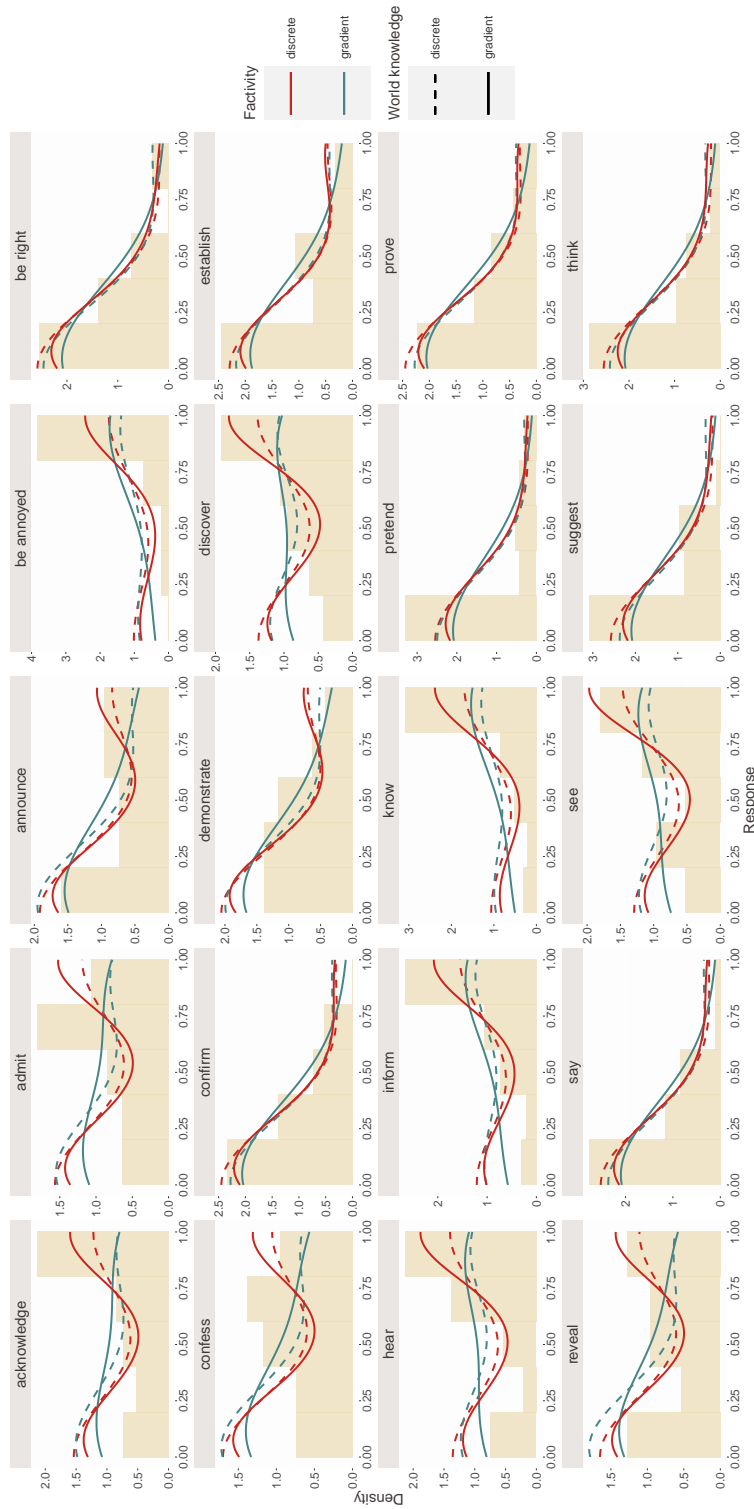


Figure 21: Posterior predictive distributions (with simulated participant intercepts) of all four model evaluations for all predicates in Degen and Tonhauser’s (2021) projection experiment. Complement clause: *a particular thing happened*. Empirical distributions are represented by density histograms.

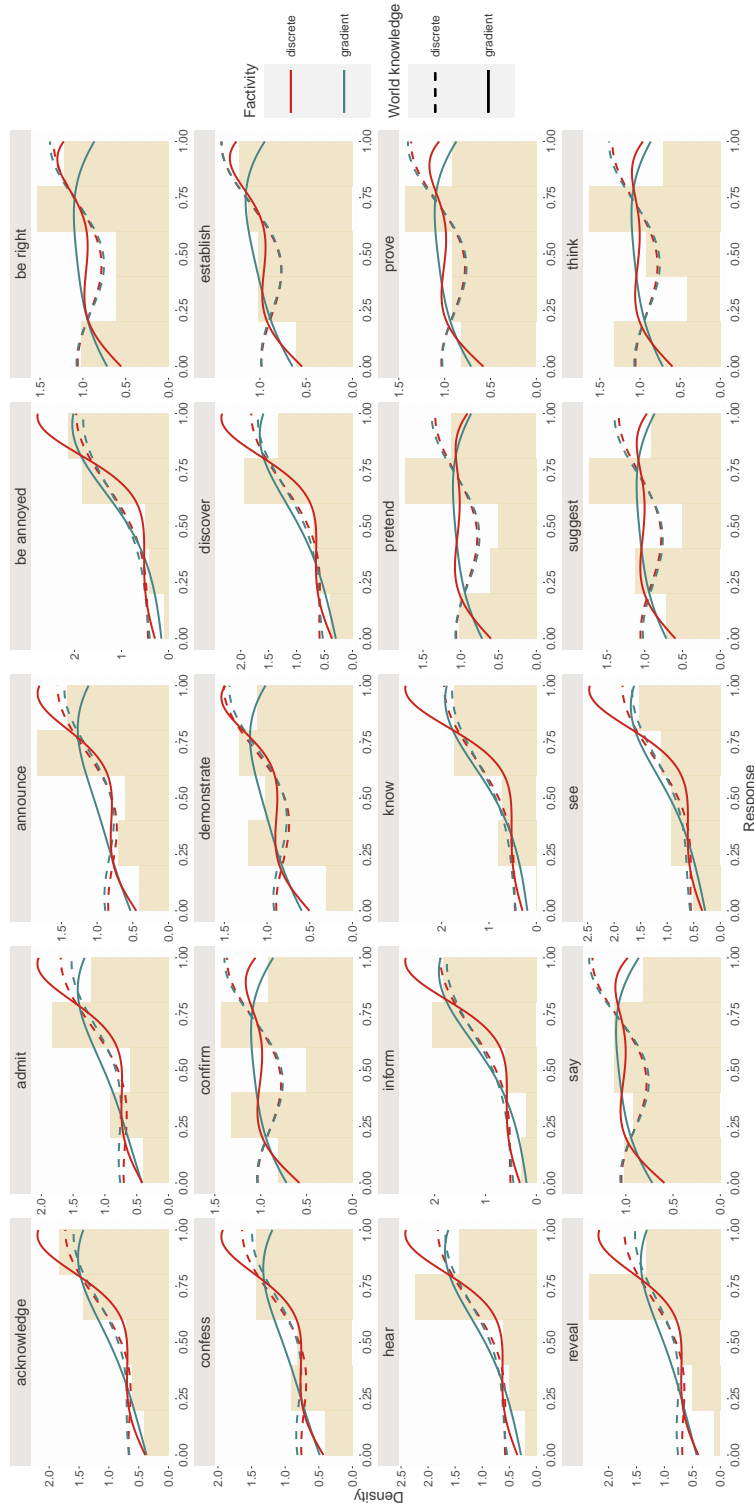


Figure 22: Posterior predictive distributions (with simulated participant intercepts) of all four model evaluations for all predicates in Degen and Tonhauser’s (2021) projection experiment. Complement clause: *X happened*. Empirical distributions are represented by density histograms.